Meteorology 6160
Example Problems: Heat Capacity

1. What is the mass per unit area of a column of the atmosphere extending from the surface, where $p = 1000$ hPa, to the top where $p = 0$?

   Solution: The mass M of a column of air above a level exerts a downwards force equal to Mg at that level, where g is the acceleration of gravity. Because pressure is force per unit area, the pressure p at a given level is equal to Mg/A, where M/A is the mass per unit area. Therefore,

$$M/A = \frac{p}{g} = \frac{1000 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 10204 \text{ kg m}^{-2}.$$

2. On a clear summer afternoon, the atmosphere is receiving an energy flux of 600 W m^{-2} from the underlying land surface, due to conduction and thermal radiation from the ground, which is being warmed by solar radiation. If this energy flux is uniformly distributed through the layer of air between the surface, at $p_1 = 900$ hPa, and $p_2 = 800$ hPa, how much will the average temperature of this layer change over 3 hours?

   Solution: The ratio of the energy ΔH supplied to a body by heating to its corresponding temperature change ΔT is called the *heat capacity* C of the body:

$$C = \frac{\Delta H}{\Delta T}.$$

   The heat capacity per unit mass of a body, called the *specific heat capacity* c, is

$$c = \frac{C}{M} = \frac{\Delta H}{M \Delta T}. \tag{1}$$

   The specific heat capacity is characteristic of the material of which the body is composed and the conditions under which the heating occurs. In this case, the material is dry air and the heating is isobaric, so $c = c_p = 1004$ J kg^{-1} K^{-1}.

   Solve (1) for the temperature change ΔT, with $c = c_p$:

$$\Delta T = \frac{\Delta H}{c_p M}. \tag{2}$$

   From the information given, we can obtain ΔH per unit area and M per unit area. Therefore we rewrite (2) as

$$\Delta T = \frac{\Delta H/A}{c_p M/A}. \tag{3}$$

   Calculate

$$\Delta H/A = P/A \times t = 600 \text{ W m}^{-2} \times 3 \text{ h} \times 3600 \text{ s h}^{-1} = 6.48 \text{ MJ m}^{-2},$$

where P/A is the power per unit area (energy flux) and t is the time interval, and

$$M/A = \frac{p_1 - p_2}{g} = \frac{(900 - 800) \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 1020 \text{ kg m}^{-2}.$$

Then

$$\Delta T = \frac{\Delta H/A}{c_p M/A} = \frac{6.48 \text{ MJ m}^{-2}}{1004 \text{ J kg}^{-1} \text{ K}^{-1} \times 1020 \text{ kg m}^{-2}} = 6.3 \text{ K}.$$

3. During a cold air outbreak from Siberia over the Sea of Japan, the temperature of the lowest 300 hPa of the atmosphere rises by 12° C, due to heating by the upper 30 m of the ocean. How much does this ocean layer cool as a result?

   Solution: In this case, (3) applies to the atmospheric layer:

$$\Delta T_a = \frac{\Delta H_a/A}{c_p M_a/A},$$

   and to the ocean layer:

$$\Delta T_o = \frac{\Delta H_o/A}{c_w M_o/A},$$

   where $c_w = 4186$ J kg^{-1} K^{-1} is the specific heat capacity of water.

   The key to solving this problem is recognizing that the energy gained by the atmosphere is equal to that lost by the ocean layer:

$$\Delta H_a/A = -\Delta H_o/A.$$

   We can now solve for ΔT_o:

$$\Delta T_o = -\Delta T_a \frac{c_p M_a/A}{c_w M_o/A}. \tag{4}$$

   Calculate the mass per unit of the atmospheric layer:

$$M_a/A = \frac{\Delta p}{g} = \frac{300 \times 100 \text{ Pa}}{9.8 \text{ m s}^{-2}} = 3061 \text{ kg m}^{-2}$$

   and of the ocean layer:

$$M_o/A = \rho_w \, \Delta z = 1000 \text{ kg m}^{-3} \times 300 \text{ m} = 30000 \text{ kg m}^{-2},$$

   where ρ_w is the density of water.

   Substitute into (4):

$$\Delta T_o = -12 \text{ K} \times \frac{1004 \text{ J kg}^{-1} \text{ K}^{-1} \times 3061 \text{ kg m}^{-2}}{4186 \text{ J kg}^{-1} \text{ K}^{-1} \times 30000 \text{ kg m}^{-2}} = -0.29 \text{ K}.$$