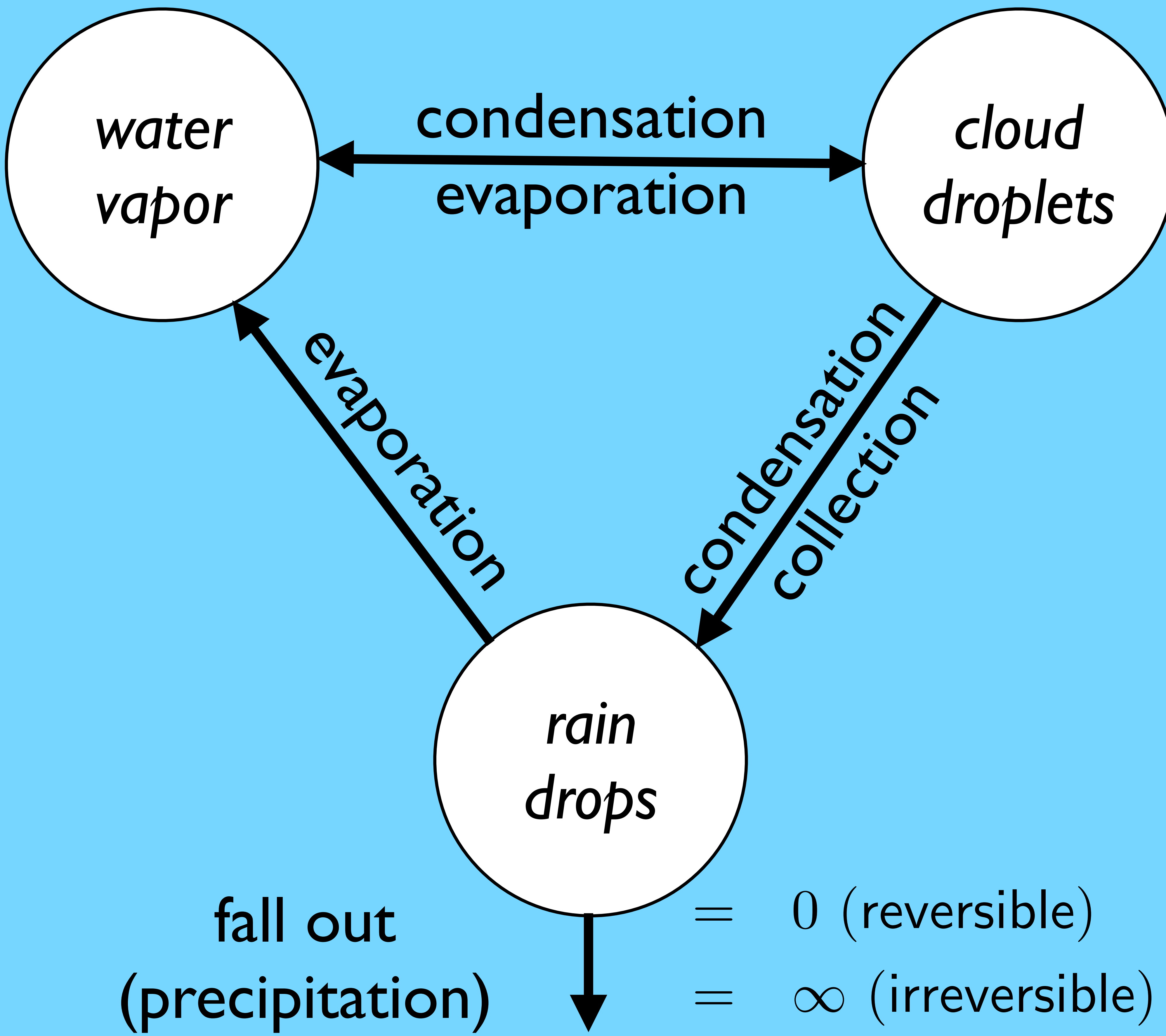
Simplified Microphysics



Simplified Microphysics

$$\frac{d\theta}{dt} = \frac{L}{c_p\bar{\pi}}C$$

$$\frac{dw}{dt} = -C + E_r$$

$$\frac{dl}{dt} = C - A_r$$

$$\frac{dr}{dt} = P_r + A_r - E_r$$

$$P_r = 0 \text{ (reversible)}$$

$$P_r = \infty \text{ (irreversible)}$$

More Simplified Microphysics

$$\frac{d\theta}{dt} = \frac{L}{c_p \bar{\pi}} C$$

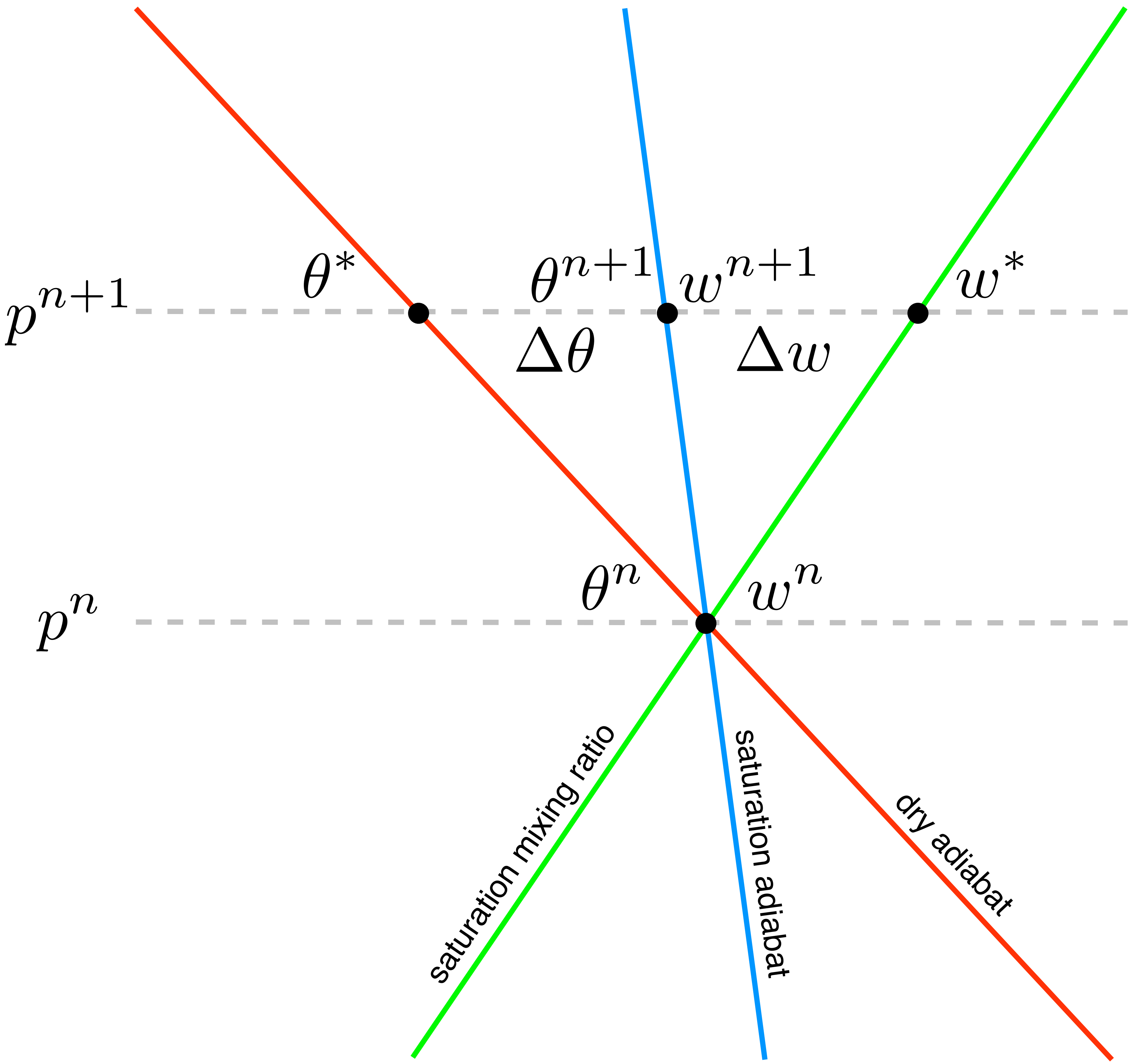
$$\frac{dw}{dt} = -C$$

$$\frac{dl}{dt} = C - A_r$$

$$A_r = 0 \ \text{(reversible)}$$

$$A_r = \infty \ \text{(irreversible)}$$

Saturation Adjustment Algorithm



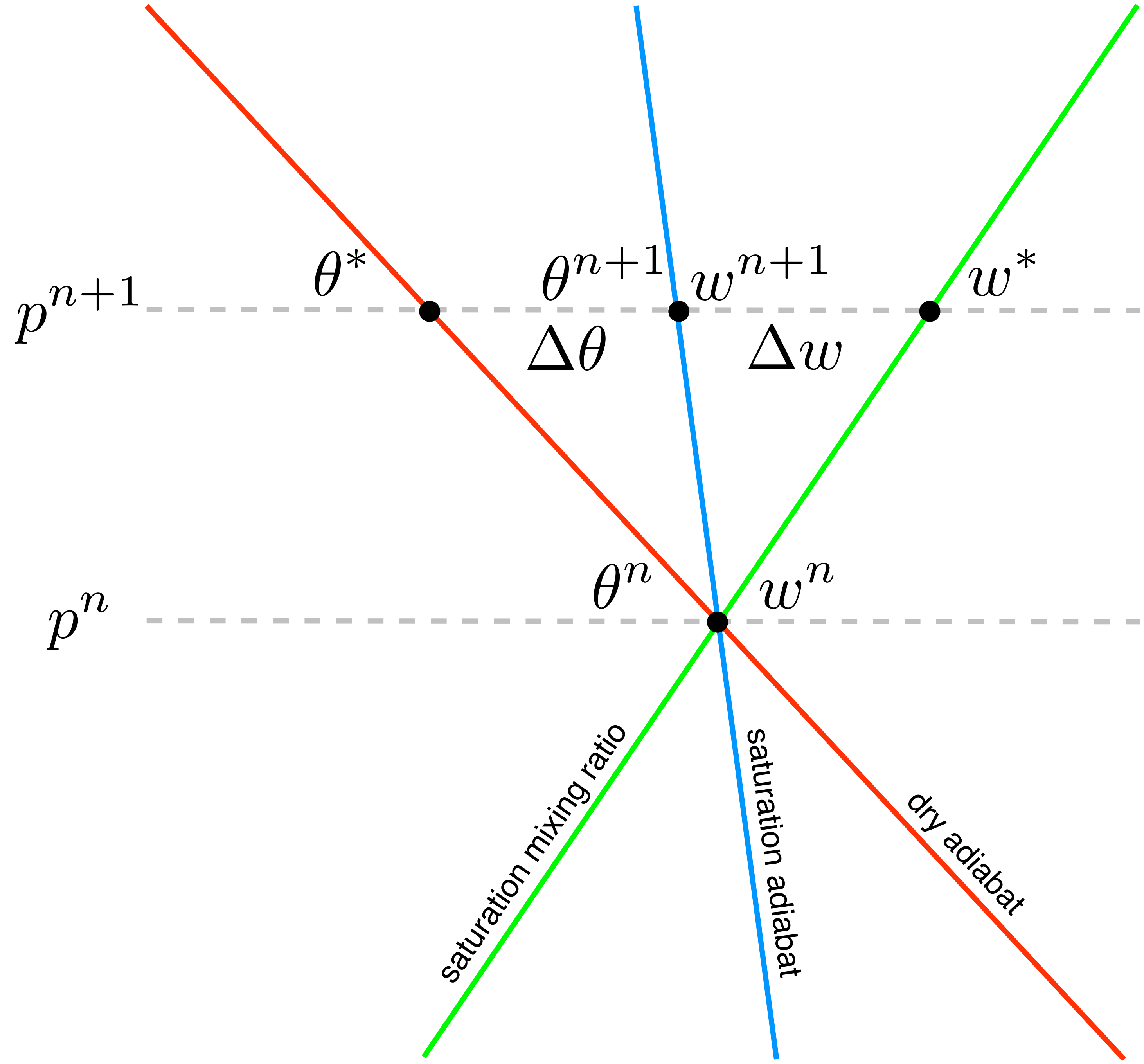
Saturation Adjustment Algorithm

1. **Adiabatic.** No phase changes involving cloud droplets (C=0):

$$\theta^n,\ w^n \rightarrow \theta^*,\ w^*$$

2. **Isobaric.** Only phase changes involving cloud droplets operate (|C| > 0):

$$\theta^*,\ w^* \rightarrow \theta^{n+1},\ w^{n+1}$$



Saturation Adjustment Algorithm

1. **Adiabatic.** No phase changes involving cloud droplets (C=0):

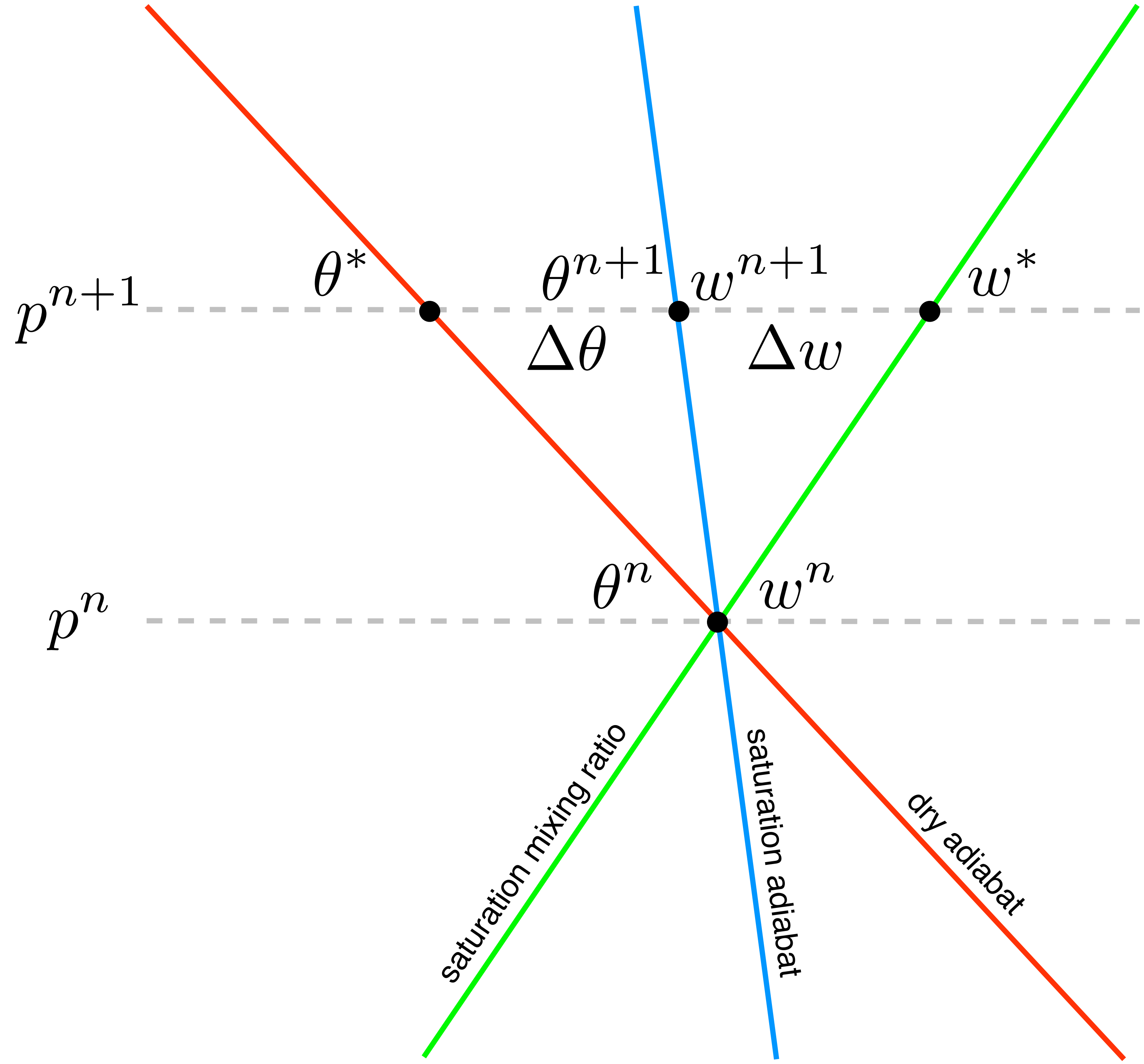
$$\theta^n,\ w^n \rightarrow \theta^*,\ w^*$$

$$(\theta^*,\ w^*) = (\theta^n,\ w^n)$$

2. **Isobaric.** Only phase changes involving cloud droplets operate (|C| > 0):

$$\theta^*,\ w^* \rightarrow \theta^{n+1},\ w^{n+1}$$

$$\theta^{n+1},\ w^{n+1} = (\theta^* + \Delta\theta,\ w^* + \Delta w)$$



Saturation Adjustment Algorithm

2. **Isobaric.** Only phase changes involving cloud droplets operate (|C| > 0):

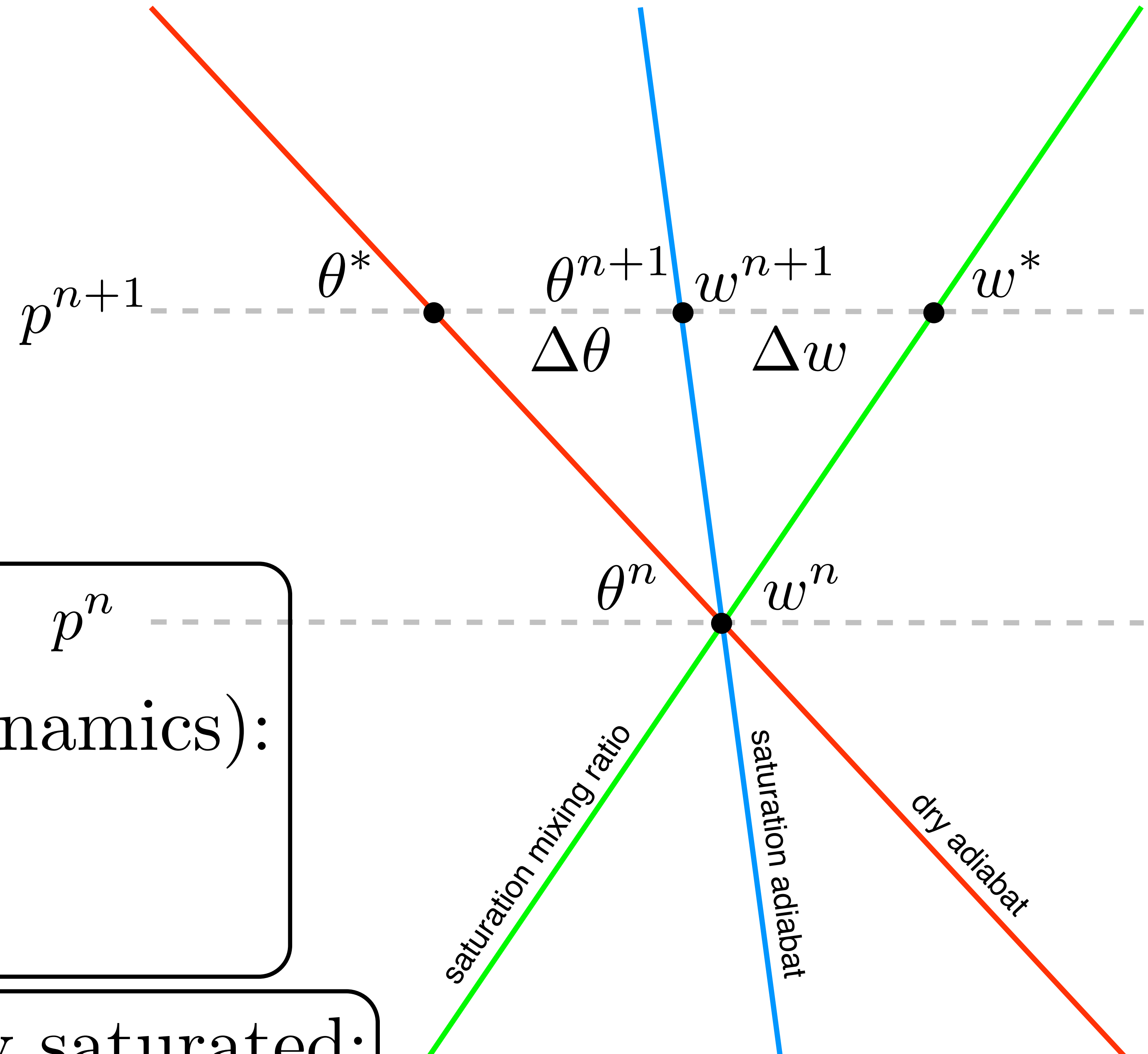
$$\theta^{n+1},\, w^{n+1} = (\theta^* + \Delta\theta,\, w^* + \Delta w)$$

Conservation of energy (First Law of Thermodynamics):

$$0 = c_p \pi \Delta\theta + L\Delta w \text{ or}$$
$$0 = c_p \Delta T + L\Delta w$$

Adjusted state is exactly saturated:

$$w^{n+1} = w_s(\pi\theta^{n+1}, p^{n+1}) \text{ or}$$
$$w^{n+1} = w_s(T^{n+1}, p^{n+1})$$



Saturation Adjustment Algorithm

$$0 = c_p \pi \Delta\theta + L \Delta w$$

$$w^{n+1} = w_s(\pi\theta^{n+1}, p^{n+1})$$

which can be written as

$$0 = c_p \pi(\theta^{n+1} - \theta^*) + L(w^{n+1} - w^*)$$

$$w^{n+1} = w_s(\pi\theta^{n+1}, p^{n+1})$$

Saturation Adjustment Algorithm

$$\theta^{n+1} + \gamma w^{n+1} = \theta^* + \gamma w^* \tag{5}$$

where $\gamma \equiv L/(c_p\bar{\pi})$.

conservation of suspended water mixing ratio (vapor and cloud droplets),

$$w^{n+1} + l^{n+1} = w^* + l^*. \tag{6}$$

We first assume that the air will be exactly saturated after adjustment, so that

$$w^{n+1} = w_s(T^{n+1}, p^{n+1}), \tag{7}$$

where $w_s(T, p)$ is the saturation mixing ratio,

$$w_s(T, p) = 0.622 \frac{e_s(T)}{p - e_s(T)}, \tag{8}$$

and $e_s(T)$ is the saturation vapor pressure. One may use Bolton's (1980) formula for $e_s(T)$:

$$e_s(T) = 6.112 \exp\left(\frac{17.67T_c}{T_c + 243.5}\right), \tag{9}$$

where e_s is in mb, $T_c = T - T_0$, and $T_0 = 273.15$ K.

Saturation Adjustment Algorithm

Equation (7) closes the set (5), (6), and (7). However, this set must be solved iteratively because w_s is a non-linear function of T. To obtain a direct (non-iterative) solution, expand w_s in a Taylor series in T about $w_s(T^*, p^{n+1})$ and neglect all terms of second and higher order:

$$w^{n+1} \approx w_s(T^*, p^{n+1}) + \left(\frac{\partial w_s}{\partial T}\right)_{T=T^*, p=p^{n+1}} (T^{n+1} - T^*). \tag{10}$$

The set (5), (6), and (10) can now be solved algebraically for θ^{n+1}, w^{n+1}, and l^{n+1}.

Saturation Adjustment Algorithm

To solve the set, we first write (10) in terms of θ instead of T:

$$w^{n+1} = w_s^* + \alpha^*(\theta^{n+1} - \theta^*), \tag{11}$$

where $w_s^* \equiv w_s(T^*, p^{n+1})$, $\alpha^* \equiv \alpha(T^*, p^{n+1})$, and

$$\alpha(T, p) \equiv 0.622 \frac{\pi p}{(p - e_s(T))^2} \left(\frac{de_s}{dT} \right)_T . \tag{12}$$

For de_s/dT, one may use the Clausius-Clapeyron equation:

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}, \tag{13}$$

where $L = 2.5 \times 10^6$ J/kg and $R_v = 461.5$ J/(kg K).

Now use (11) in (5) to eliminate w^{n+1}. Then solve for θ^{n+1}:

$$\theta^{n+1} = \theta^* + \frac{\gamma}{1 + \gamma \alpha^*}(w^* - w_s^*). \tag{14}$$

Once θ^{n+1} is known from (14), we can immediately obtain w^{n+1} from (11), and l^{n+1} from (6).

Saturation Adjustment Algorithm

If $w^{n+1} \leq w^* + l^*$, then (6) implies that $l^{n+1} \geq 0$.

If $w^{n+1} > w^* + l^*$, (6) implies that $l^{n+1} < 0$, which means that our assumption of saturation is incorrect. Therefore,

$$l^{n+1} = 0$$

replaces (10). Then (5) and (6) become

$$w^{n+1} = w^* + l^*,$$

$$\theta^{n+1} = \theta^* - \gamma(w^{n+1} - w^*).$$