

## The Surface Layer (from Holton 5.3.5)

Maintained entirely by vertical momentum transfer by turbulence.

Friction velocity,  $u_*$ :

$$u_*^2 \equiv \overline{(u'w')}_s$$

where wind near surface is parallel to x-coordinate.

$$|\overline{(u'w')}_s| \sim 0.1 \text{ m}^2 \text{ s}^{-2}, \text{ so } u_* \sim 0.3 \text{ m s}^{-1}.$$

Variation of  $\overline{u'w'}$  with  $h$  near sfc.

Scale analysis ~~is~~ in midlatitudes of

$$\frac{\overline{D\bar{u}}}{Dt} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + f\bar{v} - \frac{\partial \overline{u'w'}}{\partial z} \quad (5.16)$$

$$\begin{array}{ccc} 10^{-4} & 10^{-3} & 10^{-3} \\ \frac{U^2}{L} & \frac{\delta p}{\rho L} & f_0 U \end{array} \quad \text{m s}^{-2}$$

$$f_0 = 2\Omega \sin \phi_0 \sim 10^{-4} \text{ s}^{-1}$$

$$\delta p / \rho \sim 10^3 \text{ m}^2 \text{ s}^{-2}$$

$$U \sim 10 \text{ m s}^{-1}$$

$$L \sim 10^6 \text{ m}$$

For balance, must have

$$\frac{\partial \overline{u'w'}}{\partial z} = \frac{\delta(u_*^2)}{\delta z} \leq 10^{-3} \text{ m s}^{-2}$$

$$\text{For } \delta z = 10 \text{ m}, \delta(u_*^2) \leq 10^{-2} \text{ m}^2 \text{ s}^{-2}$$

This is  $\leq 10\%$  of  $\overline{(u'w')}_s \sim 0.1 \text{ m}^2 \text{ s}^{-2}$ .

So  $u_*$  is approx. constant near sfc.

## Log wind profile (Holton, S.3.5)

Parameterize  $\overline{(u'w')}_s$  as

$$K_m \frac{\partial \bar{u}}{\partial z} = u_*^2,$$

mixing length model:

$$K_m = l^2 \left| \frac{\partial \bar{u}}{\partial z} \right|.$$

Near surface,

$$l = kz,$$

$k = (\text{von Karman}) \text{ const.}$   
 $= 0.4$

so

$$K_m = (kz)^2 \left| \frac{\partial \bar{u}}{\partial z} \right|, \text{ and}$$

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz}, \text{ and } K_m = u_* kz.$$

Integrate  $dz$  to get log wind profile:

$$\bar{u}(z) = \frac{u_*}{k} \log(z/z_0)$$

$z_0 = \text{roughness length, chosen so}$

$$\bar{u}(z_0) = 0.$$

Obtain  $u^*$  from  $u$  measured at several heights:

Arya Fig 10.4 is an example.

$$\begin{aligned}\bar{u}(z) &= \frac{u^*}{k} \log(z/z_0) = \frac{u^*}{k} (\log z - \log z_0) \\ &= \frac{u^*}{k} \log z + c\end{aligned}$$

Plot of  $\bar{u}(z)$  vs  $\log z$ : line w/ slope  $\frac{u^*}{k}$ .

When  $z=z_0$ ,  $\bar{u}(z) = 0$ , so  $z_0$  is height for which line intersects  $u=0$ .

Numerical:  $\bar{u}$  at 2 hts  $z_1, z_2$

$$\bar{u}(z_1) = \frac{u^*}{k} \log(z_1/z_0)$$

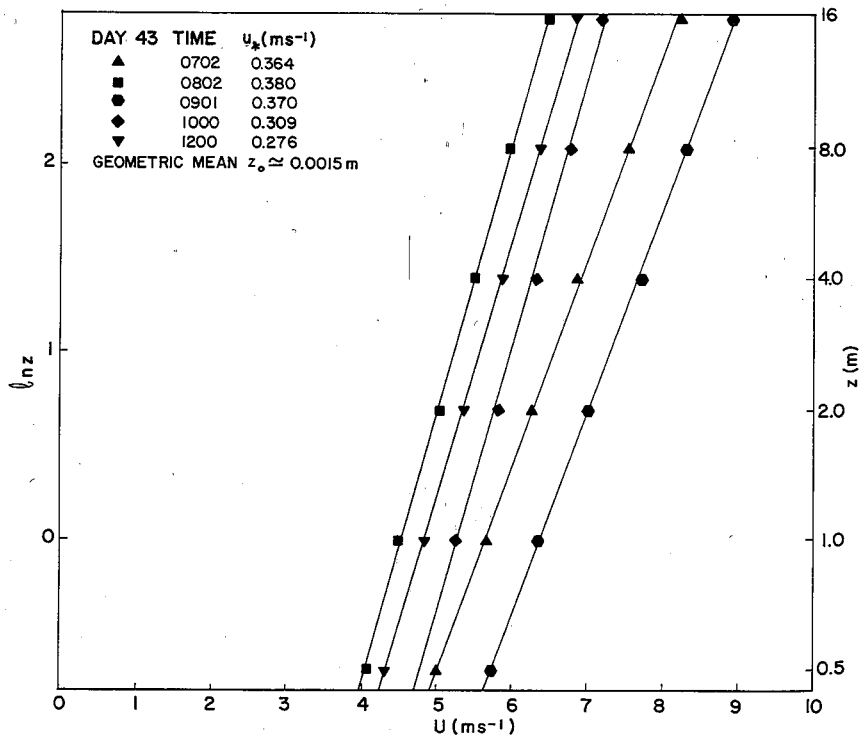
$$\bar{u}(z_2) = \frac{u^*}{k} \log(z_2/z_0)$$

$$u_2 - u_1 = \frac{u^*}{k} \log(z_2/z_1). \quad \text{Solve for } u^*.$$

Obtain  $z_0$ :

If  $u_1 = 0$ , then  $z_1 = z_0$ , so

$$u_2 = \frac{u^*}{k} \log(z_2/z_0), \quad \text{solve for } z_0.$$



**Fig. 10.4** Comparison of the observed wind profiles in the neutral surface layer of day 43 of the Wangara Experiment with the log law [Eq. (10.6)] (solid lines). [Data from Clarke *et al.* (1971).]

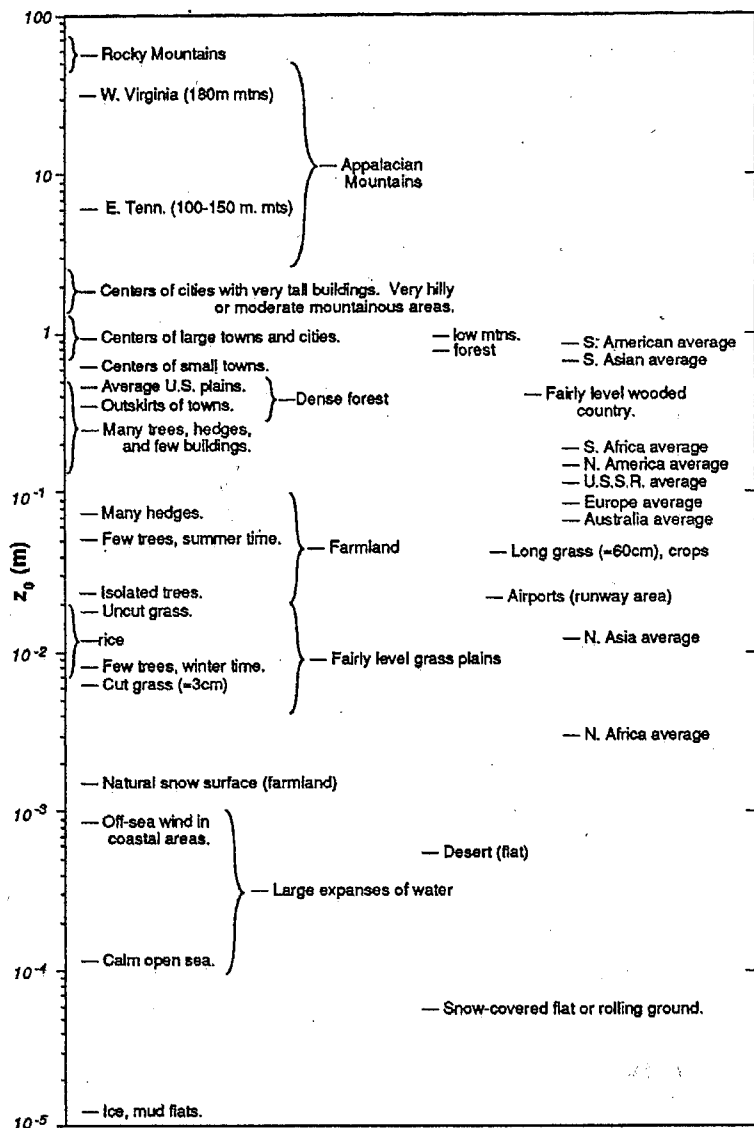


Fig. 9.6 Aerodynamic roughness lengths for typical terrain types. (After Garratt 1977, Smedman-Högström & Högström 1978, Kondo & Yamazawa 1986, Thompson 1978, Napo 1977, and Hicks 1975).