Meteorology 3510
Example Problems: Thermodynamic Processes

1. A parcel of dry air rises and expands adiabatically from $p = p_1$ where $T = T_1$ to $p = p_2$. What is $T_2 = T(p_2)$? What is θ?

   Solution:

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{R/c_p}, \qquad \theta = T_1 \left(\frac{p_0}{p_1}\right)^{R/c_p}$$

   where $p_0 = 1000$ hPa.

2. A parcel of dry air at p =600 mb has T =-10° C.
   (a) The parcel descends to 800 mb dry adiabatically; calculate its temperature there.
   (b) The parcel acends to 400 mb dry adiabatically; calculate its temperature there.
   (c) Calculate the parcel's potential temperature.

   Answers: (a) 285.5 K, (b) 234.2 K, (c) 304.3 K.

3. Near-surface air flows into a hurricane over a sea surface with uniform temperature.
   (a) If the air temperature remains 28° C and the pressure decreases from 1020 to 920 hPa, what is the increase in potential temperature?
   (b) How much energy is transferred to the air by heating during this process?

   Answers: (a) 9.0 K, (b) 8.9 kJ kg^{-1}.

4. A jet airplane cruises in the lower stratosphere where the air pressure is 150 mb and the temperature is -60° C. The ventilation system brings outside air into the cabin where the pressure is 700 mb.
   (a) The air is first adiabatically compressed to cabin pressure. What is the air's temperature after this step?
   (b) In order to reach the cabin temperature of 20° C, how much (isobaric) heating per unit mass is required after the air has been compressed to cabin pressure?

   Answers: (a) 330.8 K or 57.7° C, (b) -38.0 kJ kg^{-1}.

5. A radiosonde measures T, p, and RH. How can you obtain T_d and w from these quantities (mathematically)?

   Solution: The dew-point temperature T_d may be calculated from

$$T_d = \frac{T}{1 - \frac{TR_v}{L_e} \ln r},$$

   where $R_v \equiv R^*/m_v = R/\epsilon$, L_e is the latent heat of evaporation, and r is the relative humidity.

   The mixing ratio w may be obtained from $e = r\, e_s(T)$ and

$$w = \epsilon \frac{e}{p - e} \approx \epsilon \frac{e}{p}.$$

6. A radiosonde measures $T = 280$ K, $p = 900$ mb, and $r = 0.5$. Calculate T_d and w from these quantities. You can read $e_s(T)$ from the plots below.

   Answers: $T_d = 270.3$ K, $w = 3.5$ g kg^{-1}.

7. How much is a kilogram of air cooled by isobarically evaporating 5 g of water into it?

   Solution: $dh = -L\,dw = c_p dT$ so

$$\Delta T = -\frac{L}{c_p}\Delta w = \frac{2.5 \times 10^6 \text{ J kg}^{-1}}{1004 \text{ J kg}^{-1}\text{K}^{-1}}\ 5 \times 10^{-3} \text{ kg kg}^{-1} = 12.45 \text{ K}$$