Atmospheric Sciences 5300
Exercise: Precipitation Rate

We can derive an expression for the cloud-base precipitation rate, P, for a steady-state cloudy (saturated) updraft with cloud-base temperature = T_b, cloud-base pressure = p_b, cloud-top temperature = T_t, cloud-top pressure = p_t, and constant vertical mass flux = M ($\equiv \rho w$, where ρ is air density and $w = dz/dt$ is vertical velocity). In the cloudy updraft, air is rising from cloud base to cloud top at a constant rate M (kg m^{-2} s^{-1}). The updraft's temperature decreases from T_b at cloud base to T_t at cloud top.

We assume that water vapor condenses as air ascends in the cloudy updraft at a rate that maintains the updraft's relative humidity at exactly 100 percent with respect to liquid water. We also assume that the cloud droplets or ice crystals formed by condensation rapidly form precipitation particles. Under these assumptions, a *steady state* is achieved in which the cloud-base precipitation rate equals the vertically integrated condensation rate.

The vertically integrated condensation rate is just the condensation per unit mass of air during ascent from cloud base to cloud top times the vertical mass flux:

$$P = [w_s(T_b, p_b) - w_s(T_t, p_t)]\, M,$$

where $w_s(T, p)$ is the saturation mixing ratio, which can be read directly from a skew T-log p diagram.

(a) Use this result to calculate P in units of mm/h for the cloud types and their associated parameters listed in the table. Obtain the needed saturation mixing ratios from a skew-T log p diagram and enter them into the table. The cloud types are Stratocumulus (Sc), Cumulus congestus (Cu con), Cumulonimbus (Cb), and Nimbostratus (Ns).

(b) For a given set of values of cloud-base temperature, pressure, and vertical velocity, what cloud-top pressure produces the largest value of P, denoted $P_{\max}$? What is the mathematical expression for $P_{\max}$?

(c) Calculate $P_{\max}$ for each of the cloud types described in (a) and enter the values in the table.

(d) Assume that for the last two cloud types, Ns (Fall) and Ns (winter), the precipitation reaches the ground as snow. Convert P to snowfall rate (inches per hour); see *Useful Constants* below for the necessary conversion factors.

Cloud Type	Sc	Cu con	Cb	Ns	Ns (Fall)	Ns (Winter)
cloud-base temperature (T_b, °C)	15	15	15	15	0	-10
cloud-base pressure (p_b, hPa)	950	950	950	950	900	900
cloud-top pressure (p_t, hPa)	900	750	300	300	300	300
mass flux (M, kg m^{-2} s^{-1})	0.5	2	8	0.1	0.2	0.2
cloud-base saturation mixing ratio ($w_{s,b}$, g/kg)						
cloud-top saturation mixing ratio ($w_{s,b}$, g/kg)						
cloud-base precipitation rate (P, mm hr^{-1})						
maximum cloud-base precipitation rate ($P_{\max}$, mm hr^{-1})						
cloud-base snowfall rate (inches hr^{-1})	X	X	X	X		

Useful Constants

1 mb = 1 hPa = 100 Pa
gas constant for dry air = R = 287 J kg^{-1} K^{-1}
specific heat capacity of dry air at constant pressure = c_p = 287 J kg^{-1} K^{-1}
density of liquid water = 1000 kg m^{-3}
density of snow ≈ 100 kg m^{-3}
depth of 1 kg m^{-2} of liquid water = 1 mm
1 inch = 2.54 cm