Turbulence Closure, Scales, and Similarity Theory

Review Terms

- Statically stable
- Dynamically unstable
- Isotropic / anisotropic
- Turbulence tends to homogenize

“Unstable” profiles

- How does one even get an unstable potential temperature profile?
  - Because of outer space
    - Radiative – Convective (Dis)Equilibriua
- What profiles of wind velocity (shear) produce turbulence?
  - On the board
- Richardson # : A balance between mechanical production and buoyant dissipation of turbulence

$$\mathrm{Ri} = \frac{-B}{M} \approx \frac{\frac{g}{\overline{\theta_v}} \frac{\partial \overline{\theta_v}}{\partial z}}{\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2}$$

For $\mathrm{Ri} < 0.25$, laminar flow becomes turbulent.
For $\mathrm{Ri} > 1$, turbulent flow becomes laminar.
For $0.25 < \mathrm{Ri} < 1$, the existence of turbulence depends on the flow's history.

9.1.4 Turbulent transport and fluxes

- Kinematic heat flux: $\overline{w'\theta'}$

- For a layer of air: $\dfrac{\partial\overline{\theta}}{\partial t} = -\dfrac{\partial\overline{w'\theta'}}{\partial z} + \ldots$

9.1.4 Turbulent transport and fluxes

- For a layer of air:

$$\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial\overline{w'\theta'}}{\partial z} + \ldots$$

- The turbulent flux of the heat flux:

$$\frac{\partial\overline{w'\theta'}}{\partial t} = -\frac{\partial\overline{w'w'\theta'}}{\partial z} + \ldots$$

9.1.4 Turbulent transport and fluxes

- This is the turbulence closure problem
  - Make a “closure assumption”
  - Can such a parameterization ever be perfect?

9.1.5 Turbulence Closure

- Statistical Order and Non-localness
- An example:
- "Gradient-transfer theory" or "K-theory" or "Eddy-diffusivity theory"

$$F_H = \overline{w'\theta'} = -K\frac{\partial\overline{\theta}}{\partial z}$$

9.1.5 Turbulence Closure

- This is a first-order closure
- K is an "Eddy diffusivity" – work out units on board

$$F_H = \overline{w'\theta'} = -K\frac{\partial\overline{\theta}}{\partial z}$$

Turbulence Closure

- A local closure for K
  - Relate to local TKE and a turbulence length scale:

$$K = le,$$

where

$$e = (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)^{1/2}$$

9.1.5 Turbulence Closure

- A local closure for K
  - Related to the local wind shear and a mixing length "l"

$$K = l^2 |\partial V / \partial z|$$

- L is an average eddy size
  - At very low levels (in the surface layer) L can be approximated by kz where k=0.4

$$l = (\overline{z'^2})^{1/2}$$

$$l \approx kz$$

9.1.5 Turbulence Closure

- What would a non-local closure for K be?

$$F_H = \overline{w'\theta'} = -K\frac{\partial\overline{\theta}}{\partial z}$$

9.1.6 Scales, scales, scales

- Velocity Scales

❑ Friction velocity:

$$u_* = \left[\overline{u'w'}^2 + \overline{v'w'}^2\right]^{\frac{1}{4}} \qquad u_*^2 = \left(\overline{u'w'}\right) \qquad \text{For one-dimensional case}$$

❑ Convective velocity scale (Deardorff velocity):

$$w_* = \left[\frac{g \cdot z_i}{T_v} \overline{w'\theta_S'}\right]^{\frac{1}{3}}$$

z_i – height of capping inversion (PBL height)
T_v – virtual temperature

Friction velocity relevant to statically neutral conditions in the surface layer

Deardorff velocity relevant to unstably stratified boundary layer

9.1.6 Scales, scales, scales

- Length Scales
  - z_i : the altitude of the inversion. Relevant length scale for statically unstable and neutral conditions
  - z_0: the roughness length
  - The Obukhov length L
    - For statically non-neutral conditions: This is the height in the surface layer below which mechanical production of turbulence dominates

$$L = \frac{-u_*^3}{k \frac{g}{T_v} \left(\overline{w'\theta'} \right)_s}$$

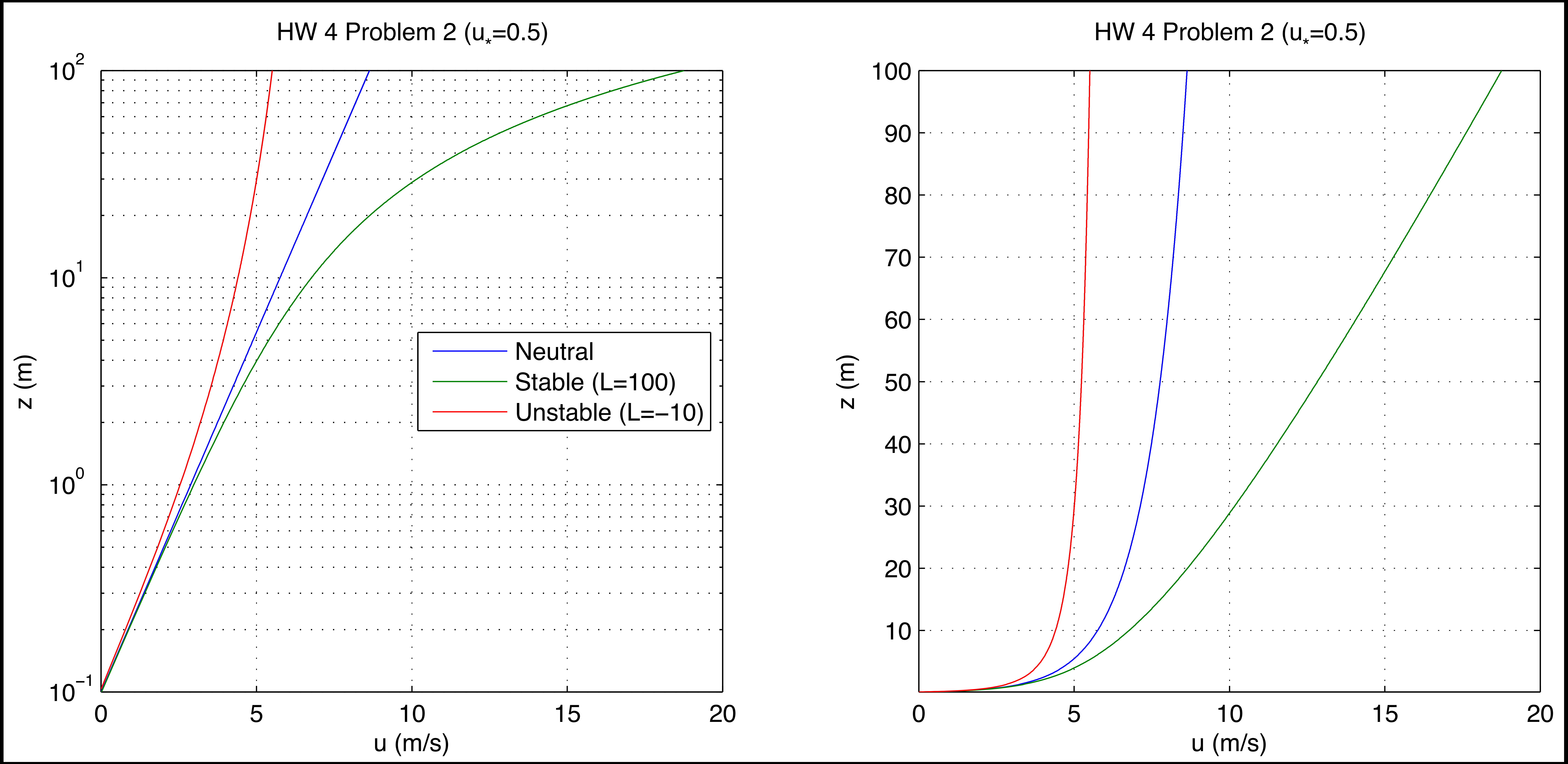
9.1.6 Scales, scales, scales

- Length Scales
  - z_0: the roughness length

10-2

$$U(z) = \frac{u_*}{k} \log\left(\frac{z}{z_0}\right)$$

Logarithmic wind profile valid for **neutral** conditions


HW 4 Problem 2 (u_*=0.5)
z (m)
u (m/s)
Neutral
Stable (L=100)
Unstable (L=−10)
HW 4 Problem 2 (u_*=0.5)
z (m)
u (m/s)

9.1.6 Scales, scales, scales

- Time Scales
  - For convective boundary layer (turnover time for largest eddies)

$$t* = \frac{z_i}{w*}$$

  - For neutral surface layer: (faster times possible)

$$t*_{SL} = \frac{z}{u*}$$

9.1.6 Scales, scales, scales

- Summary:
  - For the convective boundary layer
    - w* and z_i
  - For the surface layer
    - If neutral: u* and z_0
    - If non-neutral: u* and z_0 and L

9.1.6 Similarity

- Example: Through the depth of a convective boundary layer:

$$\frac{\overline{w'^2}}{w_*^2} = a\left(\frac{z}{z_i}\right)^b \left(1 - c\frac{z}{z_i}\right)^d$$

- Example: For a statically stable surface layer (compare with neutral log wind profile)

$$\frac{V}{u_*} = 2.5\left[\ln\left(\frac{z}{z_0}\right) + 8.1\frac{z}{L}\right]$$