Exercises

9.7 Explain and interpret the following:

(a) Air pollution is often trapped within the boundary layer.

(b) Clear-air turbulence (CAT) experienced by airplane passengers is widespread, but usually not very strong in the boundary layer. It tends to be more sporadic and also more intense near the tropopause.

(c) Air within the free troposphere exhibits only a small diurnal temperature range.

(d) Boundary layers over deserts are often much deeper than over vegetated terrain.

(e) The warmest and highest-rising thermals often have the highest LCLs.

(f) Boundary-layer turbulence in the Earth's atmosphere decays during a solar eclipse.

(g) Birds soar during daytime over land, not usually at night.

(h) Turbulence kinetic energy is dissipated into internal energy.

(i) Convective overturning creates fat thermals with a mean diameter of order of the boundary-layer depth, as opposed to thin thermals of the width of smoke-stack plumes.

(j) The theoretical height of the LCL based on surface temperature and dew point is a very accurate predictor of the height of cloud base for convective clouds.

(k) The LCL is a poor predictor for the base height of stratus clouds.

(l) Deterministic forecasts of any scale of turbulent phenomenon are accurate out to duration roughly equal to the lifetime of individual eddies.

(m) The covariance between u and w is both the vertical flux of horizontal momentum and the horizontal flux of vertical momentum.

(n) The covariance of θ and q does not represent a flux.

(o) During day, boundary layers are often shallow near shorelines and increase with distance inland.

(p) Kelvin-Helmholtz breaking waves and turbulence occur much more often than the occurrence of billow clouds.

(q) The method of estimating turbulent transport demonstrated in Fig. 9.8 does not necessarily work for larger eddies.

(r) Zeroth-order similarity theories give useful important information, even though they contain no dynamics.

(s) Gradient-transfer theory fails in the interior of the mixed layer.

(t) What would be the opposite of the oasis effect, and could it occur in nature?

(u) Significant vertical turbulent fluxes can exist near the surface even in the limit of zero wind speed.

(v) The drag coefficient is related to the roughness length.

(w) During some nights dew will form on the ground, but on others fog forms instead.

(x) Hot-air balloonists like to take off in near-calm conditions at the surface, but can find themselves in fast-moving air a short distance above the surface.

(y) Hot-air balloonists who take off in the near-calm winds of early morning, find it much more difficult to land by mid morning because of faster winds.

(z) On a sunny day, the air in the trunk space below a forest canopy is stably stratified.

(aa) Turbulence in the residual layer decays quickly after sunset.

(bb) Greater subsidence does not inject more air into the top of the mixed layer, but has the opposite effect of making the mixed layer more shallow.

(cc) The rapid-rise phase of mixed-layer growth happens only after the nocturnal inversion has been "burned off."

(dd) The boundary layer is poorly defined near fronts.

(ee) When air flows over a change in roughness, an internal boundary layer develops. The depth of this layer grows with increasing distance from the change.

(ff) Latent and sensible heat fluxes over land surfaces tend to be larger on clear days than on cloudy days.

(gg) Daytime temperatures tend to be higher over deserts than over vegetated land surfaces.

(hh) Other conditions being the same, alpine glaciers and snow-fields lose more mass on a humid summer day than on a dry summer day.

(ii) On a clear, calm day, the surface sensible heat flux into the air does not usually become positive until 30 to 60 min after sunrise.

(jj) In fair weather, the heat and momentum fluxes at the top of the mixed layer due to entrainment are usually downward, but the moisture flux is positive.

(kk) You can estimate the static stability of the boundary layer by looking at the shape of the smoke plume from a smoke stack.

(ll) In Fig. 9.40, why do the surface wind speed and the cloudiness increase as the air flows northward across the sharp front in the sea-surface temperature field that lies along 1 °N?

9.8 Estimate the temperature variance for the velocity trace at the bottom of Fig. 9.6.

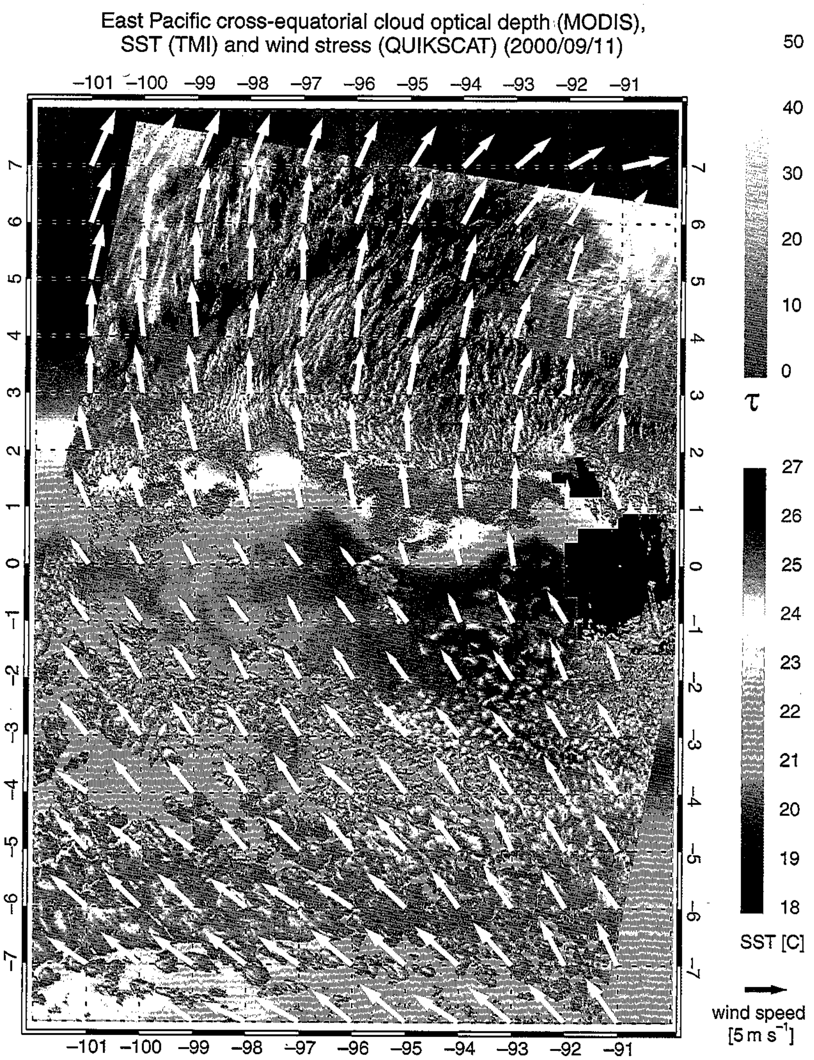

Fig. 9.40 Sea surface temperature (colored shading) surface winds (arrows) and clouds (gray shading) over the equatorial Pacific at a time when the equatorial front in the sea surface temperature field is well defined along 1 °N. The scalloped appearance of the front is due to the presence of tropical instability waves in the ocean. [Based on NASA QUIKSCAT, TMI, and MODIS imagery. Courtesy of Robert Wood.]

9.9 Prove that the definition of covariance reduces to the definition of variance for the covariance between any variable and itself.

9.10 Given the following variances in $m^2 s^{-2}$

Where:	Location A		Location B	
When (UTC):	1000	1100	1000	1100
σ_u^2	0.50	0.50	0.70	0.50
σ_v^2	0.25	0.50	0.25	0.25
σ_w^2	0.70	0.50	0.70	0.25

When, where, and for which variables is the turbulence (a) stationary, (b) isotropic, and (c) homogeneous?
Some, but not all, of the answers Homogeneous for u wind at 1100 UTC. Stationary for v wind at location B. Isotropic at location A at 1100 UTC.

9.11 Given the following synchronous time series for T(°C) and w(m/s), find (a) mean temperature, (b) mean velocity, (c) temperature variance, (d) velocity variance, and (e) kinematic heat flux.

T	21	22	20	25	25	15	18	23	21	24	16	12	19	22
w	1	−2	0	−3	2	−2	−3	3	0	0	1	4	−2	−3

9.12 Under what conditions is the Richardson number not likely to be a reliable indicator for the existence of turbulence? Why?

9.13 If the dissipation length scale is L_ϵ, what is the e-folding time for the decay of turbulence (i.e., time for TKE/m to equal $1/e$ of its initial value), assuming you start with finite TKE but there is no production, consumption, transport, or advection?

9.14 The upward vertical heat flux in soil and rock due to molecular conduction is given by

$$F = -K\frac{\partial T}{\partial z} \qquad (9.34)$$

where K is the *thermal conductivity* of the medium, in units of $W\,m^{-1}\,K^{-1}$, which ranges in value from 0.1 for peat to 2.5 for wet sand. From the first law of thermodynamics, we can write

$$C\frac{\partial T}{\partial t} = -\frac{\partial F}{\partial z} = \frac{\partial}{\partial z}K\frac{\partial T}{\partial z} \qquad (9.35)$$

where C is the heat capacity per unit volume in $J\,m^{-3}\,K^{-1}$ (i.e., the product of the specific heat times the density). If K is assumed to be independent of depth

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial z^2} \qquad (9.36)$$

where $D = K/C$ is called the *thermal diffusivity*. Consider the response of the subsurface temperature to a sinusoidal variation in surface temperature with amplitude T_s and period P. (a) Show that the amplitude of the response drops off exponentially with depth below the surface with an e-folding depth of

$$h = \sqrt{\frac{DP}{2\pi}}$$

(b) The e-folding depth of the annual cycle, as estimated from Fig. 9.12, is ~2 m. Estimate the e-folding depth of the diurnal cycle at the same site.

9.15 For the situation shown in Fig. 9.12 in the text: (a) Making use of Eq. (9.36), derive a relationship for the phase lag Δt between the temperature wave at two depths differing by height Δz, given P the wave period and D the thermal diffusivity of the soil as defined in the previous exercise. (b) Using the annual cycle data given in Fig. 9.12 in the text, estimate the phase lag between the temperature curves at 1.5 and 6 m and then use it to find the value of D.

9.16 Given the heat-flux profile of Fig. 9.22, extend the method of Fig. 9.8 to estimate the sign of the triple correlation $\overline{w'w'\theta'}$ in the middle of the mixed layer, which is one of the unknowns in Eq. (9.11).

9.17 Given: $F_{HS} = 0.2$ K m s^{-1}, $z_i = 1$ km, $u_* = 0.2$ m s^{-1}, $T = 300$ K, $z_0 = 0.01$ m, find and explain the significance of the values of the (a) Deardorff velocity scale; (b) Obukhov length; (c) convective time scale; and (d) wind speed at $z = 30$ m.

9.18 The *flux form of the Richardson number* is

$$R_f = \frac{(g/\overline{T}_v)\,\overline{w'\theta'_v}}{\overline{w'u'}\dfrac{\partial \overline{u}}{\partial z} + \overline{w'v'}\dfrac{\partial \overline{v}}{\partial z}}$$

Use gradient transfer theory to show how R_f is related to R_i.

9.19 If the wind speed is 5 m s^{-1} at $z = 10$ m and the air temperature is 20 °C at $z = 2$ m, then (a) what is the value of the sensible heat flux at the surface of unirrigated grassland if the skin temperature is 40 °C? (b) What is the value of the latent heat flux? [**Hint:** $\rho c_p \approx 1231$ (W m^{-2})/(K m s^{-1}) for dry air at sea level.]

9.20 (a) Confirm that the following expression is a solution to Eq. (9.26), and (b) that it satisfies the boundary condition that $V = 0$ at $z = z_0$.

$$\frac{V}{u_*} = \frac{1}{k}\left\{\ln\frac{z}{z_0} - 2\ln\left[\frac{1+x}{2}\right] - \ln\left[\frac{1+x^2}{2}\right] + 2\arctan x - \pi/2\right\}$$

where

$$x = \left[1 - 15\frac{(z - z_0)}{L}\right]^{1/4}$$

9.21 Use the expressions for surface-layer wind speed from Eq. (9.22), from Exercise 9.4 in Section 9.3.3, and from the previous exercise to plot curves of V vs. z for (a) neutral ($L = \infty$), (b) stable ($L = 100$ m), and (c) unstable ($L = -10$ m) stratifications and confirm that their relative shapes are as plotted in Fig. 9.17a and 9.17b. Use $z_0 = 0.1$ m.

9.22 (a) If boundary-layer divergence is constant during fair weather, the mixed layer can stop growing during midafternoon even though there are strong sensible heat fluxes into the boundary layer at the ground. Why? (b) If w_e = constant, derive an equation showing the growth of z_i with time in a region where divergence β is constant.

9.23 If F^* is known, and if $|F_G| = 0.1 \cdot |F^*|$, then show how knowledge of time-averaged temperature at two heights in the surface layer and of mean humidity at the same two heights is sufficient to estimate the sensible and latent heat fluxes in the surface layer.

√ 9.24 As a cold, continental air mass passes over the Gulf Stream on a winter day, the temperature of the air in the atmospheric boundary layer rises by 10 K over a distance of 300 km. Within this interval the average boundary layer depth is 1 km and the wind speed is 15 m s^{-1}. No condensation is taking place within the boundary layer and the radiative fluxes are negligible. Calculate the sensible heat flux from the sea surface.

9.25 (a) If drag at the ground represents a loss of momentum from the mean wind, determine the sign of $\overline{u'w'}_s$ if the mean wind is from the west. Justify your result.

(b) Do the same for a wind from the east, remembering that drag still represents a momentum loss.

9.26 Given the following temperature profile, determine and justify which layers of the atmosphere are (a) statically stable, (b) neutral, and (c) unstable.

z (km)	θ (°C)
2	21
1.8	23
1.6	19
1.4	19
1.2	13
1.0	16
0.2	16
0	10

9.27 If the total accumulated surface heat flux from sunrise through sunset is 5100 km, then use the sunrise sounding in the previous exercise to estimate the depth and potential temperature of the mixed layer just before sunset.

9.28 Given a smoke stack half the height of a valley: (a) describe the path of the centerline of the smoke plume during day and night during fair weather and (b) describe the centerline path of the smoke on a strongly windy day.

9.29 If you know the temperature and humidity jumps across the top of the mixed layer and if you know only the surface heat flux (but not the surface moisture flux), show how you can calculate the entrained heat and moisture fluxes at the top of the mixed layer.

9.30 Show that over flat terrain the large-scale vertical velocity at the top of the boundary layer is approximately equal to

$$w_i \simeq -z_i\{\nabla \cdot \mathrm{V}\} \tag{9.28}$$

where

$$\{\nabla \cdot \mathrm{V}\} \equiv \frac{\int_{p_i}^{p_s} (\nabla \cdot \mathrm{V})\, dp}{(p_s - p_i)}$$

is the mass-weighted divergence in the boundary layer and p_s and p_i are the pressures at the Earth's surface and at the top of the boundary layer.