Atmospheric Sciences 5310
Precipitation Rate

We can derive an expression for the cloud-base precipitation rate, P, for a steady-state cloudy (saturated) updraft with cloud-base temperature = T_b, cloud-base pressure = p_b, cloud-top temperature = T_t, cloud-top pressure = p_t, and constant vertical mass flux = M ($\equiv \rho w$, where ρ is air density and $w = dz/dt$ is vertical velocity). In the cloudy updraft, air is rising from cloud base to cloud top at a constant rate M (kg m^{-2} s^{-1}). The updraft's temperature decreases from T_b at cloud base to T_t at cloud top.

We assume that water vapor condenses as air ascends in the cloudy updraft at a rate that maintains the updraft's relative humidity at exactly 100 percent with respect to liquid water. We also assume that the cloud droplets or ice crystals formed by condensation rapidly form precipitation particles. Under these assumptions, a *steady state* is achieved in which the cloud-base precipitation rate equals the vertically integrated condensation rate.

The vertically integrated condensation rate is just the condensation per unit mass of air during ascent from cloud base to cloud top times the vertical mass flux:

$$P = [w_s(T_b, p_b) - w_s(T_t, p_t)]\, M,$$

where $w_s(T, p)$ is the saturation mixing ratio, which can be read directly from a skew T-log p diagram.

For a given set of values of cloud-base temperature, pressure, and vertical velocity, what cloud-top conditions produce the largest value of P, $P_{\max}$? What is the expression for $P_{\max}$?

One can use these results to calculate P and $P_{\max}$ for the following cloud types.

Cloud Type	Sc	Cu con	Cb	Ns	Ns (Fall)	Ns (Winter)
cloud-base temperature (T_b, °C)	20	20	20	20	0	-10
cloud-base pressure (p_b, hPa)	950	950	950	950	800	800
cloud-top pressure (p_t, hPa)	900	750	300	300	300	300
mass flux (M, kg m^{-2} s^{-1})	0.5	2	8	0.1	0.2	0.2
cloud-base saturation mixing ratio ($w_{s,b}$, g/kg)						
cloud-top saturation mixing ratio ($w_{s,b}$, g/kg)						
cloud-base precipitation rate (P, mm hr^{-1})						
maximum cloud-base precipitation rate ($P_{\max}$, mm hr^{-1})						

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cloud-top pressure (p_t, hPa)	900	750	300	300	300	300
mass flux (M, kg m^{-2} s^{-1})	0.5	2	8	0.1	0.2	0.2
cloud-base saturation mixing ratio ($w_{s,b}$, g/kg)	16	16	16	16	4.8	2.2
cloud-top saturation mixing ratio ($w_{s,b}$, g/kg)	15	11.5	0.8	0.8	0.05	0
cloud-base precipitation rate (P, mm hr^{-1})	1.8	32	440	5.5	3.5	1.6
maximum cloud-base precipitation rate ($P_{\max}$, mm hr^{-1})	29	115	460	5.8	3.5	1.6