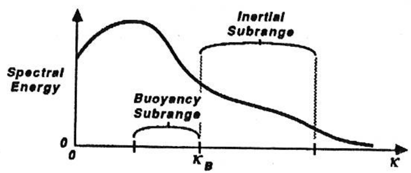
2.12 Exercises

1) It has been suggested that in regions of strong static stability, the lower (long wavelength, small wavenumber) end of the inertial subrange occurs at a wavenumber, κ_B, given by $\kappa_B \equiv N_{BV}^{3/2}\,\varepsilon^{-1/2}$, where N_{BV} is the Brunt-Vaisala frequency, and ε is the turbulence dissipation rate. Between this wavenumber and lower wavenumbers is a region called the buoyancy subrange, where the gravitational effects (i.e., buoyancy) are important. Within the buoyancy subrange sketched below, would you expect turbulence to be isotropic?



2) Given the following instantaneous measurements of potential temperature (θ) and
U vertical velocity (w) in this table, fill in all the remaining blanks in the table. Also,
G verify with the answers from above that $\overline{w\theta} = \overline{w}\,\overline{\theta} + \overline{w'\theta'}$.

	Measurements:		Calculations:					
index	w	θ	w'	θ'	(w')²	(θ')²	wθ	w'θ'
0	0.5	295						
1	-0.5	293						
2	1.0	295						
3	0.8	298						
4	0.9	292						
5	-0.2	294						
6	-0.5	292						
7	0.0	289						
8	-0.9	293						
9	-0.1	299						
Average:								

3) Given the data in problem (2), find the biased standard deviation for w and θ, and find
UG the linear correlation coefficient between w and θ.

4) Using your results from problems (2) and (3), is the data characteristic of a stable,
UG neutral, or unstable boundary layer?

5) Let: c = constant, s ≠ function of time, and $A = \overline{A} + a'$, $B = \overline{B} + b'$, and $E = \overline{E} + e'$. Expand the following terms into mean and turbulent parts, and apply Reynold's averaging rules to simplify your expression as much as possible:

a) $\overline{(c\,A\,B)} = ?$

b) $\overline{(A\,B\,E)} = ?$

c) $\overline{\left(A \cdot \frac{\partial B}{\partial s}\right)} = ?$

d) $\overline{\left(\frac{\partial A}{\partial s}\right)\cdot\left(\frac{\partial B}{\partial s}\right)} = ?$

e) $\overline{\left(c\ \nabla^2 A\right)} = ?$

6) The following terms are given in summation notation. Expand them (that is, write out each term of the indicated sums).

a) $\frac{\partial\left(\overline{u_i' u_j'}\right)}{\partial x_j}$

b) $\overline{u_i' \frac{\partial \theta'}{\partial x_i}}$

c) $\overline{U_j}\frac{\partial\left(\overline{u_i' u_k'}\right)}{\partial x_j}$

d) $\overline{u_i' u_j'}\ \frac{\partial \overline{U_k}}{\partial x_j}$

e) $\frac{\partial\left(\overline{u_i' u_j' u_k'}\right)}{\partial x_j}$

f) $\left(\frac{\partial u_i}{\partial x_j}\right)\left(\frac{\partial u_k}{\partial x_j}\right)$

7) Express the following terms in summation notation. They are given to you in vector notation.

a) Gradient of a scalar: ∇s

b) Curl of a vector: $\nabla \times V$

c) Total derivative: dV/dt

d) Laplacian: ∇^2

✓8) Consider a 100 m thick layer of air at sea level, with an initial potential temperature of
UG 290 K. If the kinematic heat flux into the bottom of this layer is 0.2 K m/s and the flux out of the top is 0.1 K m/s, then what is the potential temperature of that layer 2 hours later? Assume that the potential temperature is constant with height in the layer. (This is a thought question, meant to stimulate the student's ability to interpret a physical situation in an acceptable mathematical framework.)

9) Given the typical variation of wind speed with height within the surface layer (see Chapt 1), and using a development similar to that in section 2.7:

5.12 Exercises

1) Why doesn't turbulent energy cascade from small to large eddies (or wavelengths) in the boundary layer?

2) Refer to the TKE equation. Which term(s), if any, represent the production of turbulence during a day when there are light winds and strong solar heating of the boundary layer?

3) Given the following wind speeds measured at various heights in the boundary layer:

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z (m)	U (m/s)
2000	10.0
1000	10.0
500	9.5
300	9.0
100	8.0
50	7.4
20	6.5
10	5.8
4	5.0
1	3.7

Assume that the potential temperature increases with height at the constant rate of 6 K/km. Calculate the bulk Richardson number for each layer and indicate the static and dynamic stability of each layer. Also, show what part of the atmosphere is expected to be turbulent in these conditions.

4) Derive an expression for the kinematic heat flux $\overline{w'\theta'}$ in terms of the dimensionless wind shear ϕ_M and dimensionless lapse rate ϕ_H.

5) Given the following TKE equation:

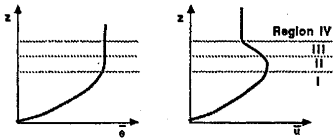
$$\underset{A}{\frac{\partial \bar{e}}{\partial t}} + \underset{B}{\overline{U}_j \frac{\partial \bar{e}}{\partial x_j}} = \underset{C}{- \overline{u'w'} \frac{\partial \overline{U}}{\partial z}} + \underset{D}{\frac{g}{\overline{\theta}_v}\left(\overline{w'\theta_v'}\right)} \underset{E}{- \frac{\partial\left(\overline{w'e}\right)}{\partial z}} \underset{F}{- \frac{1}{\overline{\rho}} \frac{\partial\left(\overline{w'p'}\right)}{\partial z}} \underset{G}{- \varepsilon}$$

a. Which terms are always loss terms?
b. Which terms neither create nor destroy TKE?
c. Which terms can be either production or loss?
d. Which terms are due to molecular effects?
e. Which production terms are largest on a cloudy, windy day?
f. Which production terms are largest on a calm sunny day over land?
g. Which terms tend to make turbulence more homogeneous?
h. Which terms tend to make turbulence less isotropic?
i. Which terms describe the stationarity of the turbulence?
j. Which terms describe the kinetic energy lost from the mean wind?

23) Given the TKE equation with terms labeled A to E below:

$$\underset{A}{\frac{\partial \bar{e}}{\partial t}} = \underset{B}{-\overline{u'w'}\frac{\partial \bar{U}}{\partial z}} + \underset{C}{\frac{g}{\bar{\theta}_v}\overline{w'\theta_v'}} - \underset{D}{\frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\bar{\rho}}+e\right)}} - \underset{E}{\varepsilon}$$

and given 4 regions of the stable boundary layer, labeled I to IV in the figure below, determine the sign (+ , - , or near zero) of each term in each region. (Assume: that term A is always zero; i.e., steady state.)



24) The dissipation rate of TKE is sometimes approximated by $\varepsilon = \bar{e}^{3/2}/l$, where l is the ***dissipation length scale***. It is often assumed that $l = 5$ z in statically neutral conditions (Louis, et al., 1983). If the TKE shown in Fig 2.9b is assumed as an initial condition, there is no shear or buoyancy production or loss, and no redistribution nor turbulent transport, then at z = 100m:

a) What is the initial value of the dissipation rate?

b) How long will it take the TKE to decay to 10% of its initial value?

25) Given $\overline{w'\theta_v'} = 0.3$ K m/s, $\overline{u'w'} = -0.25$ m^2 s^{-2}, and $z_i = 1$ km, find:

a) u_* e) θ_*^{SL}

b) w_* f) R_f (assume $\partial U/\partial z = 0.1$ s^{-1})

c) t_*^{ML} g) Obukhov length (L)

d) θ_*^{ML}

√ 26) Given the following sounding, indicate for each layer the

UG a) static stability

b) dynamic stability

c) existence of turbulence (assuming a laminar past history).

z (m)	$\bar{\theta}_v$ (K)	$\bar{U}$ (m/s)
80	305	18
70	305	17
60	301	15
50	300	14
40	298	10
30	294	8
20	292	7
10	292	7
0	293	2

√ 27) Which Richardson number (flux, gradient, bulk) would you use for the following application? (Give the one best answer for each question).

a) Diagnose the possible existence of clear air turbulence using rawinsonde data.

b) Determine whether turbulent flow will become laminar.

c) Determine whether laminar flow will become turbulent in the boundary layer.

28) What is the difference between free and forced convection?

√ 29) What is the Reynolds number? Of what importance is it to boundary layer flows?

30) What is the closure problem?

31) Given isotropic turbulence with $u_* = 0.5$ m/s and TKE/m = 0.9 m^2 s^{-2}, find the correlation coefficient, r, between w and u.

32) Given the TKE equation, name each term and describe how you could determine the value of each term.

√ 33) Indicate the nature of the flow (laminar or turbulent) for each cell in the table below:

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