The Atmospheric Boundary Layer

- Turbulence (9.1)
- The Surface Energy Balance (9.2)
- Vertical Structure (9.3)
- Evolution (9.4)
- Special Effects (9.5)
- The Boundary Layer in Context (9.6)

Turbulence Kinetic Energy

The kinetic energy of an object with mass m and speed V is

$$KE = \frac{1}{2} m V^2.$$

The *specific kinetic energy* is KE/m. The specific kinetic energy associated with turbulence is

$$\frac{TKE}{m} = \frac{1}{2}\left[\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right],$$

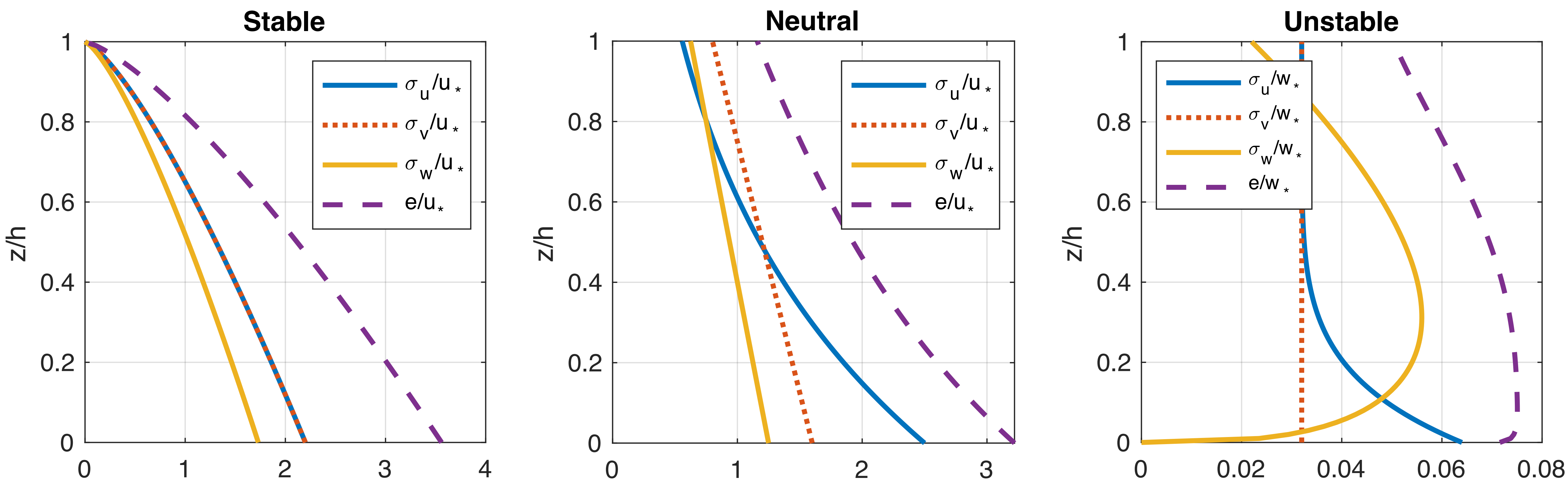
or

$$\frac{TKE}{m} = \frac{1}{2}\left[\sigma_u^2 + \sigma_v^2 + \sigma_w^2\right],$$

where TKE is *turbulence kinetic energy*.

Profiles of velocity standard deviations in the ABL for various stability conditions

(from Stull, *Practical Meteorology*, Eqs. 18.24-26)



$$e = (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)^{1/2}$$

or

$$e^2 = \sigma_u^2 + \sigma_v^2 + \sigma_w^2$$

Turbulence Kinetic Energy

Exercise
What is the specific turbulence kinetic energy if the turbulence is isotropic and $\sigma_w = 0.5\ \mathrm{m\ s^{-1}}$?

The *Eulerian* prognostic equation for TKE is

$$\frac{\partial(TKE/m)}{\partial t} = Ad + M + B + Tr - \epsilon,$$

where

$$Ad = -\bar{u}\frac{\partial(TKE/m)}{\partial x} - \bar{v}\frac{\partial(TKE/m)}{\partial y} - \bar{w}\frac{\partial(TKE/m)}{\partial z}$$

is the *advection* of TKE by the mean wind velocity, which has components $(\bar{u},\ \bar{v},\ \bar{w})$.

Turbulence Kinetic Energy

The *Eulerian* prognostic equation for TKE is

$$\frac{\partial(TKE/m)}{\partial t} = Ad + M + B + Tr - \epsilon.$$

M is *mechanical generation* of TKE.
B is *buoyancy generation or consumption* of TKE.
Tr is *transport* of TKE by turbulence.
ϵ is the viscous *dissipation rate* of TKE.

Turbulence Kinetic Energy

$$M \equiv -\overline{u'w'}\,\frac{\partial\overline{u}}{\partial z} - \overline{v'w'}\,\frac{\partial\overline{v}}{\partial z}$$

$$B \equiv \frac{g}{\theta_0}\,\overline{w'\,\theta'}$$

$$Tr \equiv -\frac{\partial\,\overline{w'(u'u' + v'v' + w'w')/2}}{\partial z} - \rho_0\,\frac{\partial\,\overline{w'\,p'}}{\partial z}$$

Turbulence Kinetic Energy

If $U = (TKE/m)^{1/2}$ is a turbulent eddy velocity scale, and L is a turbulent eddy length scale, then $T = L/U$ is a turbulent eddy time scale.

If there is no generation of TKE, then TKE will decay due to dissipation with time scale T:

$$\epsilon \sim \frac{(TKE/m)}{T} = \frac{U^2}{L/U} = \frac{U^3}{L} = \frac{(TKE/m)^{3/2}}{L}.$$

Let's look at mechanical generation and buoyancy generation or consumption in more detail.

See the slides "TKE Equation."

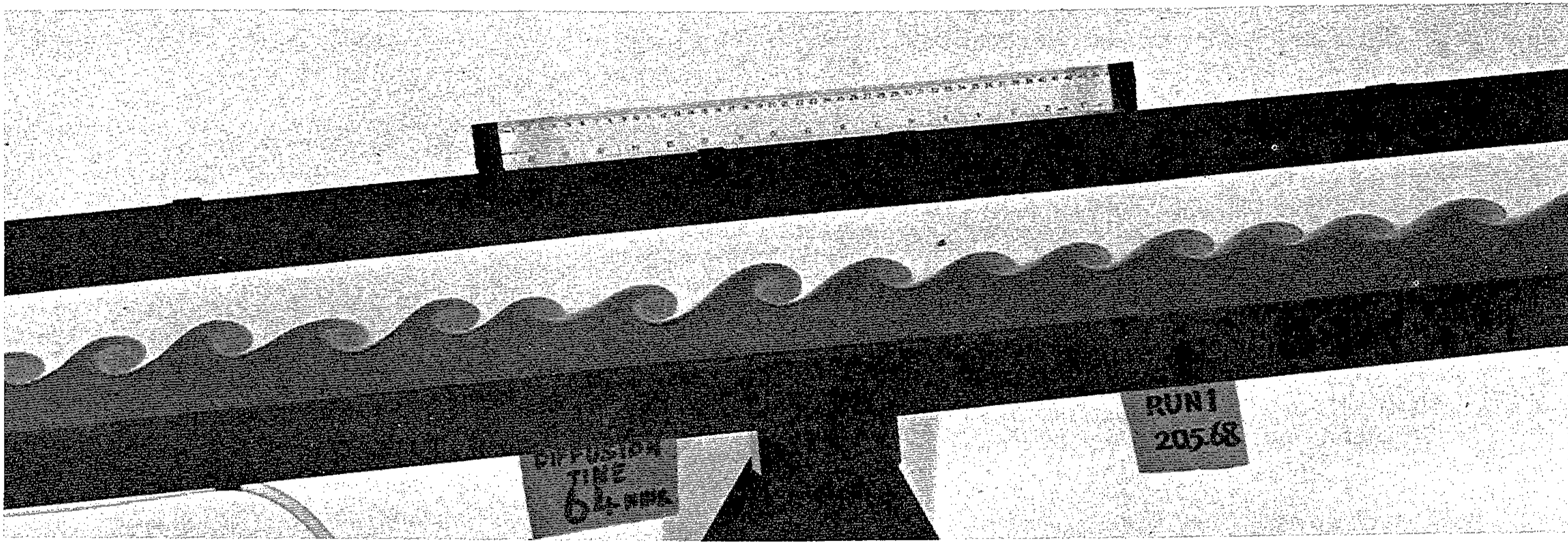
Turbulence Kinetic Energy

In a statically stable environment, the existence of TKE depends on the ratio of buoyancy consumption (B) and mechanical generation (M), which define the *Richardson number*, Ri:

$$\mathrm{Ri} = \frac{-B}{M} \approx \frac{\frac{g}{\overline{\theta_v}}\frac{\partial \overline{\theta_v}}{\partial z}}{\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2}$$

For $\mathrm{Ri} < 0.25$, laminar flow becomes turbulent.
For $\mathrm{Ri} > 1$, turbulent flow becomes laminar.
For $0.25 < \mathrm{Ri} < 1$, the existence of turbulence depends on the flow's history.

Kelvin-Helmholtz instability of stratified shear flow



145. Kelvin-Helmholtz instability of stratified shear flow. A long rectangular tube, initially horizontal, is filled with water above colored brine. The fluids are allowed to diffuse for about an hour, and the tube then quickly tilted six degrees, setting the fluids into motion. The brine accelerates uniformly down the slope, while the water above similarly accelerates up the slope. Sinusoidal instability of the interface occurs after a few seconds, and has here grown nonlinearly into regular spiral rolls. *Thorpe 1971*

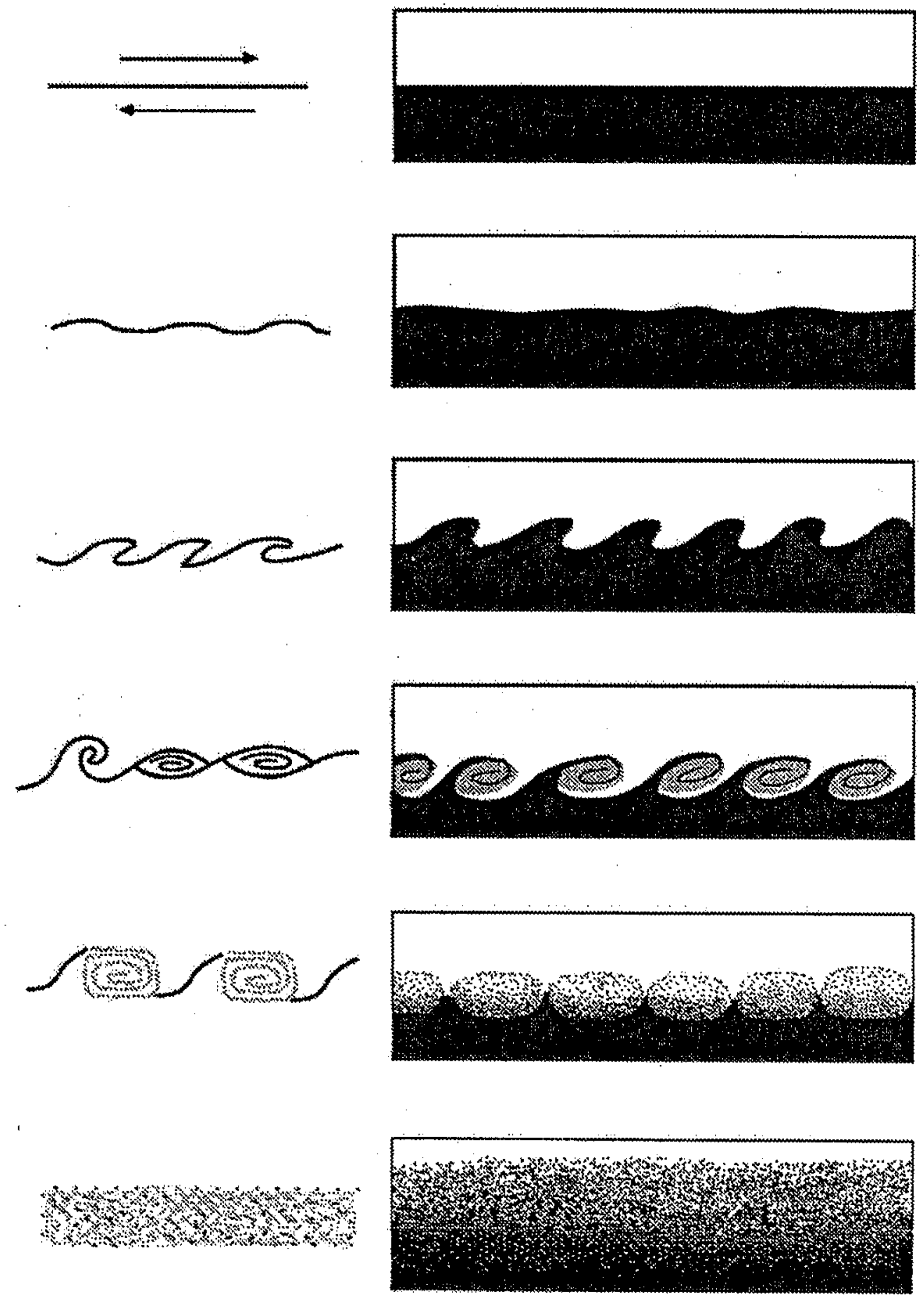


Fig. 5.18 Schematic diagram of Kelvin-Helmholtz instability in a laboratory experiment where shear flow has been generated. The upper layer, water, flows to the right, and the lower more dense fluid, dyed brine, flows to the left. The figures are about half a second apart. After Thorpe (1969,1973) and Woods (1969).

2D numerical simulation of Kelvin-Helmholtz instability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Turbulence Kinetic Energy

- The shapes of eddies in a turbulent flow depend on the static stability.
  - Unstable with rising thermals:
    anisotropic turbulence with larger TKE in vertical component; smoke plumes *loop*.
  - Statically neutral:
    isotropic turbulence; smoke plumes *cone*.
  - Statically stable but dynamically unstable:
    anisotropic turbulence with larger TKE in horizontal components; smoke plumes *fan*.
  - Statically and dynamically stable:
    No turbulence and no dispersion.

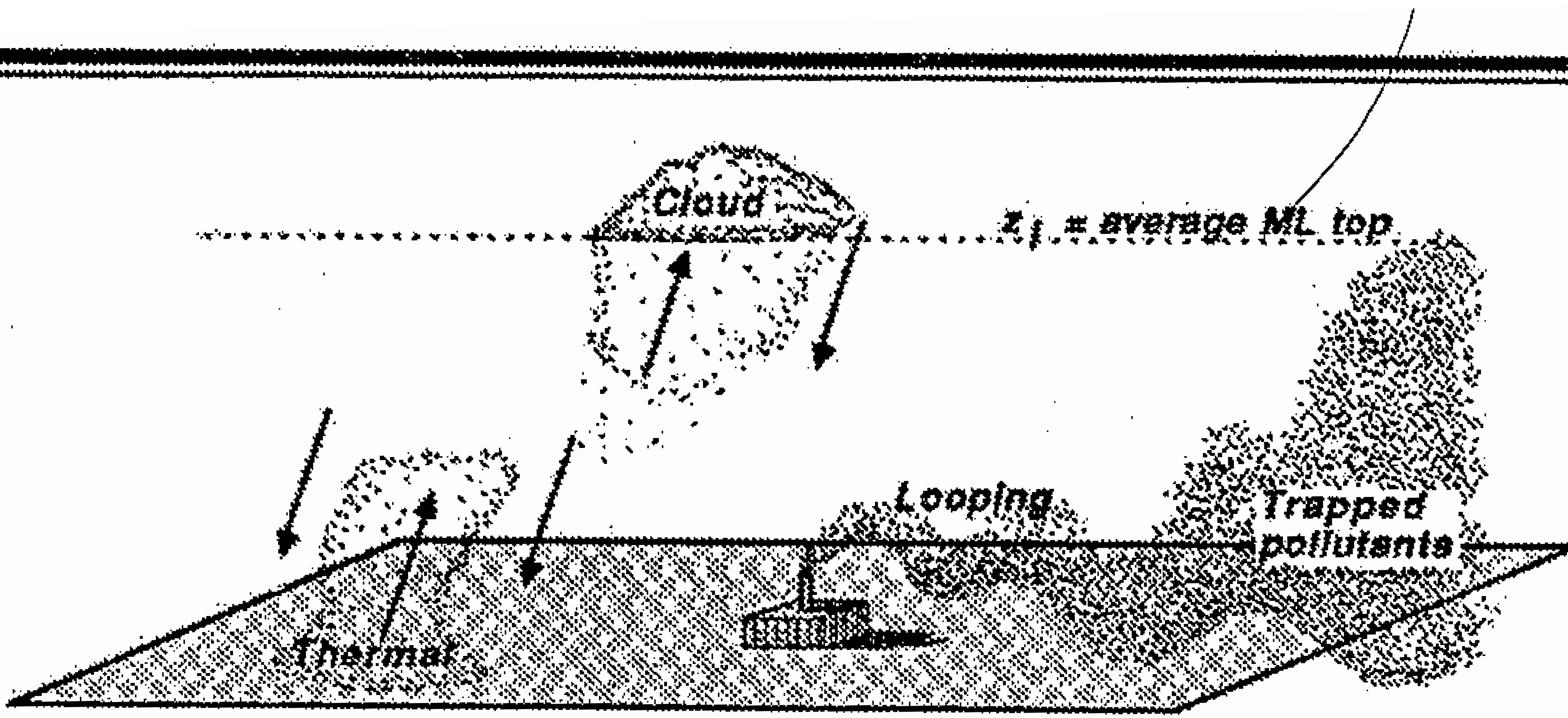

Fig. 1.8 Idealization of thermals in a mixed layer. Smoke plumes loop up and down in the mixed layer eventually becoming uniformly distributed.

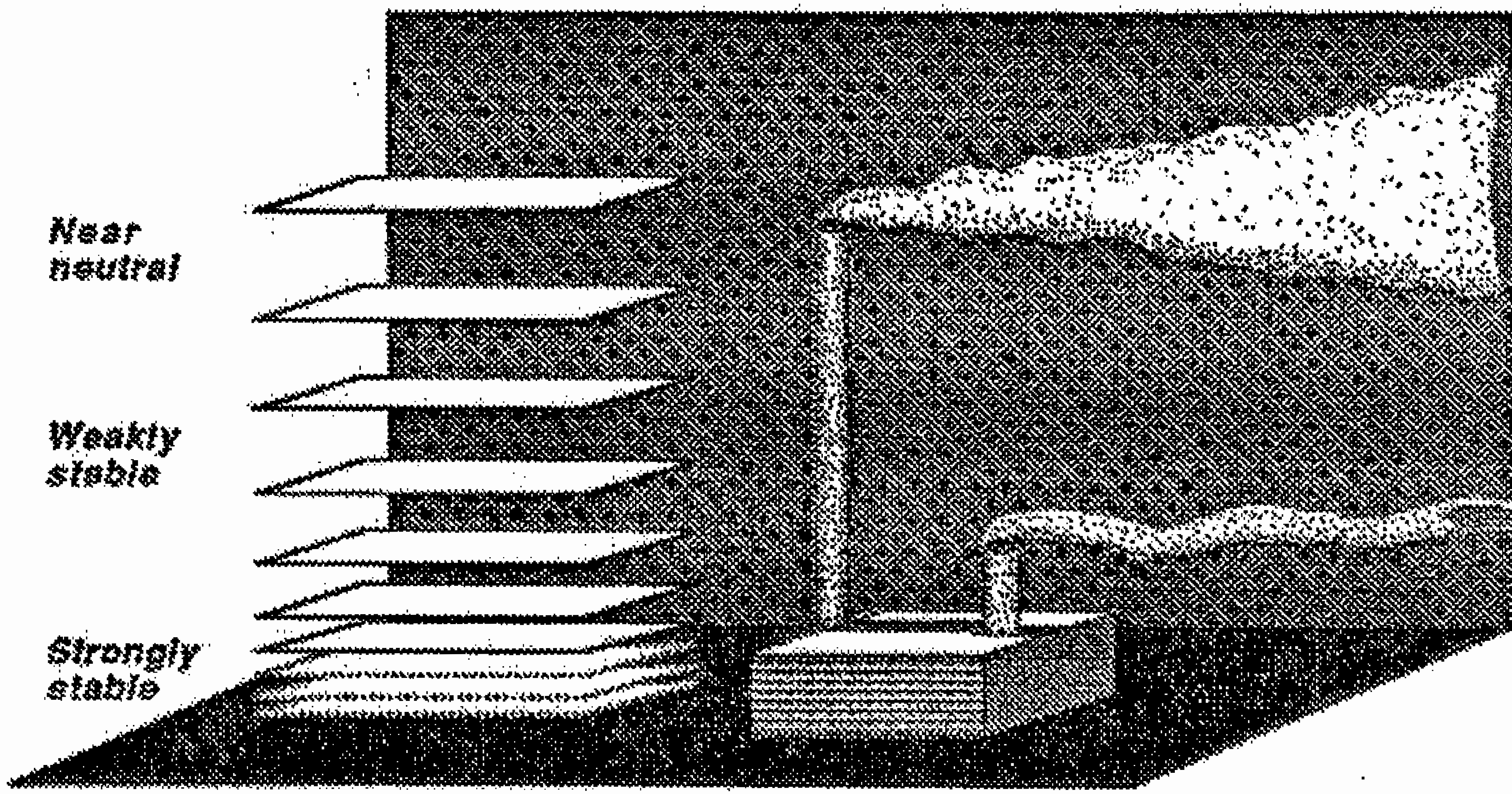

Fig. 1.10 The static stability decreases with height in the nocturnal boundary layer, gradually blending into the neutrally-stratified residual layer aloft, as indicated by the isentropic surfaces sketched on the left. Smoke emissions into the stable air fan out in the horizontal with little vertical dispersion other than wavelike oscillations. Smoke emissions in the neutral residual-layer air spread with an almost equal rate in the vertical and horizontal, allowing the smoke plume to assume a cone-like shape.

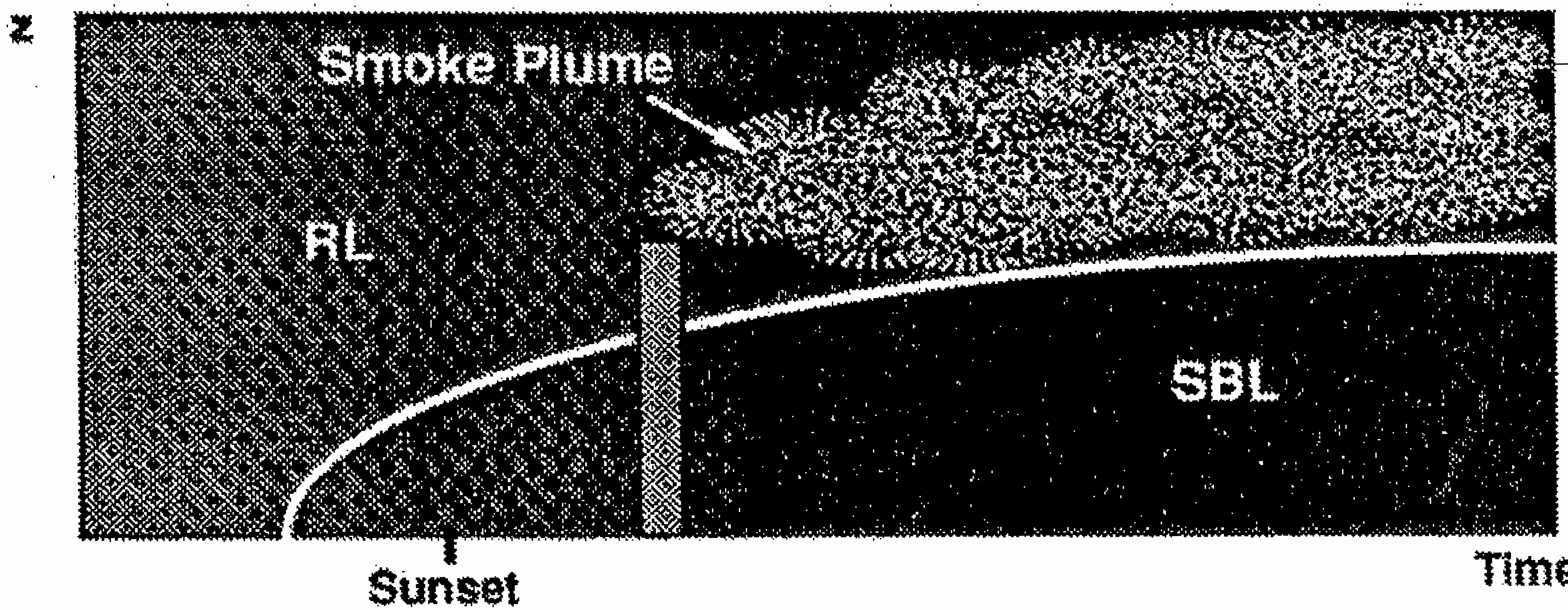

Fig. 1.13 Lofting of a smoke plume occurs when the top of the plume grows upward into a neutral layer of air while the bottom is stopped by a stable layer.

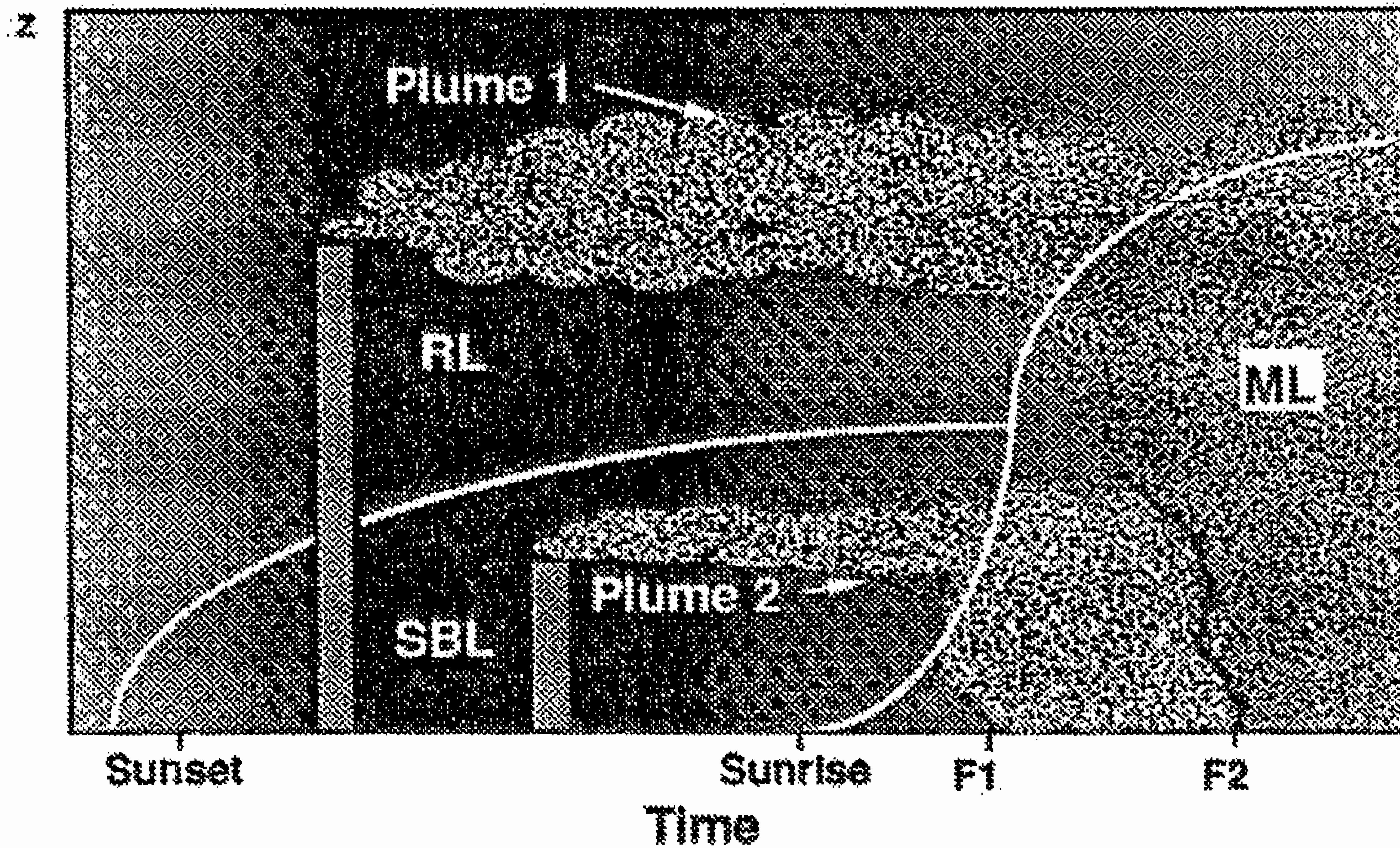

Fig. 1.14 Sketch of the fumigation process, where a growing mixed layer mixes elevated smoke plumes down to the ground. Smoke plume 1 is fumigated at time F1, while plume 2 is fumigated at time F2.

Turbulent Transport and Fluxes

Q. How does turbulence affect the mean profiles?

A. Through turbulent fluxes.

Turbulent fluxes appear in the equations for the *mean* profiles which are derived from the equations for the *total* (or instantaneous or local) profiles using *Reynolds averaging*.

Reynolds Averaging

In a turbulent flow, a field variable, such as velocity or temperature, measured at a point generally fluctuates rapidly as eddies of various scales pass the point.

To be truly representative of the large-scale flow, an average over an interval of time long enough to average out small–scale eddy fluctuations [denoted by $(\ )'$], but short enough to preserve trends in the large–scale flow field [denoted by $\overline{(\ )}$] is necessary.

This is called *Reynolds Averaging.*

Reynolds Averaging

Following the scheme introduced by Reynolds, a field variable α can be written as

$$\alpha = \overline{\alpha} + \alpha'$$

where $\overline{\alpha}$ is a running mean (or running time-average)

$$\overline{\alpha} = \frac{1}{\Delta t} \int_{t-\frac{\Delta t}{2}}^{t+\frac{\Delta t}{2}} \alpha(x, y, z, t)\, dt.$$

Here Δt is chosen so that it is long enough to average out the short term fluctuations, but short enough to retain the long term fluctuations.

Reynolds Averaging

We generally associate the slowly-varying quantities as corresponding to the synoptic-scale, whereas the turbulent fluctuations are due to small-scale processes.

This distinction between mean flow and turbulence is justified by the existence of a *spectral gap*, a region in frequency space in which there is relatively little variability on time scales between about 10 minutes (turbulence scales) and 10 hours (synoptic scales).

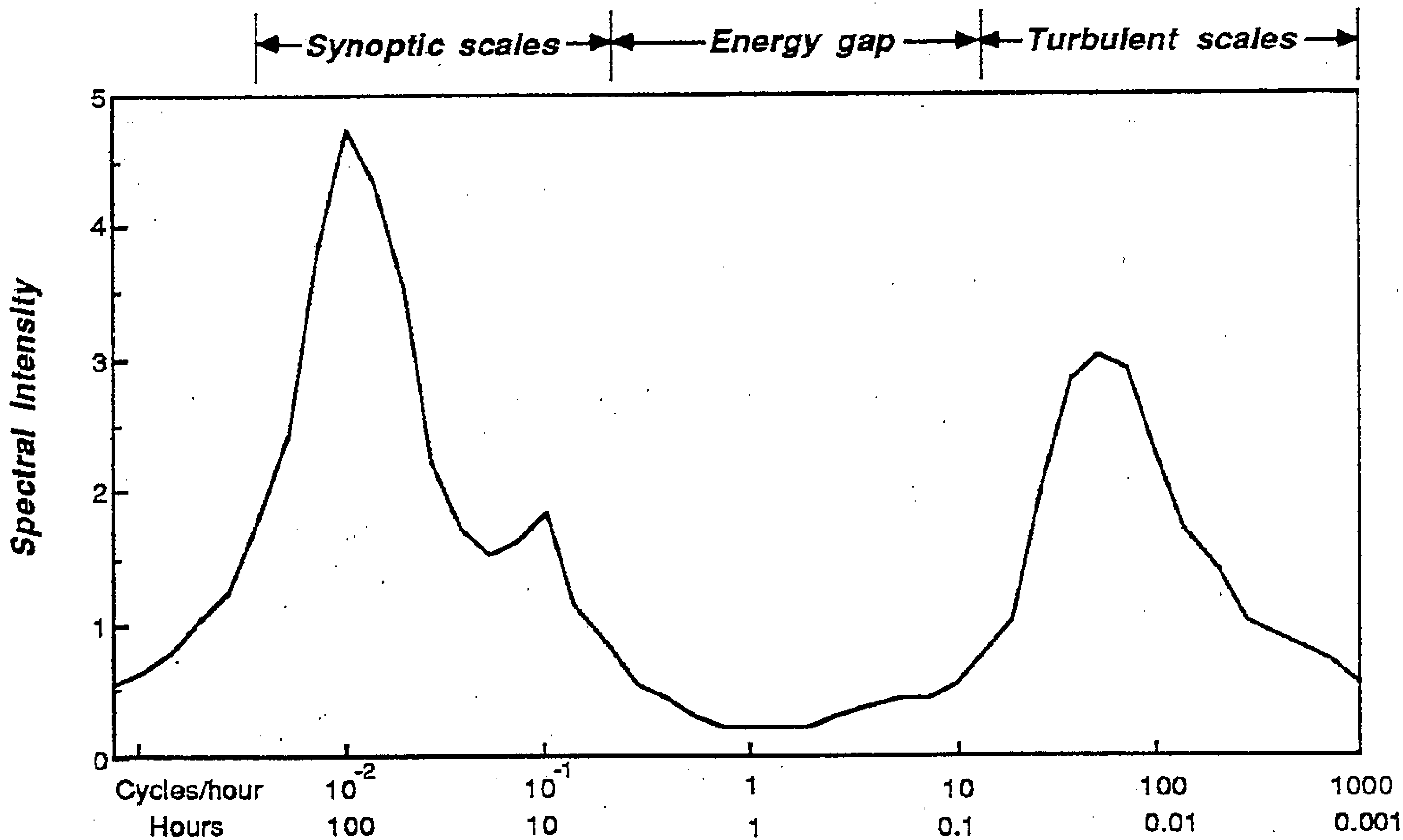

Fig. 2.2 Schematic spectrum of wind speed near the ground estimated from a study of Van der Hoven (1957).

Reynolds Averaging

Examples

$$
\begin{aligned}
\overline{u\,v} = \overline{(\overline{u}+u')(\overline{v}+v')} \quad &= \quad \overline{\overline{u}\,\overline{v}} + \overline{\overline{u}v'} + \overline{u'\overline{v}} + \overline{u'v'} \\
&= \quad \overline{u}\,\overline{v} + \overline{u}\,\underbrace{\overline{v'}}_{=0} + \underbrace{\overline{u'}}_{=0}\,\overline{v} + \overline{u'v'} \\
&= \quad \overline{u}\,\overline{v} + \overline{u'v'}
\end{aligned}
$$

Similarly,

$$
\overline{w'\overline{\theta}} = \overline{\overline{w'}\,\overline{\theta}} = 0,
$$

so that

$$
\overline{w\,\theta} = \overline{(\overline{w}+w')(\overline{\theta}+\theta')} = \overline{w}\,\overline{\theta} + \overline{w'\theta'}.
$$

Reynolds Averaging

$$\frac{Du}{dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + fv + F_{rx}$$

Before applying Reynolds decomposition, we rewrite the total derivative in flux form:

$$\begin{aligned}\frac{Du}{Dt} &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + u\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \\ &= \frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z}.\end{aligned}$$

Reynolds Averaging

Separating each dependent variable into mean and fluctuating parts and then averaging yields

$$\overline{\frac{Du}{Dt}} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x}\left(\overline{u}\,\overline{u} + \overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\overline{u}\,\overline{v} + \overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\overline{u}\,\overline{w} + \overline{u'w'}\right)$$

Noting that the mean velocity fields satisfy the continuity equation, we can rewrite this as

$$\overline{\frac{Du}{Dt}} = \frac{\overline{D}\overline{u}}{dt} + \frac{\partial}{\partial x}\left(\overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\overline{u'w'}\right)$$

where

$$\frac{\overline{D}}{dt} = \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z}$$

is the rate of change following the mean motion.

Reynolds Averaging

The mean equations thus have the form

$$\frac{\overline{D}\overline{u}}{dt} = -\frac{1}{\rho_0}\frac{\partial\overline{p}}{\partial x} + f\overline{v} - \left[\frac{\partial\overline{u'u'}}{\partial x} + \frac{\partial\overline{u'v'}}{\partial y} + \frac{\partial\overline{u'w'}}{\partial z}\right] + F_{rx},$$

$$\frac{\overline{D}\overline{v}}{dt} = -\frac{1}{\rho_0}\frac{\partial\overline{p}}{\partial y} - f\overline{u} - \left[\frac{\partial\overline{u'v'}}{\partial x} + \frac{\partial\overline{v'v'}}{\partial y} + \frac{\partial\overline{v'w'}}{\partial z}\right] + F_{ry},$$

$$\frac{\overline{D}\overline{w}}{dt} = -\frac{1}{\rho_0}\frac{\partial\overline{p}}{\partial z} + g\frac{\overline{\theta}}{\theta_0} - \left[\frac{\partial\overline{u'w'}}{\partial x} + \frac{\partial\overline{v'w'}}{\partial y} + \frac{\partial\overline{w'w'}}{\partial z}\right] + F_{rz},$$

$$\frac{\overline{D}\,\overline{\theta}}{dt} = -\overline{w}\frac{d\theta_0}{dz} - \left[\frac{\partial\overline{u'\theta'}}{\partial x} + \frac{\partial\overline{v'\theta'}}{\partial y} + \frac{\partial\overline{w'\theta'}}{\partial z}\right],$$

$$\frac{\partial\overline{u}}{\partial x} + \frac{\partial\overline{v}}{\partial y} + \frac{\partial\overline{w}}{\partial z} = 0.$$

Reynolds Averaging

The horizontal convergences of the turbulent fluxes can usually be neglected. This is called the *boundary-layer approximation.* Also, $\overline{w}$ can be obtained from the equation for mass conservation.

$$\frac{\overline{D}\overline{u}}{dt} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \frac{\partial \overline{u'w'}}{\partial z} + F_{rx},$$

$$\frac{\overline{D}\overline{v}}{dt} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial y} - f\overline{u} - \frac{\partial \overline{v'w'}}{\partial z} + F_{ry},$$

$$\frac{\overline{D}\,\overline{\theta}}{dt} = -\overline{w}\frac{d\theta_0}{dz} - \frac{\partial \overline{w'\theta'}}{\partial z},$$

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0.$$

Concept of Flux

$\bar{\rho}\, c_p\, \overline{w'\theta'}$ is the turbulent flux of sensible heat.

Units are:

$(\mathrm{kg\ m^{-3}})\ (\mathrm{J\ kg^{-1}\ K^{-1}})\ (\mathrm{m\ s^{-1}})\ \mathrm{K}$

$= \mathrm{J\ s^{-1}\ m^{-2}} = \mathrm{W\ m^{-2}}$

Concept of Flux

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$= \mathrm{J\ s^{-1}\ m^{-2}} = \mathrm{W\ m^{-2}}$

$\bar{\rho}\, L\, \overline{w'q'}$ is the turbulent flux of latent heat.

Units are:

$(\mathrm{kg\ m^{-3}})\ (\mathrm{J\ kg^{-1}})\ (\mathrm{m\ s^{-1}})\ (\mathrm{kg\ kg^{-1}})$

$= \mathrm{J\ s^{-1}\ m^{-2}} = \mathrm{W\ m^{-2}}$

Concept of Flux

$\bar{\rho}\, c_p\, \overline{w'\theta'}$ is the turbulent flux of sensible heat.

Units are:

$$(\mathrm{kg\ m^{-3}})\ (\mathrm{J\ kg^{-1}\ K^{-1}})\ (\mathrm{m\ s^{-1}})\ \mathrm{K}$$

$$= \mathrm{J\ s^{-1}\ m^{-2}} = \mathrm{W\ m^{-2}}$$

$\bar{\rho}\, L\, \overline{w'q'}$ is the turbulent flux of latent heat.

Units are:

$$(\mathrm{kg\ m^{-3}})\ (\mathrm{J\ kg^{-1}})\ (\mathrm{m\ s^{-1}})\ (\mathrm{kg\ kg^{-1}})$$

$$= \mathrm{J\ s^{-1}\ m^{-2}} = \mathrm{W\ m^{-2}}$$

$\bar{\rho}\, \overline{w'q'}$ is the turbulent flux of water vapor.

Units are:

$$(\mathrm{kg\ m^{-3}})\ (\mathrm{m\ s^{-1}})\ (\mathrm{kg\ kg^{-1}})$$

$$= \mathrm{kg\ s^{-1}\ m^{-2}}.$$

Concept of Flux

$\overline{u'p'}$ is the flux of peanuts.

u is trips into store per unit time.

p is kg of peanuts per trip.

Units are:

$$(\text{trips s}^{-1})\ (\text{kg trip}^{-1}) = \text{kg s}^{-1}.$$

Example: Unload a truck with containers of peanuts.
Each trip between truck and store takes 30 seconds.
Each trip from truck to store carries 50 kg of peanuts.
Each trip from store to truck carries no peanuts.
What is flux of peanuts?

Concept of Flux

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Each trip between truck and store takes 30 seconds.
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What is flux of peanuts?
Answer: What are mean u and mean p during all trips?
For each trip (into or out of) store:
u' = u - u_mean and p' = p - p_mean
u_mean = ?
p_mean = ?

Concept of Flux

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Answer: What are mean u and mean p during all trips?
For each trip (into or out of) store:
u' = u - u_mean and p' = p - p_mean
u_mean = 0
p_mean = 50/2 = 25 kg
u' = ?
p' = ?

Concept of Flux

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What is flux of peanuts?
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For each trip (into or out of) store:
u' = u - u_mean and p' = p - p_mean
u_mean = 0
p_mean = 50/2 = 25 kg
into store: u' = 1/30; out of store u' = -1/30
into store: p' = 50-25 = 25; out of store p' = 0 - 25 = -25
flux due to trips into store = (u'p')in = ?
flux due to trips out of store = (u'p')out = ?

Concept of Flux

$\overline{u'p'}$ is the flux of peanuts.

u is trips into store per unit time.

p is kg of peanuts per trip.

Units are:

(trips s^{-1}) (kg $trip^{-1}$) = kg s^{-1}.

Example: Unload a truck with containers of peanuts.
Each trip between truck and store takes 30 seconds.
Each trip from truck to store carries 50 kg of peanuts.
Each trip from store to truck carries no peanuts.
What is flux of peanuts?
Answer: What are mean u and mean p during all trips?
For each trip (into or out of) store:
u' = u - u_mean and p' = p - p_mean
u_mean = 0
p_mean = 50/2 = 25 kg
into store: u' = 1/30; out of store u' = -1/30
into store: p' = 50-25 = 25; out of store p' = 0 - 25 = -25
flux due to trips into store = (u'p')in = 1/30 * 25 = 5/6
flux due to trips out of store = (u'p')out = -1/30 * (-25) = 5/6
average flux into store = 1/2 *(5/6 + 5/6) = 5/6 kg/s
common-sense flux into store per round trip: 50 kg per 60 s = 5/6 kg/s