PBL Momentum Eqs. (Holton 5.3)

B.L. approx. + neglect of viscosity:

$$\frac{\bar{D}\bar{u}}{Dt} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x} + f\bar{v} - \frac{\partial \overline{u'w'}}{\partial z} \quad (1)$$

$$\frac{\bar{D}\bar{v}}{Dt} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial y} - f\bar{u} - \frac{\partial \overline{v'w'}}{\partial z} \quad (2)$$

Unknowns: $\overline{u'w'}$, $\overline{v'w'}$

(Assume $\partial \bar{p}/\partial x$, $\partial \bar{p}/\partial y$ known.)

Approx. mid-lat. balance:

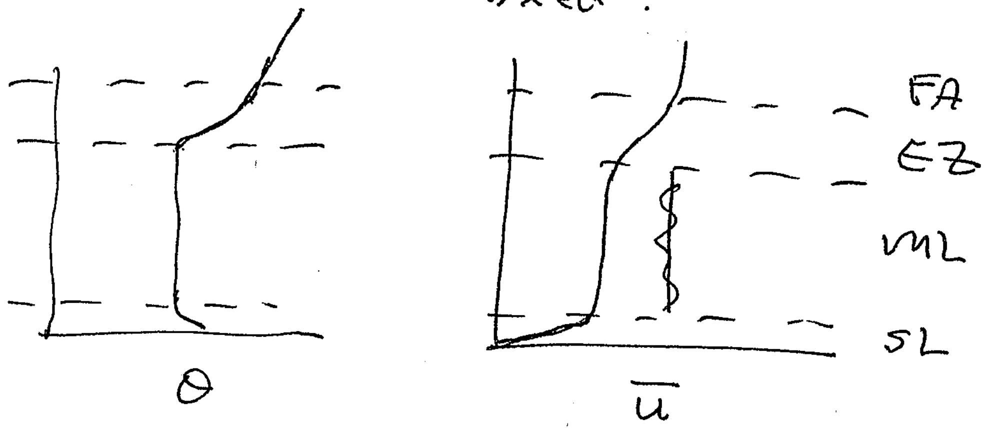
$$f(\bar{v} - \bar{v}_g) - \frac{\partial \overline{u'w'}}{\partial z} = 0 \qquad (3)$$

$$-f(\bar{u} - \bar{u}_g) - \frac{\partial \overline{v'w'}}{\partial z} = 0 \qquad (4)$$

whee $-f\bar{v}_g \equiv -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x}$, $f\bar{u}_g \equiv -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial y}$.

Mixed Layer Model

CBL is well-mixed:



PBL mom. Eqs.

Occurrence: over land during sunny days, over oceans when SST > air T_s.

M.L. model:

- $\bar{\theta}(z)$, $\bar{u}(z)$, $\bar{v}(z)$ const.
- Fluxes must be linear in z.
- Assume turbulence ($\Rightarrow$ fluxes) $\to 0$ at BL top.

Linear flux profile:

$$F(z) = F_s(1 - z/h).$$

Need F_s.

Use <u>bulk aerodynamic formula</u> (observationally based):

$$(\overline{u'w'})_s = -C_d\,|\underset{\sim}{\bar{V}}|\,\bar{u} \qquad (5)$$

(skip) $$(\overline{v'w'})_s = -C_d\,|\underset{\sim}{\bar{V}}|\,\bar{v} \qquad (6)$$

C_d: drag coeff. (non-dim.) $\sim 1.5\times 10^{-3}$ over oceans; much larger ~~more~~ over rough ground.

Can now integrate (3), (4) using (5), (6):
From surface to $z=h$:

$$f(\bar{v}-\bar{v}_g) = -\frac{(\overline{u'w'})_s}{h} = C_d\,|\underset{\sim}{\bar{V}}|\,\bar{u}/h \qquad (7)$$

(skip) $$-f(\bar{u}-\bar{u}_g) = -\frac{(\overline{v'w'})_s}{h} = C_d\,|\underset{\sim}{\bar{V}}|\,\bar{v}/h \qquad (8)$$

choose axes so $\bar{v}_g = 0 \Rightarrow$

$$\bar{v} = \varkappa_s |\underset{\sim}{\bar{V}}| \, \bar{u}$$

$$\bar{u} = \bar{u}_g - \varkappa_s |\underset{\sim}{\bar{V}}| \, \bar{v}$$

where

$$\varkappa_s \equiv c_d / (fh).$$

wind speed $|\underset{\sim}{V}| = (\bar{u}^2 + \bar{v}^2)^2$ is less than $|\underset{\sim}{\bar{V}}_g| = u_g$, and vector balance gives cross-isobar flow toward low pr.

Table 9.2 The Davenport classification, where z_o is aerodynamic roughness length and C_{DN} is the corresponding drag coefficient for neutral static stability[a]

z_0 (m)	Classification	Landscape	C_{DN}
0.0002	Sea	Calm sea, paved areas, snow-covered flat plain, tide flat, smooth desert.	0.0014
0.005	Smooth	Beaches, pack ice, morass, snow-covered fields.	0.0028
0.03	Open	Grass prairie or farm fields, tundra, airports, heather.	0.0047
0.1	Roughly open	Cultivated area with low crops and occasional obstacles (single bushes).	0.0075
0.25	Rough	High crops, crops of varied height, scattered obstacles such as trees or hedgerows, vineyards.	0.012
0.5	Very rough	Mixed farm fields and forest clumps, orchards, scattered buildings.	0.018
1.0	Closed	Regular coverage with large size obstacles with open spaces roughly equal to obstacle heights, suburban houses, villages, mature forests.	0.030
≥2	Chaotic	Centers of large towns and cities, irregular forests with scattered clearings.	0.062

[a] From Preprints *12th Amer. Meteorol. Soc. Symposium on Applied Climatology*, 2000, pp. 96–99.

Ex.

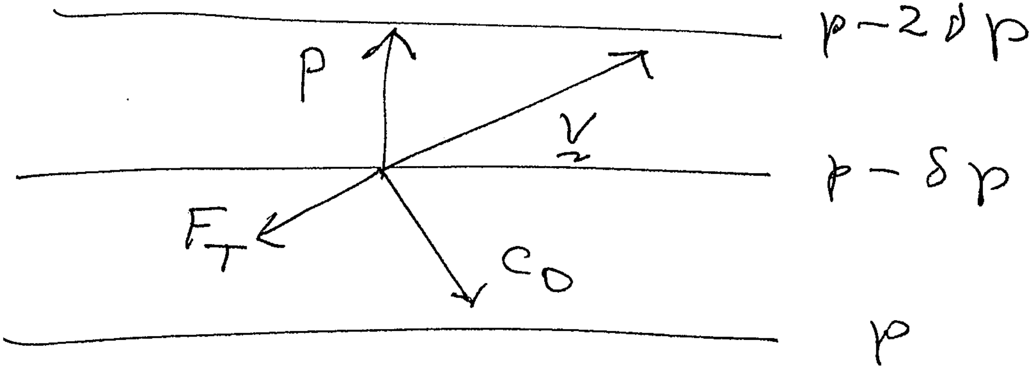
$$\left.\begin{aligned} \bar{u}_g &= 10\ m/s \\ K_s &= .05\ m^{-1}\, s \end{aligned}\right\} \quad \begin{aligned} \bar{u} &= 8.28\ m/s \\ \bar{v} &= 3.77\ m/s \end{aligned}$$

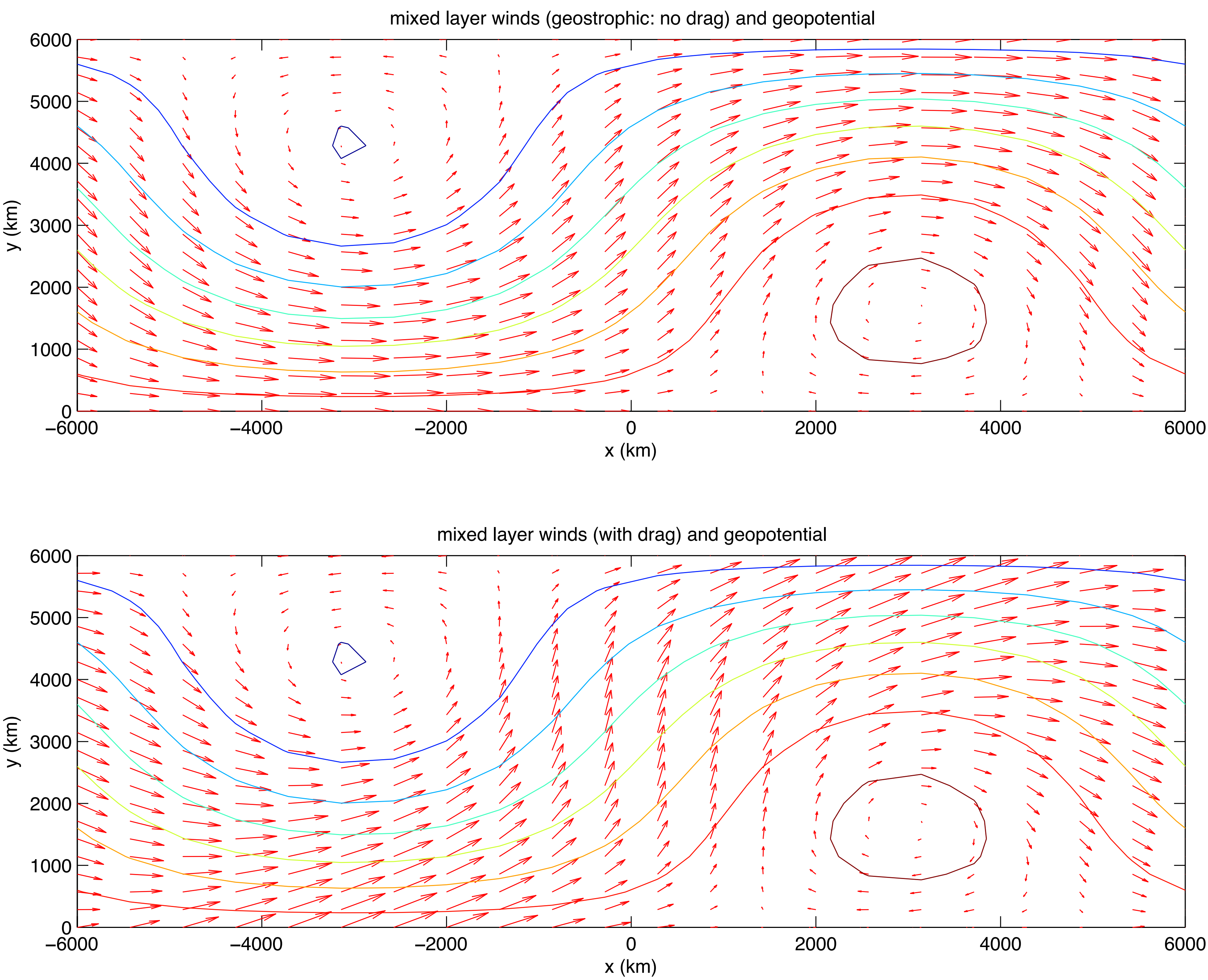
$$|\bar{\underset{\sim}{V}}| = 9.10\ m/s.$$

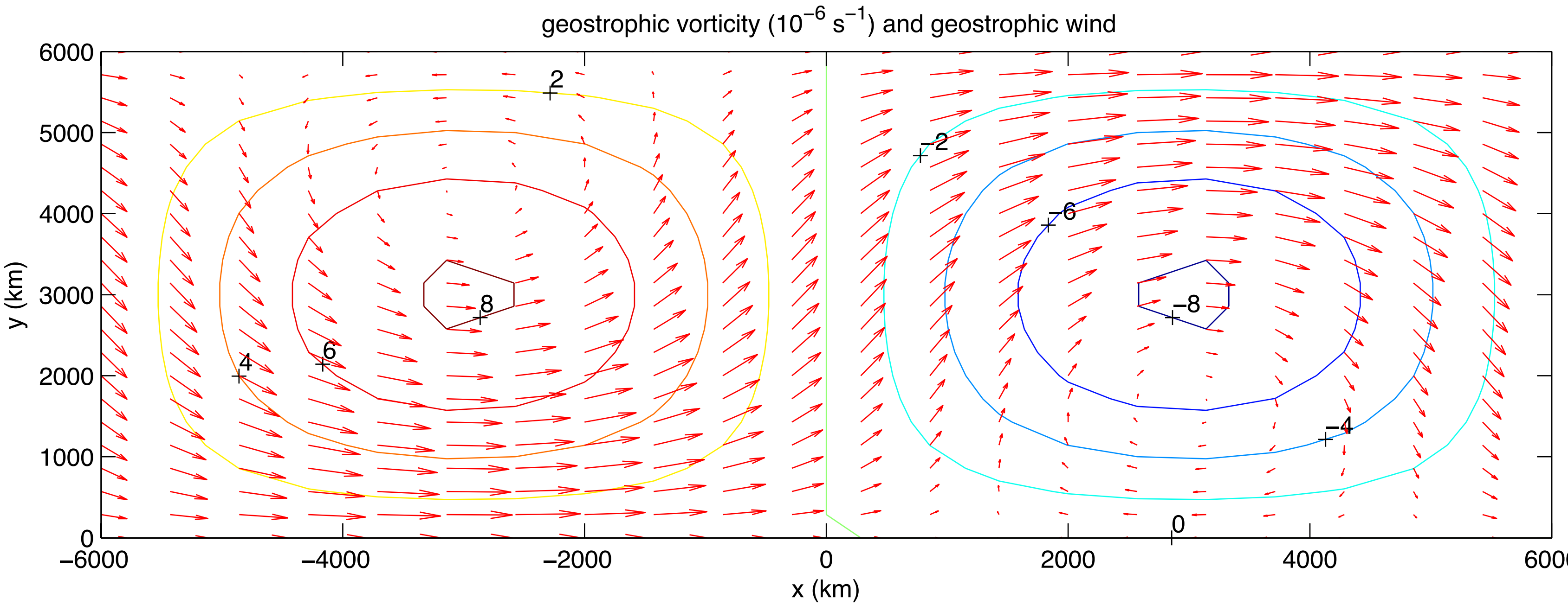
What is cross-isobar angle?

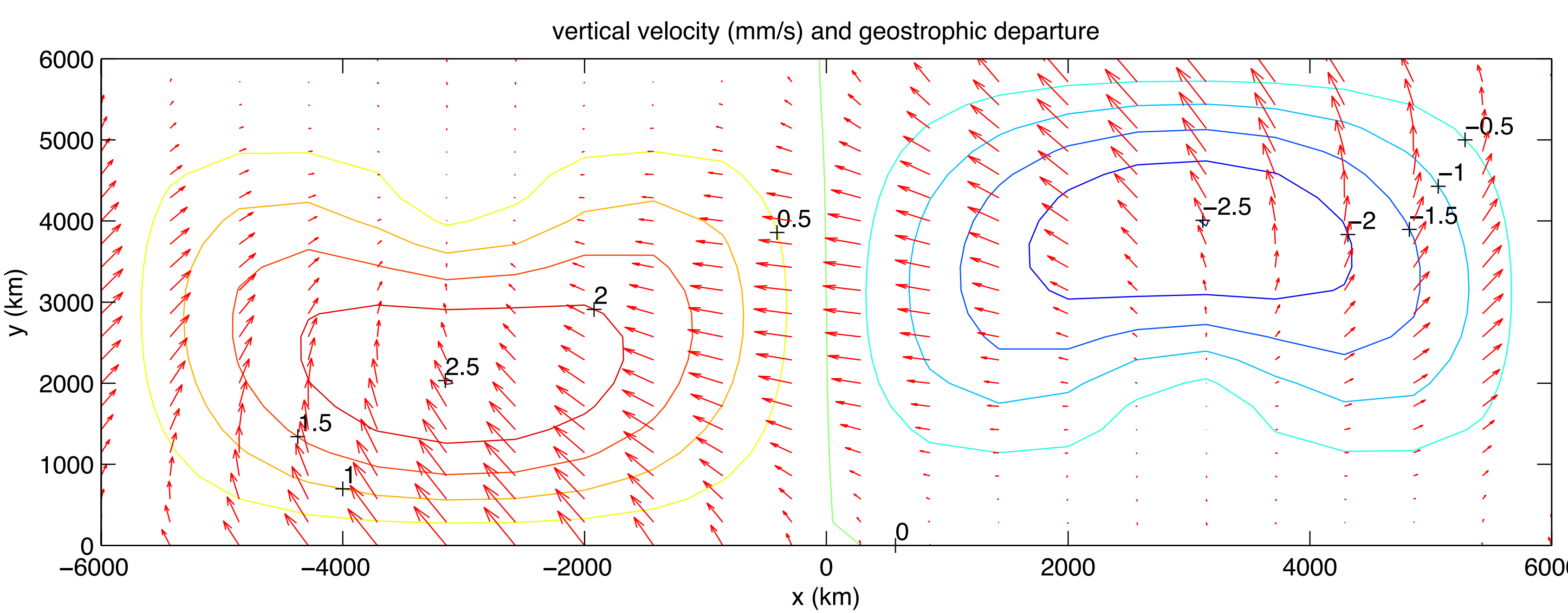
Work is done by PGF to balance KE loss due to <u>friction</u>.

Balance




mixed layer winds (geostrophic: no drag) and geopotential
6000
5000
4000
3000
2000
1000
0
y (km)
−6000
−4000
−2000
0
2000
4000
6000
x (km)
mixed layer winds (with drag) and geopotential
6000
5000
4000
3000
2000
1000
0
y (km)
−6000
−4000
−2000
0
2000
4000
6000
x (km)


geostrophic vorticity (10^{-6} s^{-1}) and geostrophic wind
y (km)
x (km)
2
−2
−6
8
−8
6
4
−4
0


vertical velocity (mm/s) and geostrophic departure
y (km)
x (km)
−0.5
−1
0.5
−2.5
−2
−1.5
2
2.5
1.5
1
0