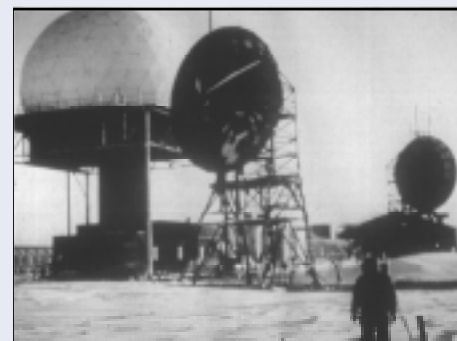# Snow Mechanics
Review of the State of Knowledge and Applications

Lewis H. Shapiro, Jerome B. Johnson, Matthew Sturm, and George L. Blaisdell

August 1997

Abstract: A review of snow mechanics indicates that, with the exception of avalanche studies, it is seldom used. In this report we give our interpretation of why this is the case, and suggest ways to help expand the range of problems to which snow mechanics can be applied. Until the late 1960s, most experimental work in snow mechanics was devoted to finding values of the parameters for equations of linear elasticity, viscosity, and viscoelasticity. In about 1970, work on that approach stopped and since then the emphasis has been on 1) the development of nonlinear theories to describe the deformation and fracture of snow, and 2) attempts to develop constitutive relationships based on the study of the microstructural aspects of snow deformation. We believe that the best hope of encouraging more applications for snow mechanics in the near term lies in improving and expanding the database on the response of snow to applied loads, and organizing it in a manner that makes it easy for potential users to determine the anticipated deformational behavior of snow in any particular application. To do this, we suggest developing a classification of snow based on physical properties and index parameters that give information about the bonding and microstructure. Mechanical properties, constitutive relations under various loading conditions, and other relevant information can then be associated with each class.

Cover (clockwise from top right): Exploration—Blasting to open snow-bridged crevasses in shear zone between Ross and McMurdo Ice Shelves, Antarctica, in search of a safe route for heavy tractor trains (R.G. Alger, Michigan Technological University); Buildings—Compacted snow foundation supports huge Defense Early Warning radar station on Greenland Ice Cap (W. Tobiasson, CRREL); Utilities construction—Milling machine bores long unlined tunnel and vacuum system removes snow chips 10 m below snow surface for Amundsen-Scott South Pole Station sewer line (M.R. Walsh, CRREL); Snow control—Snow fence array on steep slopes above village in Swiss Alps retains snow and controls avalanches (E. Wengi, Swiss Federal Institute for Snow and Avalanche Research); Surface transportation—Military vehicles in deep seasonal snowfields in Alaska rely on efficient compaction and shearing of snow to be mobile; Logistics—LC-130 Hercules (ski-wheel) aircraft provide the only means of supplying Amundsen-Scott South Pole Station with fuel, food, and other cargo; they operate from a groomed skiway (G.L. Blaisdell, CRREL).

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Cold Regions Research &
Engineering Laboratory

Snow Mechanics
Review of the State of
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Lewis H. Shapiro, Jerome B. Johnson, Matthew Sturm,
and George L. Blaisdell

August 1997

PREFACE

This report was prepared by Dr. Lewis H. Shapiro, Geologist and Consultant; Dr. Jerome B. Johnson, Geophysicist; Dr. Matthew Sturm, Research Physical Scientist; and George L. Blaisdell, Research Civil Engineer, Applied Research Division, Research and Engineering Directorate, U.S. Army Cold Regions Research and Engineering Laboratory.

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Snow Mechanics
Review of the State of Knowledge and Applications

LEWIS H. SHAPIRO, JEROME B. JOHNSON,
MATTHEW STURM, AND GEORGE L. BLAISDELL

INTRODUCTION

Overview

In this document we describe the state of knowledge of the field of snow mechanics* and suggest studies to help expand its range of applications. The work was motivated by a review of the literature, which showed that, aside from avalanche studies (i.e., release mechanisms, particle movement and impact effects, and effects on avalanche defenses), relatively infrequent use is made of snow mechanics. However, possible applications in other areas include 1) design of equipment for snow removal, 2) calculating loads on structures (roofs, towers, snow fences, cables, walls, etc.), 3) the use of snow as a construction material for roads, airstrips and foundations, 4) over-snow vehicle traffic including sinkage, shear strength, resistance to motion and design of tracks and tread patterns for vehicles, and 5) military applications such as those based on the capability of snow to absorb projectile impacts, and the problems presented by snow-covered minefields.

We agree with Brown (1989), who pointed out that "...the properties of snow are not yet well enough known for use with a high degree of confidence. Snow, as a natural geological material, is found in a wide range of densities, stages of metamorphism, free water content, etc., and its properties have been determined only for a few cases."

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*We define "snow mechanics" in a similar fashion to Jaeger and Cook (1976, p.1) but change the word "rock" to "snow": "Snow mechanics is the theoretical and applied science of the mechanical behavior of snow; it is that branch of mechanics concerned with the response of snow to the force fields of its environment."

Perhaps the most important reason why this is so is that there are few commercial or governmental activities that *absolutely* require knowledge of snow properties and processes. For example, in the case of snow removal, heavy equipment designed for road construction and maintenance is available, although it may be significantly overpowered for the task of clearing snow (Minsk 1989). Similarly, in the design and construction of structures that must contend with snow loading, overdesign can be substituted for knowledge of snow properties because the additional construction costs are a small fraction of the total for any single project. In general, the economic incentives for any one project or agency are insufficient to encourage the research necessary to improve the body of snow mechanics information. However, the economic benefits would be significant when the entire range of potential applications are considered. Minsk (1989) noted that an average saving of 10% in the cost of snow removal alone would save about $100,000,000 per year in the U.S.A.

A further impediment to progress has been that the community of researchers in the field has always been relatively small and scattered. As a result, there has never been broad awareness nor interest in the field within the scientific community in general. The breadth of intellectual activity that could lead to expanded financial support for basic research on snow mechanics has frequently been lacking.

Thus, despite the potential for practical, economically viable applications, development of snow mechanics has been limited. Our purpose is to determine why this is so and what can be done to extend the range of applications.

Plan of the report

In the following sections we review the literature on descriptive and experimental studies of snow mechanics and snow deformation, and give our view of the current state of the subject. We conclude that the field is relatively static at present, particularly in the area of applications to engineering problems. We then argue that there is little hope for improvement in the near future, unless special efforts are made to make data on the deformational behavior of snow available to potential users in an accessible format. A prime source of difficulty is that data on mechanical properties and deformational behavior* have usually been organized and presented as functions of the snow density. However, we will show from the literature that snow density is not a reliable indicator of these properties. Instead, for a given temperature and loading condition, the response to load depends primarily on the bonding and microstructure, and the geometric characteristics of the grains. This was recognized in early studies of snow deformation, but developing a method of using microstructural properties as an indicator of deformational response to load still remains to be done. We propose that this can be done by building a classification of snow based on a combination of microstructural properties and physical characteristics, with the classes then correlated to characteristic deformational behavior. We argue that the critical microstructural properties cannot be established by stereologic work (App. C). Instead, we suggest that index properties (the results of tests designed to be sensitive to the state of the microstructure) are the best way to represent the critical microstructure. We describe several possible index tests, but suggest that a modification of a blade penetration measure of snow hardness (Fukue 1979) may be most useful. With a classification established, tests can be run to obtain stress–strain–time–strength data to establish the characteristic deformational behavior for each class of snow.

We have limited this report to the properties of dry snow in order to avoid dealing with the problems introduced by the presence of free water. For brevity, we have not considered friction between snow and other materials, acoustic properties, properties of snow in motion, and shock waves in snow.

REVIEW OF PREVIOUS WORK

Background

Most of the literature on snow mechanics has been summarized in reviews by Bader (1962a), Mellor (1964, 1975, 1977) and Salm (1982). We used these extensively. For discussion, we separate the field into two areas. The first area includes the descriptive and experimental studies that established the basic ideas about snow deformation and snow as a material, and efforts to establish constitutive relationships for snow. In much of this work, the objectives were to describe how snow responds to applied loads, to measure the strengths of various types of snow under different loading conditions, and to find numerical values of the parameters required by the various constitutive relationships. Early experiments and constitutive relations were based on measurements of macroscopic deformation. Later, recognition of the importance of snow microstructural influences on deformational behavior led to the second area of research: studies of microstructural scale processes that operate during deformation. The purpose of these studies has been to describe and quantify the changes in grain and bond relationships that occur during deformation as the grains rearrange, fracture, recrystallize or sinter, and then to use the results as the basis for developing constitutive relationships for snow.

Constitutive equations
and parameters

Most of the descriptive and experimental studies were done between 1930 and 1980 and are described in the reviews by Bader (1962a), Mellor (1964, 1975) and Salm (1982). The early studies in western Europe were primarily motivated by the need to understand and predict the occurrence of avalanches, and to mitigate their hazards. Similarly, the problems posed by the heavy seasonal snow cover in parts of Japan provided the incentive for the systematic studies of snow properties by researchers at the Institute of Low Temperature Science at the University of Hokkaido. In the U.S.A., the work by SIPRE and CRREL investigators between the late 1940s and continuing

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*We use "mechanical properties" to refer to parameters such as the constants of the familiar stress–strain relationships of elastic, viscous and viscoelastic materials, or the strength in various loading modes. The expression "deformational behavior" is intended to mean the nature of the response to load in a general sense. For example, Young's modulus is a mechanical property, while deformational behavior is the bulk deformation of the material.

to the late 1960s was largely in response to needs arising from the expansion of U. S. military activities in the polar regions. The sheer size of the Soviet Union, and its range of arctic, subarctic and alpine environments, made the study of snow mechanics important in that nation, although, unfortunately, only a small fraction of the resulting literature is available in translation.

The objective of most of the work through this period was to determine the parameters required for application of linear elasticity, viscosity and viscoelasticity to problems involving snow mechanics. The effort followed the recognition that some patterns of deformational behavior in snow samples in a laboratory or field setting could be described by linear relationships. For example, Bader et al. (1939) discussed the creep of snow (they used the term "plasticity") in connection with investigations of snow settlement. They did experiments on samples in both uniaxial confined and unconfined compression, but since they did not attempt to formulate a constitutive relationship to describe the process there was no framework within which parameters could be defined. Thus, they made no mention of any particular mechanical property or constitutive relationship, although the patterns of deformation certainly suggested a combination of linear elastic and viscous behavior. In fact, Yosida et al. (1956) were able to use data from Bader et al. (1939) to calculate values for the coefficient of Newtonian viscosity of snow.

The most general constitutive relationship used for snow prior to about 1970 was the equation for a four-parameter viscoelastic fluid with linear elements (App. A). According to Yosida et al. (1956), it was first used in snow mechanics by de Quervain (1946) to interpret the results of torsion experiments.* Bucher (1948) included a sketch of a Maxwell model (a spring and dashpot in series as shown in Fig. A1 in App. A) and used the constitutive relationship for a linear viscous fluid to find the coefficient of Newtonian viscosity for compacted snow as a function of temperature, duration of loading and a variety of types of snow, grain sizes, and ages. Interestingly, although the Maxwell model includes a spring element, Bucher made no mention of the elastic properties (or lack of them) of snow, although Yosida et al. (1948)
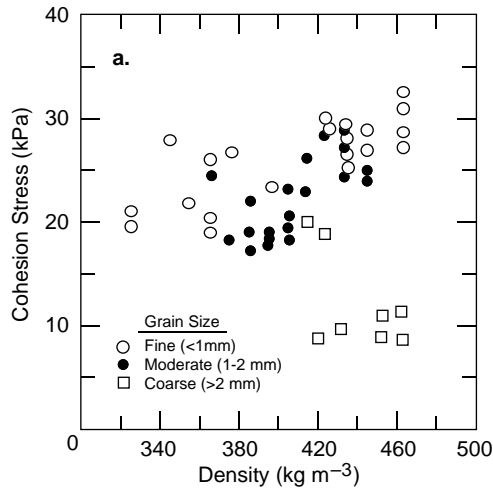
did measure Young's modulus of snow in static uniaxial compression tests. Later, Yosida et al. (1956) discussed the interpretation of the four-parameter model and found the parameters for it from creep tests on snow under uniaxial compressive stress. Bader (1962a) also suggested that the one-dimensional hyperbolic sine relationship:

$$\frac{d\varepsilon}{dt} = \varepsilon_0 \sinh(A\sigma) \tag{1}$$

(where ε is the strain, σ is the stress and t is time and ε_o and A are constants) might be used to describe creep in snow; that is, it could replace the linear relationship for the dashpot of the Maxwell element of the four-parameter model. Mellor (1964) introduced an additional term into eq 1 by dividing the coefficient of the hyperbolic sine by a viscosity coefficient, η. He also discussed the use of exponential and power relationships to represent compactive viscosity (i.e., the viscosity determined from the compaction of natural snow-packs, or from confined compression experiments in the laboratory) in terms of the snow density as derived from data sets collected by various investigators. Other determinations of the constants for the four-parameter model from creep test data have been done in Russia by Kuvaeva et al. (1967) and by Shinojima (1967). Parameters for these linear relationships, along with the available values, are summarized in Appendix B.

Even as efforts continued to find parameters for linear relationships, it was apparent that the ranges were too limited to solve many problems in snow mechanics. Bader (1962a) recognized the problem and suggested that the ranges of the linear relationships might be extended if they were applied incrementally, as the values of the parameters change with deformation. We have found no references in which attempts to use this approach were made, although Desrues et al. (1980) did devise a similar method involving simple nonlinear relationships. Mellor (1975) stated that there were still no alternatives to linear relationships, and that 1) there were no constitutive relationships for use in solving problems involving multiaxial stress states, and 2) the data to develop such relationships did not exist. He credited B. Salm with initiating efforts to address the need for such relationships. In fact, Salm (1967) did consider the extension of the hyperbolic sine relationship to cases of the creep of snow in triaxial stress states. Later Salm (1971) used the relationships in exponential form to develop a failure criterion based on energy storage and dissipa-

*Kuvaeva et al. (1967) reported that the viscosity of snow was first determined by "the group of K. S. Zavriev in 1937." Unfortunately, the reference they gave for this work appears to be incorrect and we could not locate the paper.

a. Cohesion stress vs. density.
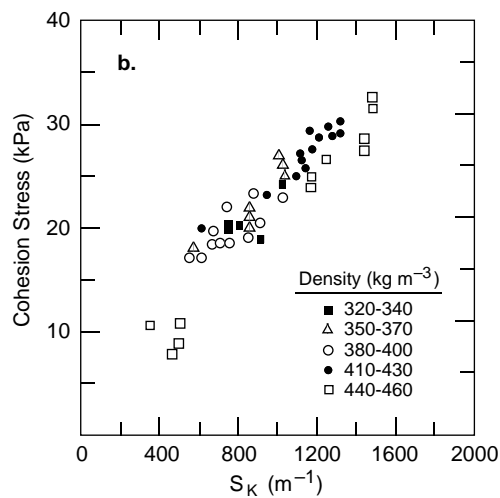
b. Cohesion stress vs. specific grain contact surface.

Figure 1. Cohesion stress vs. density and specific grain contact surface S_k for snow samples with grain sizes as indicated. S_k is defined as the net area of intergrain contacts per unit volume. The cohesion shows little dependence on the density but is approximately linearly related to S_k. Figure modified from Voitkovsky et al. (1975).

tion. Work by Brown et al. (1973), Brown and Lang (1975) and Brown (1976, 1977) had the objective of deriving constitutive relationships and failure criteria for snow from theories of nonlinear viscoelastic materials. However, little additional work in this direction has been published since these papers appeared. Instead, the effort appears to have shifted to finding constitutive relationships for snow from studies of its microstructural properties and processes.

Microstructural studies

When the parameters of linear constitutive relationships or the strength of snow are plotted against density, typically the scatter is large (App. B). This situation prompted a question to M. Mellor, following his presentation at the 1974 Grindelwald International Symposium on Snow Mechanics (Mellor 1975) as to whether the large scatter might be reduced if the influence of the snow "texture" could be accounted for. Mellor agreed, and there are many reports in the literature that indicate it is necessary to characterize the microstructure along with determining the density in order to derive indicators of the mechanical properties of snow. Yosida et al. (1956) used plate penetration experiments to demonstrate the differences in deformation between snow samples of the same density but different degrees of bonding. Voitkovsky et al. (1975) showed that the cohesion of a particular suite of samples was independent of the density, but linearly related to the contact area between grains (Fig. 1). Similarly, from his own experiments and a review of the available data, Fukue (1979) argued that snow density is not a reliable predictor of the uniaxial compressive strength, and showed (Fig. 2) that the unconfined compressive strength of manufactured snow samples increased by a factor of 10 as they sintered at constant density. Armstrong (1980), in a study of the densification of an alpine
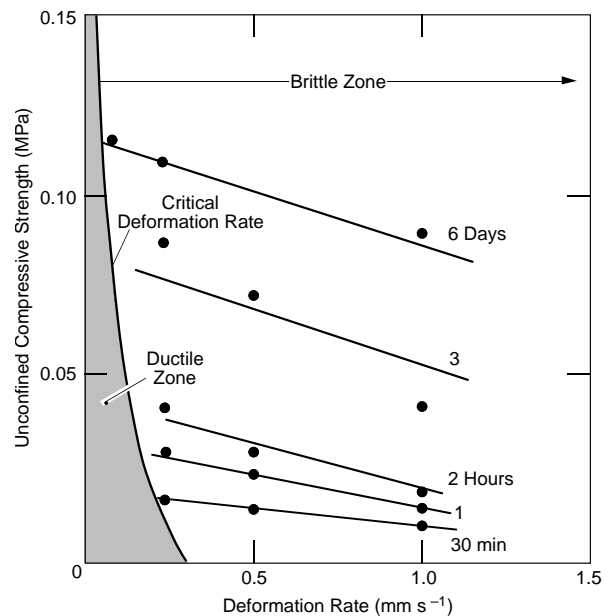

Figure 2. Unconfined compressive strength vs. deformation rate for 15-cm-long snow samples of various ages (from Fukue 1979).

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snowpack, showed that a layer of fine-grained, sintered snow deformed at a rate 10 times greater than that of an adjacent layer of depth hoar, although the densities of both layers were the same. Finally, de Montmollin (1982) argued that breaking and rapid redevelopment of bonds, even during the course of an experiment, is important in snow deformation. These examples show that regardless of the specific mechanisms involved, the bonding between snow grains (and not the density) is the critical factor in determining the response of the snow to applied loads.

The importance of snow microstructure to deformational processes has been known for many years. Bader et al. (1939) made thin sections of snow after it was deformed in order to search for changes in grain orientation that might have been attributed to deformation. Kragelski and Shakhov (1949) also recognized the importance of bonding. Yosida et al. (1956) referred to bonding in their interpretations of test results, Bader (1962a) discussed snow deformation in terms of bonding in a general way, and Kinosita (1967) showed the difference in microstructural-scale process between high- and low-rate tests in uniaxial compression. In addition, several theories based on assumptions about the processes that affect changes in bonding during deformation or over time have been derived to describe snow consolidation (Feldt and Ballard 1966, Ebinuma and Maeno 1987, Alley 1987, Wilkinson 1988) and strength (Ballard and McGaw 1965, Ballard and Feldt 1966).

Keeler (1969a,b) was the first investigator to systematically study the relationship between microstructural changes in the fabric of snow during deformation and metamorphism. He credited Eugster (1952) with doing the first thorough fabric study of snow in thin section, and Kinosita (1960) with introducing the parameter of "joint order" (the number of intersections of lines halving connecting grains), which is important in the analysis of snow fabrics. However, it was Nakaya (1961) who first tried to relate microstructure to mechanical properties when he experimentally determined the relationship between the dynamic Young's modulus and the density of processed snow, and interpreted the results in terms of the degree of bonding between grains.

Kry (1975a,b) tried to determine how the microstructure of snow changes with deformation, and how the changes affect the mechanical properties. He developed techniques and definitions to quantitatively describe the grain and bond structure of snow (see also Good 1975). In his experiments, snow samples were repeatedly deformed in uniaxial compression by rapid loading and unloading to determine values of the static Young's modulus. Next the samples were allowed to creep at constant stress and the coefficient of Newtonian viscosity was found from the relationship between stress and the "steady-state" strain rate (Kry 1975b). Each sample was deformed in stages until the strain reached about 30% and observations of the bond structure were made from samples collected at several stages. The results showed that the stress is transmitted through only a fraction of the grains, and that these are grouped into chains. Kry (1975b) hypothesized that the chains should be regarded as the basic unit of snow structure, and used that concept to interpret the variations in viscoelastic properties. At the same time, Akitaya (1974) described the "skeleton" structure of some types of depth hoar in which grains were bonded primarily in the vertical direction providing strength in vertical loading, but virtually none for lateral loads. This skeleton structure is clearly similar to the "chains" identified by Kry (1975b). Subsequently, Gubler (1978a,b) extended the idea of chains, and used it to interpret data on the tensile strength of snow.

St. Lawrence (1977, 1980), St. Lawrence and Bradley (1975) and St. Lawrence and Lang (1981) have used acoustic emissions as indirect evidence of microstructural changes to develop constitutive equations for snow. Similarly, Brown (1979, 1980) derived a constitutive relationship based on a model of collapsing pore spaces to describe the volumetric compaction of snow. Alley (1987) used a grain boundary sliding model to describe the densification of highly porous firn. Wilkinson (1988) used a density at which particle rearrangement can no longer act (about 600 kg m^{-3}) and a multi-mechanism theory of pressure sintering to describe the densification of polar firn to ice. Hansen and Brown (1986, 1987) and Hansen (1988) have also derived constitutive relationships for snow based on theoretical considerations of microscopic deformation mechanisms and measured geometric parameters of the bonds. Edens and Brown (1991), and Brown and Edens (1991) have studied the deformation of the grain bonds and produced a mathematical model to describe some of the processes involved. However, the general application of constitutive relationships based on models of the deformation on a microstructural scale is still considered to be some years in the future (Weeks and Brown 1992).

**A descriptive model for
snow deformation**

Based on the studies cited above, the microstructural deformation of a snow sample during a hypothetical experiment in uniaxial compression can be described. Our purpose is to highlight the problems involved in characterizing the deformational properties of a material like snow that can occur in many forms. For simplicity, temperature and loading rate are assumed to be constant and the grains in the sample are initially bonded into chain structures (Akitaya 1974, Kry 1975b).

There are several paths that the deformation can follow as the load increases. The bonds between grains can deform so that they thicken or thin according to their orientation with respect to the loading. Alternatively, fracture of the bonds will permit grains to be displaced with respect to each other and grains can break, changing the grain size distribution (Kinosita 1967). Bond geometry can also change by sintering at a rate that depends on the temperature and the pressure at grain contacts. In fact, for a test in which the deformation mechanisms operate at low rates, it is possible that the changes in bonding from sintering can be more important than those due to the deformation.

The overall effect of the deformation is to tighten the structure and increase the density of the snow. Concurrently, the bonding changes so that the mechanical properties of the snow can vary through a wide range of values, depending on the deformation path. For example, Salm (1977) found a 20% change in the viscosity of a snow sample due to 1% deformation in uniaxial compression.

As the deformation process continues, an *apparent* relationship between density and mechanical properties may be established. The reason this relationship seems to exist is that both the mechanical properties and the density depend on the nature of the bonding/grain contacts. Thus, it is the bonding, and not the density, that is the critical variable, suggesting that some parameter that represents the influence of the bonding should replace the density in plots of snow strength or other properties.

The macroscopic deformation of a snow sample reflects the accumulated deformation on the scale of the grain size. The relationship between the macroscopic deformation and the stress is used to determine the parameters for constitutive relationships. In general, if tests on natural snow are of short duration, then strains are small and

changes in bond structure limited. In such cases, the constitutive equations for linear-elastic, viscous or viscoelastic materials can be used to interpret the test results as described previously (see, for example, Yosida et al. 1956, Shinojima 1967, Kuvaeva et al. 1967, Kry 1975b). However, if the snow has been compacted, plowed, wind-blown, or otherwise processed and is well-bonded and of high density, then even relatively large stresses can be sustained without significant deformation or changes in bonding (Abele and Gow 1976). In these cases, linear relationships are probably applicable over a relatively large range of stresses. Unfortunately, in most applications involving natural snow, the strain is large enough that significant changes in the bonding and deformational properties occur throughout the deformational process and linear relationships apply only over a limited range of deformation. Thus, either general nonlinear constitutive relationships that span the entire range of behavior are needed or, as suggested by Bader (1962a) simple linear relationships may be used incrementally as deformation increases.

SOME CASE HISTORIES
ILLUSTRATING THE USE
OF SNOW MECHANICS

Despite the impediments that exist and make snow mechanics difficult to use, it has been applied to a diverse range of problems (vehicle mobility, foundations, tunnels, creep loading of structures, roads and runways, snow removal, impact and explosive shock loading, avalanche release, construction of snow structures, and interpretation of seismic and acoustic signals). Here we give an overview of three engineering topics that illustrate the success, and the recurring problems, in trying to apply snow mechanics.

**Snow creep forces on
avalanche structures**

One of the problems which prompted the start of formal study of snow mechanics was the determination of snow creep forces acting on fixed structures designed to prevent snow avalanches (Bader et al. 1939). Initial attempts to calculate snow forces on avalanche structures were made by assuming that a snow block, between two infinitely long containing walls, with dry sliding resistance at the base, acted on the downslope resisting structure. The model predicted a linear

increase in snow pressure with slope distance between the two containing walls because of the assumption that the downslope retaining structure supported the entire weight of the snow block less the basal frictional restraint (i.e., no frictional resistance along the side walls) (Salm 1977).

Snow deforms by viscous creep under the action of gravitational forces to increase the pressure acting on the avalanche defense structure. The contributing factors to this pressure are the static load, the creep motion of snow particles downslope, the glide of the snow cover along the ground, and the friction between the structure and the snow. Static load is produced by the transference of the vertical stresses laterally (the magnitude of the lateral stress is determined by the viscous analog to Poisson's ratio). The pressure due to creep and glide is caused by the retardation of downslope movement of the snow by a structure. The effect of this retardation is greatest at the structure and decreases with slope distance away from the structure (called the back pressure zone). Structure/snow friction retards the settlement of the snow producing a force parallel to the upright face of the structure.

Avalanche defense structures with a finite cross-slope length display three-dimensional flow of snow around the structure and will produce end-effect forces as well. End effects increase the force acting on the structure compared to an infinitely long structure (the force acting on an isolated structure asymptotically approaches that of the infinite structure as the ratio of the structure length to snow depth increases). When multiple finite length avalanche defense structures are used their force influences may overlap, allowing a reduction in the structural forces (the magnitude of force reduction depends on the separation between structures).

Initial theoretical work was done by Bader et al. (1939), Haefeli (1948, 1951) and Bucher (1948) to obtain engineering formulae to estimate the creep pressures and forces acting on infinite and finite length avalanche defense structures. These were modified as a result of later studies and empirically adjusted to include the possible range of effects due to creep, snow density, and snow depth that might occur throughout Switzerland (Salm 1960, de Quervain and Salm 1963). The theoretical developments and field measurements of forces on avalanche defense structures (Kummerli 1958) are the main basis for establishment of the Swiss guidelines for avalanche control in the starting zone (Switzerland 1990). The guidelines are a detailed engineering primer for constructing avalanche control structures and were first issued in 1955. They have been revised when warranted by new findings.

The problem of determining creep pressures on avalanche defense structures has attracted significant attention beyond the Swiss effort to establish engineering guidelines. Ziegler (1963, 1975) applied plasticity theory to determine the pressures acting on avalanche defense structures and the resulting length of the backpressure zone. Theories assuming Newtonian or non-Newtonian viscosity have been used along with field measurements to develop estimates of the pressure and pressure distribution on the upright face of avalanche defense structures and the backpressure zone around such structures (McClung 1974, 1976, 1982, 1984, McClung and Larson 1989, McClung et al. 1984, Brown and Evans 1975, Bader and Salm 1989, Larson et al. 1985, Olagne and McClung 1990).

When the Japanese tried to apply the Swiss guidelines to construct avalanche control structures in their country, they found that the Swiss guidelines were not always adequate to prevent snow creep damage to avalanche defense structures under Japanese conditions. Katakawa et al. (1992) conducted a study to determine the appropriate design factors for Japan and found that glide factors and pressure distributions on structures there were significantly higher (about 1.7 times greater) than in Switzerland. These results point to the empirical nature of avalanche structure design and the fact that the results of the extensive Swiss efforts were fully applicable only to snow very similar to that found in Switzerland.

Vehicle mobility in snow

Mobility of ground vehicles is defined as the efficiency with which a vehicle travels between two points of interest. While this may include a broad range of factors, the essence of mobility is the balance of traction and motion resistance. Traction is the ability of the vehicle's running gear to engage the terrain and the strength of the terrain to resist horizontal shear deformation. These combine to generate horizontal thrust from which a vehicle may move forward, accelerate, tow loads, climb hills, or do other useful work.

Many sources give rise to resistance and they constitute a tax on the vehicle's available power. Resistance sources internal to the vehicle are varied and generally well known (e.g., drive train gear losses, tire flexing or track bending resis-

tance), allowing vehicle manufacturers to design for them. For travel in deformable terrain, such as most snow, the dominate source of resistance is deformation and displacement of the surface. By virtue of the terrain surface not being able to support the running gear contact pressure, the vehicle must sink to a level where adequate support can be found. Thus, the vehicle is perpetually attempting to drive itself out of a rut.

The mechanical aspects of snow important to mobility are the ability to support vertical loads and its resistance to horizontal shear displacement. These two requirements are closely related, and hinge on the bearing capacity and shear strength of natural and compacted snow. The adhesion between a vehicle's running gear and the snow is also a factor in some situations, but is usually only a small contributor to traction. Effective running gear will shear the snow within the snowpack, since the thrust available there is nearly always greater than adhesion.

The bearing capacity of natural snow (density less than 400 kg m^{-3}) is usually very low compared to the needs of a vehicle for support. This results in considerable snow deformation (nearly all in the form of compaction) leaving the vehicle founded some distance below the snow surface. Compaction and vertical sinkage proceed until the pressure bulb (sharply defined zone of influence under the running gear) reaches a height that can provide enough vertical shear area to make up the difference between the natural snow's bearing capacity and the load placed by the running gear.

Occasionally, during growth, the pressure bulb encounters a firm snow layer or the base of the snowpack. This increases greatly the effective bearing capacity and thus reduces the vertical shear area required. When a very firm base (either soil, pavement, or ice) is contacted by the base of the pressure bulb, the snow depth is considered shallow (note that this depth is dependent on the combination of vehicle and snow type). For some shallow snow conditions it is possible for vehicle loads to force the pressure bulb of compacted snow beyond the confines of the vertical projection of the edges of the running gear. This only occurs when vehicle load is significantly greater than snow bearing capacity and the depth to a firm substrate is small. In all other circumstances the pressure bulb maintains essentially vertical side walls that are aligned exactly with the lateral boundaries of the running gear.

For shallow snow it has been shown the pressure bulb has virtually constant properties for a very wide range of vehicles and snow types (Blaisdell et al. 1990). Sinkage z is predicted by

$$z = h\,[1 - (\rho_0 / \rho_f)] \tag{2}$$

where h = snow depth
  ρ_0 = undisturbed snow density
  ρ_f = the pressure bulb density.

Pressure bulb density ρ_f for shallow snow is essentially a constant (critical density) at 500 kg m^{-3} (Young and Fukue 1977). A balance of forces requires that

$$L = S_{bc}\,A_h + S_s\,A_v \tag{3}$$

where L = vertical load
  S_{bc} = the natural snow's bearing capacity
  S_s = the shear strength of the natural snow/pressure bulb interface
A_h and A_v = the horizontal and vertical areas of the pressure bulb.

This describes vertical equilibrium in the snowpack. The height of the pressure bulb H can be incorporated in the A_v term; sinkage z and H are also related. However, the usefulness of eq 3 is limited by the unknowns S_{bc} and S_s. Further limitations are the unknown properties of the pressure bulb and the fact that S_s is a shear force developed between two dissimilar snow masses.

Once sinkage equilibrium is reached, the vehicle running gear can engage to produce horizontal shear to generate forward thrust. In shallow snow the available horizontal shear strength was derived empirically in Blaisdell et al. (1990). This was possible because of the "constant" pressure bulb properties found in the shallow snow condition.

Results of field mobility tests in deep snow are very limited. The deep snow case is considerably more difficult since the pressure bulb has no firm base to assist in supporting the normal and shear forces. Additionally, during horizontal shearing it is common for some portion of the top of the pressure bulb to be removed by shear displacement. This upsets vertical equilibrium (eq 3) and the vehicle suffers greater sinkage. This process is called slip sinkage and explains why tracked vehicles operating at even a small degree of slip always assume a "bow up–tail down" attitude in deep snow.

While most mobility researchers agree that motion resistance in snow is related principally to

the sinkage (volume of compacted snow) there has yet to be a reliable mathematical description. Richmond et al. (1995), Richmond et al. (1990), and Blaisdell et al. (1990) have tried many empirical and analytical possibilities but acknowledge no better than 25% accuracy on average. Some vehicle types show much larger divergence between measured and predicted resistance. Mobility measurements can even differ widely for snows that have the same density and similar physical characteristics. Closer inspection usually shows that these differences are the result of differences in the internal strength of the snows brought on by variable compaction or sintering histories. Penetrometer and direct shear tests have occasionally been used in an attempt to document snow strength. However, these are isolated attempts and none have been shown by themselves to accurately determine expected snow/vehicle behavior. Thus, numerical models have begun to appear (Mohamed et al. 1993 and Xu et al. 1993). These models hold promise for greater accuracy and insight in describing mechanical interaction between the snow and a vehicle's running gear. However, these models are currently limited by the need for complex snow load response data that in general does not exist. For use, sophisticated and case-specific tests are performed to obtain these data. No systematic library of these test data is maintained.

Snow roads and runways

The most practical, and perhaps widespread application of snow mechanics is for the creation of snow roads and runways. Animal herds produced the first snow "roads," having recognized the reduced energy expenditure associated with traveling along narrow compacted paths. Humans traveling over snow-covered terrain followed this approach and, using snow shoes, skis, or boots, packed trails to increase travel efficiency. Mechanical techniques were sought by humans to produce robust snow roads beginning when beasts were harnessed to conveyances and continuing when mechanical locomotive devices evolved. Upon the refinement of motor vehicles and the advent of aircraft, the focus moved from modifying the snow to removing the snow. Today, only persons interested in off-road travel and polar operators still required snow roads and runways.

Abele (1990) produced a thorough review of the topic of snow roads and runways. His review highlights the fact that compaction and snow milling (with snowblowers) were the only successful means used to routinely generate strong snow pavements. Using additives has always been popular; however, these rarely provide a long-term benefit (Lee et al. 1989) Studies of snow pavement technology at the time of those reports, and continuing to now, were largely empirical.

Since Abele's review, there have been a few advances. A successful experimental effort was completed to build a snow runway on deep snow at the Australian Antarctic base Casey (Russell-Head and Budd 1989). Compaction in layers was used to build up a pavement of snow that withstood proof rolling by a cart that simulated a loaded C-130 Hercules aircraft. A prototype snow runway was also produced in the Ross Sea area of Antarctica using sequential compaction efforts governed by seasonal ambient temperature changes (Blaisdell et al. 1992). This group took advantage of warming temperatures to place increasing loads on thin (10-cm) snow layers via a heavy pneumatic tire roller (glacial ice provided a rigid reaction base for the roller). Rest periods of at least 24-hours were interspersed between compaction rolling to allow new interparticle bonds to form. Densities of about 600 kg m^{-3} were the maximum attained and strengths adequate to support a test landing by C-130. Lack of near-melting temperatures and the ever-present strong temperature gradient limited bond development and thus the ultimate strength of the snow.

Lang et al. (in press) performed a series of tests using a variety of snow processing tools. They used snow tillers, of the type used by the ski industry for reconstituting ski slopes, and a snow blower. Minimal compaction was done, in contrast to the emphasis in all prior studies. This study appears to be the first to attempt to identify the intergranular processes occurring as a result of processing and subsequent aging. Using stereology and mechanical index tests (penetrometer), Lang et al. (1996) tried to correlate intergranular bond and grain size changes with strength changes. They also correlated these changes with ambient temperature changes.

The study by Lang et al. (in press) was successful in producing some of the strongest snow pavements ever recorded. However, snow strength was difficult to quantify, owing to the difficulty in using a penetrometer in hard, dense snow. Larger scatter in the data were apparent, and occasionally, the penetrometer could not be forced into the snow. In addition, it was found that the current state of stereological software is inadequate for making determinations of snow's me-

chanical properties. Some positive correlation was found among the stereology results and mechanical tests, but stereology factors were misleading.

ASSESSMENT OF THE CURRENT STATE OF SNOW MECHANICS

Our assessment of the current state of snow mechanics is pragmatic: Is snow mechanics being used for practical engineering, and if not, why not? We compare snow with other materials, both natural and man-made. For those materials where mechanics is being used, we find that there are extensive compilations of data and tabulated parameters for constitutive relationships that describe the deformational behavior under many loading conditions. No comparable compilations exist for snow, and existing parameters for constitutive relationships are either limited in range of applicability, or untested. Further, we do not see the research activity necessary for rapid improvement. In short, our view is that the field is little used and relatively stagnant at present.

The most comprehensive source of both data and parameters for linear constitutive relationships for snow are the reviews by Mellor (1975, 1977). There have been few new determinations of values since his were published (see App. B). Mellor (1975, 1977) recognized the importance of the microstructure in controlling snow's mechanical properties, but no data relating them to the microstructural features existed. Therefore, Mellor had to present the results plotted against snow density. But, as we discussed above, density is a poor predictor of the mechanical properties. Not surprisingly, the values of constitutive parameters show large scatter: commonly 100% to 300% (Fig. B1–B7; App. B). As a result of the large scatter, an engineer seeking to use linear constitutive relationships to solve problems cannot expect satisfactory solutions.

The usefulness of the nonlinear constitutive relationships, mostly developed between the 1960s and late 1970s, is also limited. While it has been demonstrated in the literature that reasonable equations can be derived to fit particular data sets, there are no examples of the resulting constitutive relationships having been shown to fit other data sets. Without such independent tests, there can be little confidence that the nonlinear relationships can be applied generally to solve problems. Also, many of the nonlinear constitutive relationships require parameters for which no compilations of numerical values exist. An engineer wishing to use the relationships for a particular application would be faced with the formidible task of having to determine the parameters for the particular type of snow of his or her application.

Developing constitutive relationships based on snow microstructure and micro-mechanical processes is still a relatively young field and the question of its ultimate utility is still open. Two points, however, are already clear: 1) stereological analysis is both difficult and tedious, and 2) there is uncertainty in how measured stereological values relate to the actual microstructural state of the snow. The latter point implies that stereology is more suited for establishing microstructural indexes than for describing the true microstructural state of the snow (App. C). Also, microstructural descriptions of snow deformational behavior suffer from the same lack of data and independent testing as the nonlinear constitutive relationships and theories. As a consequence, any possibility of deriving constitutive relationships for general use from microstructural analysis is far in the future.

In summary, our general view of the state of the field is that snow mechanics is in a relatively static condition at present. We think that this reflects the fact that the existing experimental data are limited and constitutive relationships are not sufficiently developed to describe the behavior of snow over its full range of deformation and loading conditions. This situation is partly due to the many different types of snow that exist over a wide range of environmental conditions and the broad range of deformation behaviors. In addition, the majority of existing data do not include independent variables that reflect the influence of the microstructure or include sufficient information about the characteristics of the snow to which the data apply. Finally, there is not, at present, a workable method of relating the easily observed physical features of snow (cf. Colbeck et al. 1990), to its deformational response to an applied load. Finally, funding for snow mechanics research, which has always been sparse, has further declined. This, in turn, has reduced the number of workers in the field and limited the research opportunities of those who remain. As a result, the scope of research at present is relatively narrow and the prospects for expanding applications are limited.

For the field of snow mechanics to find wider application, investigators must be able to identify and classify the type(s) of snow involved in a

problem, locate information on the expected deformational behavior for the conditions of the problem, and have access to numerical values of the parameters for constitutive relationships that are applicable. Our suggested approach to filling this need is given in the next section.

AN APPROACH TO SNOW MECHANICS RESEARCH

Introduction

We believe that for the present, the goal of engineering snow mechanics research should be to develop a comprehensive source of data on the mechanical properties of interest and analytic tools that can be used to solve engineering problems.* This would make it possible for investigators to 1) identify the types of snow involved in a particular problem, 2) anticipate the response of that snow to applied loads under the conditions of the problem using various measures or indices of the mechanical property of interest (see Abele 1990), 3) guide the selection of an appropriate constitutive relationship and test its usefulness, 4) find numerical values of the parameters of that relationship, and 5) determine the strength of snow in different loading modes if that is relevant to the problem. This clearly requires new data on snow in a format that currently does not exist in the literature. To provide it requires that a classification of snow be developed relating the physical characteristics of snow (e.g., grain size, grain size distribution, grain shape, density and other measures) to specific deformational behavior (e.g., compressive strength and deformation under load) that operate over known ranges of environmental conditions. The data on the deformational behavior will be needed to select appropriate constitutive relationships and their parameters for various snow types under the conditions of specific problems.

The classification must be based on features that can be determined objectively and repeatably by direct observation or by simple measurements. The physical characterization of snow should be familiar (e.g., *International Classification of Seasonal Snow on the Ground* [Colbeck et al. 1990]), and measures of mean grain size, grain size distribution, snow crystal morphology, bulk snow structure, and density are appropriate. To categorize snow types by their deformational behavior, the classification should also include information on microstructure and bonding that most influence deformational processes. Unfortunately, there are no suitable variables that provide unambiguous microstructural information (App. C) so it will be necessary to use index properties that depend on microstructure instead. These are not true properties of the material, but are the numerical results of simple tests that are correlated with the deformational behavior of interest.

Once a classification is established, the deformational behavior of each class of snow can be characterized. This would involve collecting representative stress–strain–time-strength data for samples of snow from each class in different loading modes and rates. When available over a sufficient range of conditions, the data would be useful for selecting constitutive relationships and determining their parameters. Initially, the testing might be restricted to a representative range of conditions to demonstrate the styles of deformation for each class of snow.

Establishing independent variables or index properties for snow microstructure

Index properties are the results of simple tests that are correlated with the deformational behavior for snow. According to Salm (as cited in Oakberg 1982) in order for the results of some test to be useful as an index property it is necessary to establish the following:

1. The results of the test depend on the microstructure of the snow, although it is not necessary to know exactly how that dependence arises.
2. The results are repeatable and can be done in a field setting, either in-situ or with portable equipment that minimizes the need to handle the snow.
3. The numerical range of the test results is large enough to discriminate across the scope of possible seasonal snow types as they appear in various environmental conditions.
4. The test results can be shown to vary systematically with the mechanical properties by demonstrating, for example, that they

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*A mechanical property of interest is defined as that property most relevant to a particular snow mechanics application (e.g., high rate uniaxial compaction for impact and explosive problems, compaction and shear deformation for mobility and avalanche release studies, creep deformation behavior to determine loads on snow fences and structures, and other data useful in dealing with a particular engineering problem).

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are correlated with a parameter such as the uniaxial compressive strength at selected rates of loading.

Several types of measurements that might serve as index properties are described in Appendix D. They include electrical properties, disaggregation energy, sonic wave propagation velocity, and various methods for measuring the penetration hardness. Based on previous experimental results, all but the disaggregation energy have some promise as index measures of microstructure, but we believe that an adaptation of the blade penetration force suggested by Fukue (1979) is the best of these. Fukue (1979) used a relatively thick, short blade to demonstrate that the penetration force was linearly related to the uniaxial compressive strength (App. D, Fig. D3). Other penetrating devices (the most common of which is the Rammsonde penetrometer) require that relatively large volumes of snow be compacted or displaced ahead of the advancing penetrometer (Huang et al. 1993). Thus, they are sensitive to the shape and rate of advance of the penetrometer, the properties of the snow, and the manner of interaction between the penetrometer and snow (which can vary during a test). Rammsonde penetration is complicated and does not appear to be consistently related to any particular mechanical property (see discussion in App. D). However, we believe that the reason that Fukue (1979) obtained good results was that his blade penetrometer interacted with the snow on a scale that was not much larger than the microstructural elements. Even better results may be possible by using a thinner, longer blade that brings the scale of the interaction closer to that of the microstructure and increases the number of bonds and grains that the blade contacts. We have done preliminary experiments which indicate that a thin-walled cylinder penetrometer may work as well as a blade, yet provide sufficient strength to penetrate hard snow.

The snow microstructure involves the properties of the bonds between grains and the manner in which they are coupled into larger structures (such as chains), the shapes, sizes and size distribution of the grains, and other variables. Thus, since no single physical property uniquely defines the microstructure, it is reasonable to expect that more than one index property will correlate with various modes of deformational behavior that are controlled by the microstructure. For example, in addition to the penetration force of blades or thin-walled tubes described above, index properties based on sonic wave propagation speed, electrical conductivity, and some stereologically derived variables may also correlate well with the deformational behavior of snow in some regimes. Because of this we anticipate that values of good index properties may correlate with each other and the microstructral factors that affect deformational behavior. This also allows for the possibility that test results related to index measures may eventually be related directly to microstructurally important variables, when accurate methods to determine them are more fully developed.

**A classification of
snow for applications**

A classification of snow for engineering applications consists of a physical classification (i.e., snow crystal size, shape, type, structure, free water content, density, and other relevant features) combined with a deformational classification. The deformational classification would be obtained from index property measurements as described in the last section and would give information about the microstructure and bonding of the snow. In practice the classification would provide a means to develop classes of snow in which deformational behavior and physical characteristics are related. Experiments to acquire stress–strain–time–strength data for the classes in the classification would then give the range of deformational behavior for each snow class.

A possible model for such a classification was given by Bader et al. (1939). They separated snow into 10 classes using qualitative measures of grain size and bond strength as discriminators (Fig. 3). In effect, this is a classification based on a physical property of snow and a parameter that may be an index property of the microstructure. Bader et al. (1939) intended the classification for use in identifying snow types in the field, and were not attempting to classify by deformational behavior. However, Kuvaeva et al. (1967) and Fukue (1979) have suggested classifications of seasonal snow types according to anticipated deformational response to applied loads. Both authors require only four classes of snow which are similar in both classifications and are comparable to some of the classes in the classification of Bader et al. (1939). Neither author reported having done any systematic work leading to establishing the classification, but the fact that they are similar and were derived independently may indicate that
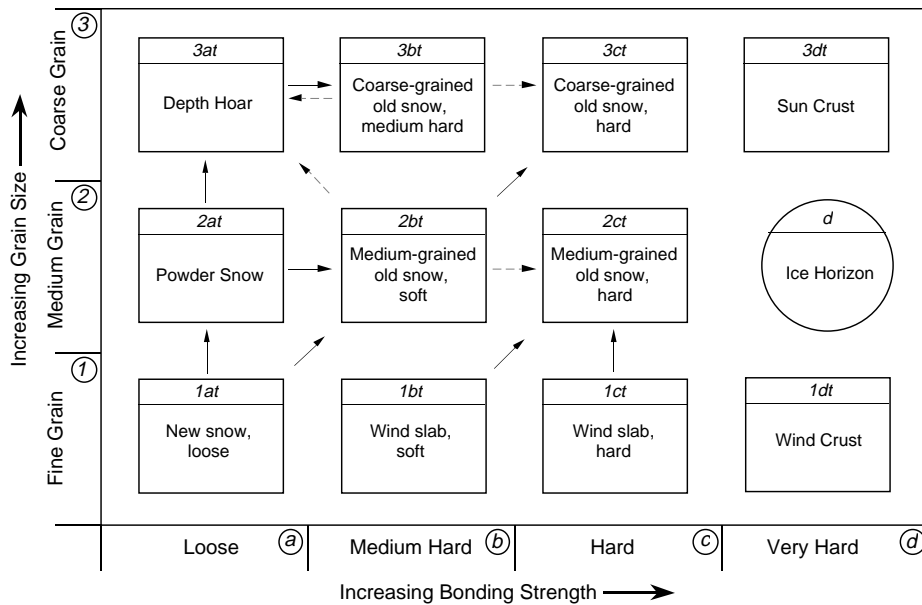
Coarse Grain (3)

| 3at — Depth Hoar | 3bt — Coarse-grained old snow, medium hard | 3ct — Coarse-grained old snow, hard | 3dt — Sun Crust |

Medium Grain (2)

| 2at — Powder Snow | 2bt — Medium-grained old snow, soft | 2ct — Medium-grained old snow, hard | d — Ice Horizon |

Fine Grain (1)

| 1at — New snow, loose | 1bt — Wind slab, soft | 1ct — Wind slab, hard | 1dt — Wind Crust |

Increasing Grain Size →

Loose (a)   Medium Hard (b)   Hard (c)   Very Hard (d)

Increasing Bonding Strength ⟶

Figure 3. Classification of snow from Bader et al. (1939)

the number of classes in a classification such as we suggest might not be excessive.

Testing and test data

We anticipate that initially the experiments to demonstrate the range of deformational behavior of the various snow classes will be unconfined and confined uniaxial compression at various loading rates. The confined compression tests would provide useful information on compaction of different types of snow, and the data from the unconfined tests (including the compressive strength) could be used for comparison with earlier work, and for determination of the parameters of some of the linear stress-strain relationships.

Regardless of which parameters are selected as discriminators in the classification, there will be gradients rather than sharp boundaries between snow classes. In addition, it is likely that stress–strain–time–strength data for some classes will overlap under some conditions, indicating that the deformational behavior of those types of snow are similar for those cases. This might reduce the number of classes needed to sort snow types according to deformational behavior, although it would still be necessary to establish the range of conditions over which the overlap occurs. The effort required to do this might ultimately be equivalent to constructing a "deformation map" for each snow class. These would be similar to the maps used to show how the defor-

mation mechanisms for specific materials vary with test conditions (as an example, Wilkinson [1988] includes a recent deformation map for pure ice). Information needed to evaluate the quality of testing includes details about the test apparatus, the sample, test procedure, measurement methods and errors, and data analysis methods. A preliminary discussion of the information that should accompany an experiment to measure mechanical properties is given in Smith et al. (1982).

RECOMMENDATIONS AND CONCLUSIONS

We believe that the most important step that can be taken to encourage the expanded use of snow mechanics in applied problems is to develop a readily accessible source of data on the response of snow to applied loads. This would provide the information to allow engineers to treat problems using available numerical data in the absence of reliable constitutive relationships and would make information available to assist in the development of constitutive relationships. We recognize that developing such a comprehensive data set is a daunting task, but we do not think the situation will improve unless such an effort is made. The approach that we recommend involves the following steps:

1. Develop methods to measure index properties that are sensitive to variables of the snow

microstructure that influence deformational behavior. The methods should be suitable for field use, and their relevance as indicators of response to load needs to be demonstrated by comparison with the results of experimental measurements of properties, such as the strength under some standard set of conditions.

2. Develop a classification system for snow in terms of familiar, descriptive physical properties and values of the index properties described above. Suitable physical properties might include mean grain size, grain size distribution, snow crystal morphology, bulk snow structure, density, and/or other properties that can be determined in the field.

3. Conduct tests to gather stress–strain–time–strength data on representative samples from the various classes in the classification.

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The motive for making many of the measurements of the mechanical properties of snow has been to find parameters for applying the constitutive relationships for ideal elastic solids and viscous fluids to problems involving snow deformation. The purpose of this appendix is to describe these "ideal" materials and show how the relationships that describe their response to stress are related to more general constitutive relationships. Note that we use the term "ideal" to describe these materials and their constitutive relationships because they are idealizations based on data from experiments on many types of materials. Thus, their origin is empirical, rather than analytical.

For our purposes we only need to consider ideal elastic solids and viscous fluids under isothermal conditions. An ideal elastic solid is defined by the property that the strain at any time depends only on the instantaneous magnitude of the stress and is independent of the stress history. Further, if the stress is removed from a deformed sample of an ideal elastic material, then the strain disappears, the sample returns to its original state and all the strain energy stored during deformation is recovered. Note that the definition does not require the stress-strain relationship to be linear. However, determining a value of Young's modulus (E) for snow from a static test in uniaxial loading includes the assumption that it is linear, because the experimental data are fit to the one-dimensional form of Hooke's law, $\sigma = E\varepsilon$ where σ is stress and ε is strain in the same direction. Similar arguments apply to the other elastic constants (the shear and bulk moduli, Poisson's ratio and Lame's constant, λ).

Ideal linear viscous behavior is represented by a constitutive equation in which the stress is proportional to the strain rate. As a result, the strain at any time depends on the complete history of the stress, rather than its instantaneous magnitude. Further, when the load is removed from a sample of a linear viscous fluid undergoing deformation, the strain rate goes to zero and none of the strain is recovered. Thus, no strain energy is stored during deformation, and all of the work done by external stresses is nonrecoverable.

The one-dimensional constitutive relationship for a homogeneous, isotropic, linear viscous fluid is $\sigma = \eta(d\varepsilon/dt)$ where η is the coefficient of Newtonian viscosity. Unlike the case for elasticity, the parameters for nonlinear constitutive equations for viscous fluids have been determined under some conditions for both snow and ice. The relationships that have been used are the power law $\left(d\varepsilon/dt = A\sigma^n\right)$, the exponential law $\left(d\varepsilon/dt = Be\sigma^{bn}\right)$ and the hyperbolic sine law $\left(d\varepsilon/dt = C\sinh c\sigma\right)$ where A, B, b, C and c are constants that may depend on the temperature, pressure and physical properties.

The idea of using the law for what is now called the four-parameter viscoelastic fluid (called the Burger's material or general linear substance by some authors) to represent the deformation of a material was apparently first proposed by Nadai (1963, p. 166*) based on observations of experimental creep curves. He defined a material that responds to loads by "three distinct types of strain ε_1, ε_2, ε_3 and two types of stress, σ_1, σ_2 ..." The strains are 1) an ideal elastic strain that responds instantly to changes in stress, 2) a component of permanent strain that changes as a function of time and load, and 3) a semi-permanent, recoverable strain which represents a time-dependent elastic response to the applied load. The familiar spring-dashpot model for this material (shown in Fig. A1 with linear springs and dashpots) illustrates how the total strain results from the summation of the "three distinct types of strain..." listed above. The first two are from the Maxwell model, while the third is the Voigt model, (called the Kelvin, firmoviscous, or Bingham material by some authors). Summing the strain components

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*The discussion of the section titled "Composite, Viscoelastic Substance Disclosing Recovery Strains" in Nadai (1963, p. 166), begins "This leads us to propose a *third, ideal, composite, viscoelastic, recovery-sensitive substance...*" (italics his). Nadai then continues to describe the model of the four-parameter viscoelastic fluid as given in the text above. Further, in a footnote on p. 170, Nadai noted the spring-dashpot models analyzed by Burgers and referred to one that "...demonstrates our composite substance having the three types of strain..." Note that Nadai (1963) is a version of a volume which was published originally in 1931, and in revised form in 1951. There is no similar discussion in either of the earlier editions nor is there reference to earlier publication of the model, although it is was included without reference in Jaeger (1962).

in this manner implies the assumption that they are independent of each other so that, for example, changes in the magnitude of the permanent strain component with time do not affect the parameter that determines the instantaneous elastic response. To our knowledge, this effect has never been studied experimentally, but it seems likely that at some strain, the assumption will no longer be satisfied. Another point of interest is that, since the three strains are independent of each other, it is easy to omit one or two of them depending upon the application. Thus, for example, for a problem in which a small stress is applied for a long period of time, the permanent strain would become much larger than the combined instantaneous and time-dependent elastic strains. As a result, the elements that contribute these strain components can be ignored. Conversely, if the loads are applied for only a short time, then the permanent strain can be neglected with only the elastic components being retained.

Note also that the "two types of stress" referred to by Nadai (1963) are simply the stresses across the arms of the Voigt model (Fig. A1). Equilibrium requires that they sum to the magnitude of the stress, σ, applied across the model.

The summation and integration of the strains for a constant stress σ_0 applied at time $t=0$ to the

model with linear elements, leads to (Fig. A1)

$$\varepsilon(t) = \sigma_0 \left\{ \frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2} \left[1 - \exp\left(-\frac{E_2 t}{\eta_2} \right) \right] \right\}. \quad \text{(A1)}$$

A plot of this equation approximates a creep curve, so that, as described in Yosida et al. (1956), Bader (1962a), Nadai (1963) and Mellor (1964), and shown in Figure A1, the parameters in eq A1 can be determined from a single experimentally derived creep curve (although, obviously, more curves would be required for accuracy). This was done by Yosida et al. (1956) and Shinojima (1967) from experimental data for snow deformed to small strains. A similar procedure could also be used (although more experiments would be required) to determine the parameters if some or all of the model elements followed nonlinear stress-strain rate relationships.

Next, we show the relationship between the four-parameter viscoelastic fluid model and the more general constitutive relationships for non-linear viscoelastic materials. The intent is to show the assumptions required to make the transition between the different constitutive relationships.

The constitutive relationships for the four-parameter model with linear elements can be derived from conditions of equilibrium and the sum-
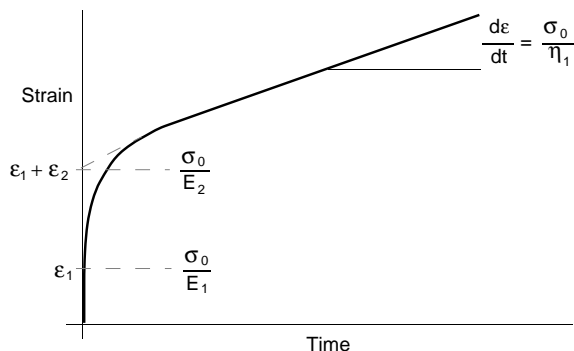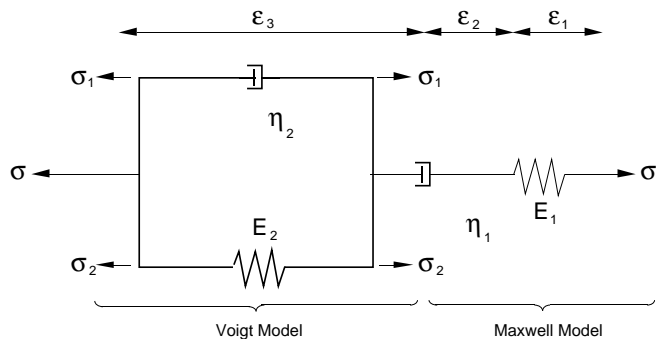

Figure A1. The four-parameter spring-dashpot model of a viscoelastic fluid showing nomenclature, stress-strain laws and relationships for determining values of the parameters from a creep curve.

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mation of the strains. As shown in Mellor (1975), in differential form it is

$$\frac{\partial^2 \sigma}{\partial t^2} + \left(\frac{E_M}{\eta_M} + \frac{E_M}{\eta_K} + \frac{E_K}{\eta_K}\right)\frac{\partial \sigma}{\partial t} + \left(\frac{E_M\,E_K}{\eta_M\,\eta_K}\right)\sigma$$

$$= E_M\,\frac{\partial^2 \varepsilon}{\partial t^2} + \left(\frac{E_M\,E_K}{\eta_K}\right)\frac{\partial \varepsilon}{\partial t} \quad . \qquad (A2)$$

in which σ and ε are understood to be functions of time. Equation A2 is a special case of the general relationship,

$$P\sigma = Q\varepsilon \qquad (A3)$$

where P and Q are the differential operators

$$P = a_0 + a_1\frac{\partial}{\partial t} + a_2\frac{\partial^2}{\partial t^2} + \ldots + a_n\frac{\partial^n}{\partial t^n}$$

$$Q = b_0 + b_1\frac{\partial}{\partial t} + b_2\frac{\partial^2}{\partial t^2} + \ldots + b_m\frac{\partial^m}{\partial t^m} \;.$$

The forms of the constants a_0, a_1,..., b_0, etc., can be determined for the four-parameter model by comparison with eq A2.

Equation A3 can be derived from the integral form of the one-dimensional stress-strain relationships for linear viscoelastic fluids,

$$\sigma = \int_{-\infty}^{t} G(t-\tau)\frac{\partial \varepsilon}{\partial \tau}\,d\tau \qquad (A4)$$

$$\varepsilon = \int_{-\infty}^{t} J(t-\tau)\frac{\partial \sigma}{\partial \tau}\,d\tau$$

where G and J are relaxation and creep functions respectively. However, there is some loss of generality in making this step (Christensen 1971) since the complete spectrum of possible creep or relaxation times in the integrals in eq A4 is replaced by a discrete number in the differential form (eq A3).

Equations A4 are the one-dimensional forms of the integrals

$$\sigma_{ij} = \int_{-\infty}^{t} G_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}\,d\tau \qquad i,j = 1,2,3. \quad (A5)$$

and its equivalent for strain in terms of stress, which are the three-dimensional forms of the constitutive relationships for linear viscoelastic materials.

The problem of defining constitutive relationships for nonlinear viscoelastic materials has been approached through both empirical and analytic means (see Lockett 1972). The former involves fitting relatively simple mathematical expressions to experimental data, which gives results with limited ranges of application. In contrast, the analytical approach is more rigorous, originating in fundamental axioms of physics. An example of an empirical relationship follows from the observation in Lockett (1972) that the equation

$$\varepsilon_t = \varepsilon_e \sinh\left(\frac{\sigma}{\sigma_e}\right) + \varepsilon_d\,t^n \sinh\left(\frac{\sigma}{\sigma_d}\right), \quad (A6)$$

where σ is stress, ε_t is the total strain, ε_e, ε_d, σ_e, and σ_d are constants, is a good representation of the creep of many plastics at the constant stress, σ. Note that if σ_e is large, then

$$\sinh\left(\frac{\sigma}{\sigma_e}\right) \approx \left(\frac{\sigma}{\sigma_e}\right),$$

so that, with the substitution $(\varepsilon_e/\sigma_e) = 1/E$, eq A6 describes a Maxwell material with a linear elastic spring and a dashpot which, for $n=1$, follows the hyperbolic sine relationship for the relationship between stress and strain rate. Equation A6 or its version with the linear spring can be extended to the four-parameter model by simply adding the contribution of the Voigt model shown in Figure A1, with either linear or nonlinear elements depending upon the data which is to be fitted to the equation. Thus, there is some flexibility to the empirical relationships with respect to fitting data, but the resulting equations could become inconvenient for solving boundary value problems.

As an illustration of the analytical approach we use the example of the stress–strain relationship for a Green-Rivlin material, because it is similar to a relationship used by Brown et al. (1973) and Brown (1976) to describe the nonlinear deformational behavior of snow. After assuming isotropy, homogeneity and deformation under isothermal conditions, the relationship can be written in matrix form as (Lockett 1972)

$$\sigma(t) = \int_{-\infty}^{t} \left\{\mathbf{I}\psi_1\,T_1 + \psi_1\,\mathbf{M}_1\right\}d\tau_1$$

$$+ \int_{-\infty}^{t}\int_{-\infty}^{t} \left\{\mathbf{I}\psi_3\,T_1\,T_2 + \mathbf{I}\psi_4\,T_{12}\right.$$

$$\left. + \psi_5\,T_1\,\mathbf{M}_2 + \psi_6\,\mathbf{M}_1\,\mathbf{M}_2\right\}d\tau_1\,d\tau_2$$

$$+ \int_{-\infty}^{t}\int_{-\infty}^{t}\int_{-\infty}^{t} \left\{\mathbf{I}\psi_7\,T_{123} + \mathbf{I}\psi_8\,T_1\,T_{23}\right.$$

$$+ \psi_9\, T_1\, T_2\, \mathbf{M}_3 + \psi_{10}\, T_{12}\, \mathbf{M}_3 + \psi_{11}\, T_1\, \mathbf{M}_2\, \mathbf{M}_3$$

$$+ \psi_{12}\, \mathbf{M}_1\mathbf{M}_2\,\mathbf{M}_3\big\}\, d\tau_1\, d\tau_2\, d\tau_3 + \ldots \qquad (A7)$$

$$\text{where} \quad \mathbf{I} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\sigma = \sigma_{ij}$$

$$\mathbf{M}_\alpha = \frac{\partial \varepsilon_{ij}(\tau_\alpha)}{\partial \tau\alpha}$$

$$T_\alpha = \delta_{ij}\, \frac{\partial \varepsilon_{ij}(\tau_\alpha)}{\partial \tau_\alpha}$$

$$T_{\alpha\beta} = \delta_{ij}\, \frac{\partial \varepsilon_{ij}(\tau_\alpha)}{\partial \tau_\alpha}\, \frac{\partial \varepsilon_{ij}(\tau_\beta)}{\partial \tau_\beta}$$

$$T_{\alpha\beta}\gamma = \delta_{ij}\, \frac{\partial \varepsilon_{ij}(\tau_\alpha)}{\partial \tau_\alpha}\, \frac{\partial \varepsilon_{ij}(\tau_\beta)}{\partial \tau_\beta}\, \frac{\partial \varepsilon_{ij}(\tau_\gamma)}{\partial \tau_\gamma}\, .$$

$$\delta_{ij} = \begin{cases} = 0 \;\; for\; i \neq j \\ = 1 \;\; for\; i - j \end{cases}$$

The functions ψ_a are relaxation functions (analogous to $G(t-\tau)$ in eq A4 in which ψ_1 and ψ_2 are functions of $t-\tau_1$, $\psi_3,\ldots,\psi_6$ are functions of $t-\tau_1$ and $t-\tau_2$, and $\psi_7,\ldots,\psi_{12}$ are functions of $t-\tau_1$, $t-\tau_2$, and $t-\tau_3$. Depending upon their form, each ψ_a could include several constants which, if the physics were known completely, might be determined analytically. However, in general, they are found by fitting the equation to experimental data. Thus, the approach is really semiempirical, rather than completely analytical. The equation could be written in inverted form, with strain as a function of stress. In that case, the functions ψ_a would be creep functions analogous to $J(t-\tau)$ above. Note that the equation is written in three dimensions although it is often reduced to one dimension for application as was done in Brown et al. (1973) and Brown (1976).

Finally, to demonstrate the connection between the relatively simple relationships in eq A3 and A4 as compared to A7, we note that, for isotropic materials, eq A5 can be simplified to (Lockett 1972)

$$\sigma_{ij} = \delta_{ij} \int_{-\infty}^{t} \lambda(t-\tau)\frac{\partial \varepsilon_{ij}}{\partial T}\, d\tau$$

$$+\, 2 \int_{-\infty}^{t} \mu(t-\tau)\frac{\partial \varepsilon_{ij}}{\partial T}\, d\tau \qquad (A8)$$

which, for constant λ and μ, reduces to the familiar relationship for a linear elastic material. However, introducing the substitutions $\psi_1 = \lambda(t-\tau)$ and $\psi_2 = \mu(t-\tau)$ puts eq A8 into the notation of eq A7 and shows that it is the linearized form of the constitutive equation for the Green-Rivlin material.

APPENDIX B: REVIEW OF THE LITERATURE ON MECHANICAL PROPERTIES OF SNOW WITH COMPILATION OF DATA

HISTORICAL DEVELOPMENT

In this appendix we use published reviews of the field of snow mechanics to trace the development of data on the mechanical properties of snow. Following the discussion, the data are given in a series of figures which have been updated from Mellor (1975, 1977).

The first attempt at a comprehensive review of the mechanical properties of ice and snow in English was presented in SIPRE Technical Report 4 (Mantis 1951). The only data given in that report were 1) the values of a static Young's modulus for snow from Yosida et al. (1948), 2) the snow viscosities from Bucher (1948), 3) some values for friction, and 4) a few tensile and shear strength magnitudes. There was also mention of unsuccessful attempts by Corps of Engineers personnel to determine some of the mechanical properties of snow by dynamic methods, although no details or references were given. SIPRE Technical Report 7 (Bader et al. 1951) included the same values of the tensile and shear strengths of snow as in Mantis (1951) but also reported the first values of the cross section number "*m*" (the reciprocal of Poisson's ratio). There was an obvious lack of quantitative information on the mechanical properties of snow available when these reports were prepared. However, there was enough information on the qualitative aspects of the behavior of snow during deformation that it was possible to outline a broad experimental program, including the design of testing equipment, to acquire additional data (Bader et al. 1951).

The next review and summary of the data on snow mechanics was published by Bader (1962a). His primary objective was to describe the general behavior of snow as a material but he also presented data on mechanical properties from static tests by Butkovich (1956) and Jellenik (1957), and from dynamic tests on polar snow by Nakaya (1959a). In addition, Bader (1962a) discussed the insights to understanding the processes of snow deformation contributed by the creep experiments under uniaxial and multiaxial stresses by Landauer (1955, 1957), and the plate indentation tests by Landauer and Royse (1956) and Yosida et

al. (1958). It is of interest that the numerical values for properties determined by Yosida et al. (1956) and some of those from Jellenick (1957) were the only new data for mechanical properties of nonpolar snow included in Bader (1962a) that had appeared in the literature since the publication of Mantis (1951).

Two years later, Mellor published his first review of the mechanical properties of snow (Mellor 1964) as part of the snow engineering section of the CRREL monograph series. In it, Mellor collected and organized data on seasonal, polar and various types of processed snow, but the only work on seasonal snow reported by Mellor (1964) and not by Bader (1962a) was from Yosida (1963). There were, however, several contributions on the properties of polar and processed snows (Butkovich 1962; Bender 1957a; Brunke 1959; Lee 1961; Nakaya 1959b, 1961; Ramseier 1963; Ramseier and Pavlak 1964; Mellor and Hendrickson 1965).

The most comprehensive review of the literature on mechanical properties of snow was done by Mellor (1975), supplemented by additional data in Mellor (1977). Aside from the results from Abele and Gow (1975, 1976) on the relationship between density and maximum principal stress, the newest reported data on the mechanical properties or strength of snow in either review paper was that by Kovacs et al. (1969). In addition, the only data on seasonal snow that had not appeared in the earlier reviews were some values of Young's modulus from Kojima (1954) that had been overlooked earlier but had appeared in Yosida et al. (1956), and the results of creep tests in torsion, uniaxial tension and uniaxial compression by Shinojima (1967).

Figures from Mellor (1975, 1977) are still the most comprehensive sources of data on the mechanical properties of snow. Some of these are presented later in this appendix with supplemental data added where possible.

The most recent review of the field of snow mechanics was done by Salm (1982). However, it was devoted mainly to describing progress in understanding the mechanics of fracture of snow and in the development of constitutive relation-

ships including fracture criteria. The only studies of snow properties cited in Salm (1982) are McClung's (1977) work on the shear strength of snow, Narita (1980) and Watanabe (1980) on the tensile deformation and strength, and an earlier paper by Tusima (1973) on the densification of snow under repeated loading.

An extensive body of work done in Russia on snow properties and processes by Kuvaeva et al. (1967) appeared in English translation in 1975 but was overlooked in the reviews described above. It is a compilation and summary of studies done between 1948 and 1962 in the Caucasus Mountains and includes material on 1) the thermal properties of a snowpack, 2) snow metamorphism, 3) avalanche forecasting, effects and defense, and 4) the mechanical properties of snow. The tests of mechanical properties were done between 1958 to 1962, and involved snow types ranging from newly fallen snow ($\rho \approx 60$–80 kg/m^3) to wind slab and firn ($\rho \approx 400$–500 kg/m^3). Representative samples were tested but the data from individual tests were not generally given. Instead, the results were presented as parameters for various constitutive relationships (i.e., as an equation for Young's modulus with density and temperature as parameters) that were stated to be within, for example, 10 or 15% of the data.

Another important work that was not reviewed earlier is the thesis by Fukue (1979), some of which was previously published in Yong and Fukue (1977). The study was broad, but emphasized the interplay between small-scale processes (friction, adhesion, sintering and creep of individual grains) and the macroscopic behavioral characteristics of snow in a variety of loading modes. The work involved experiments designed to illustrate those characteristics, but which were not comprehensive enough to define the form or parameters of constitutive relationships. However, the results were synthesized into a qualitative model of snow structure which provides a useful framework for considering and anticipating snow deformation under a variety of conditions. Fukue (1979) also proposed a classification of snow for engineering purposes, and developed and tested an experimental procedure, which we believe will be important for indexing snow properties.

The data on specific parameters are presented in the following sections. Note that in most of the figures the parameter is plotted against the density, which is the usual procedure.
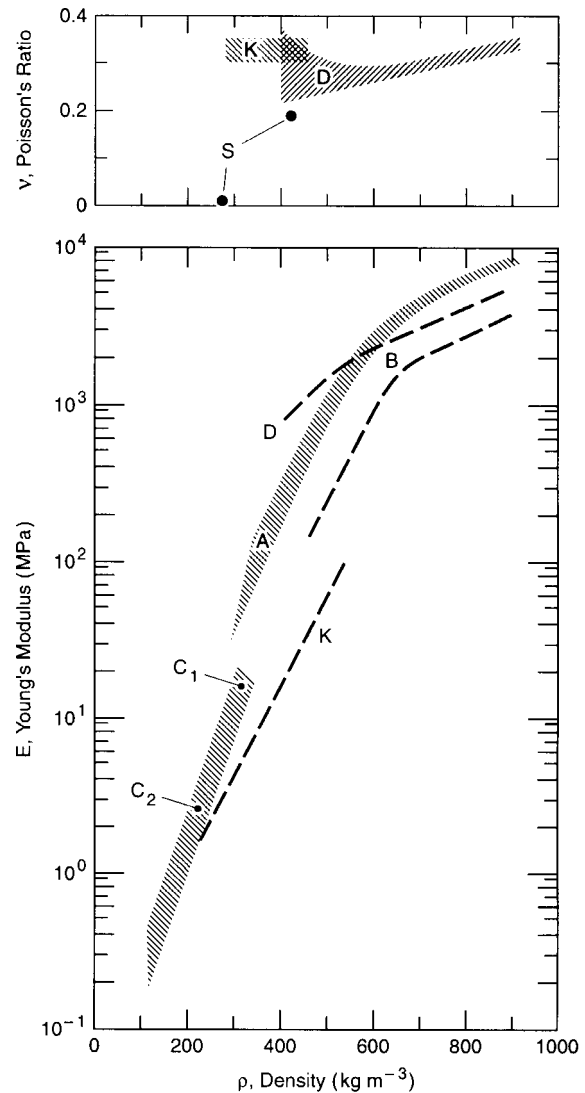

Figure B1. Young's modulus and Poisson's ratio vs. density for dry, coherent snow (modified from Fig. 2 in Mellor 1975). Data sources cited in the original figure are (A) Pulse propagation or flexural vibration at high frequencies, $-10°$ to $-25°C$ (Smith 1965; Nakaya 1959a,b; Bentley et al. 1957; Crary et al. 1962; Lee 1961; Ramseier 1963). (B) Uniaxial compression, strain rate approximately 3×10^{-3} to $2 \times 10^{-2} \text{ s}^{-1}$, temperature $-25°C$ (Kovacs et al. 1969). (C$_1$) Uniaxial compression and tension, strain rate approximately 8×10^{-6} to $4 \times 10^{-4} \text{ s}^{-1}$, $-12°$ to $-25°C$. (C$_2$) Static creep test, $-6.5°$ to $-19°C$ (Kojima 1954). (D) Complex modulus, 10^3 Hz, $-14°C$ (N. Smith 1969). (S) Quasi-static measurements of Poisson's ratio (Salm 1971). Additional data added for this report; (K) plotted from equation for best fit curve to data for static Young's modulus and quasi-static measurements of Poisson's ratio from Kuvaeva et al. (1967).

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YOUNG'S MODULUS AND POISSON'S RATIO

The plot of the data for both static and dynamic measurements of Young's modulus for snow from Mellor (1975), supplemented by additional data from Kuvaeva et al. (1967), is shown in Figure B1. In addition, the only data available from dynamic or quasistatic determinations of Poisson's ratio for snow are also included in the figure.

VISCOSITY

The difficulty of determining Poisson's ratio in rapid-loading compression tests, coupled with the extreme compressibility of low density snow, led to the introduction in Bader et al. (1951) of the parameter called the "cross section number (the reciprocal of Poisson's ratio)." Mellor (1975) identified this parameter as the viscous equivalent of Poisson's ratio and presented the available data (Fig. B2).

Mellor (1975) separated the data on viscosity into the categories of "axial" and "compactive" viscosity. The former refers to the viscosity determined from the "steady state" creep rate in experiments under constant uniaxial compressive stress. In terms of the four-parameter model,

the axial viscosity is the viscosity of the dashpot of the Maxwell model. The compactive viscosity is determined either from the results of experiments in confined compression (uniaxial strain), or from measurements of compaction as a function of time for natural snowpacks.

The data on axial viscosity are shown in Figure B3. The range of values for this parameter is large, even allowing for the differences in the physical properties of polar and seasonal snow. It should also be noted that, in addition to determining the axial viscosity, Shinojima (1967) also did creep experiments in torsion and uniaxial tension and used the results to determine the parameters for the four-parameter viscoelastic fluid model for all three loading modes. In addition, the data were used to define the viscosity of the lead dashpot of the model as a function of temperature and snow density over the range of temperatures from 0° to −40°C and densities from 125–300 kg/m^3. Note that these results were used by Lang and Sommerfeld (1977).

Mellor (1975) determined the compactive viscosity of seasonal snow from data from field measurements by Kojima (1967), and both field and laboratory studies by Keeler (1969a). He combined these results with data for polar snow from Bader (1962b), experimental work by Mellor and Hendrickson (1965) and other field studies, into his Figure 11 (p. 266). Ambach and Eisner (1985)
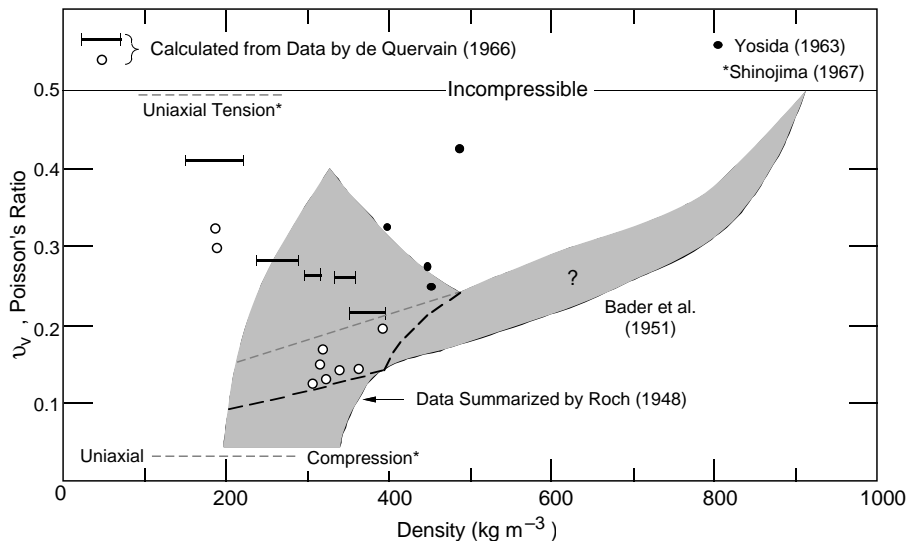

Figure B2. Mellor's (1975) summary of data on the viscous analog of Poisson's ratio. Data sources were de Quervain (1966), Roch (1948), Shinojima (1967), Yosida (1963) and Bader et al. (1951).

(a)   (b)
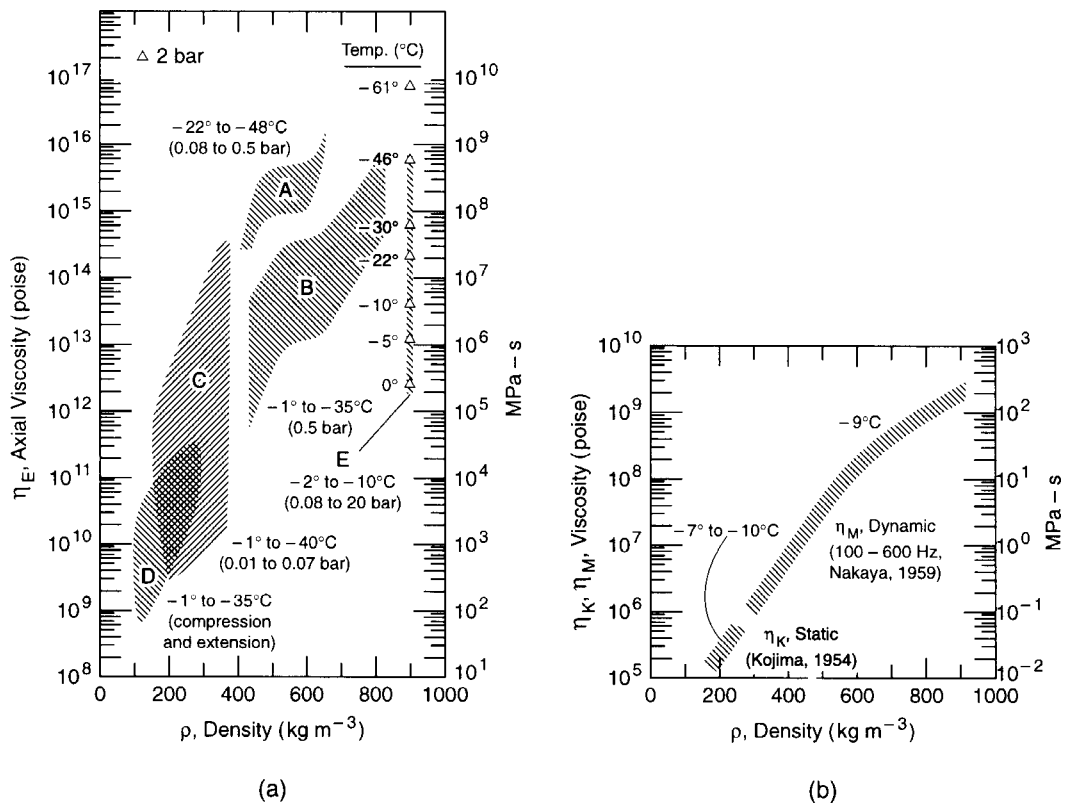
Figure B3. Axial viscosity of snow vs. density from Mellor's (1975) Figures 8 and 9. The data sources in (a) are (A) Ramseier and Pavlak (1964), (B) Mellor and Smith (1967), (C) Bucher (1948), (D) Shinojima (1967), (E) Mellor and Smith (1967) and Mellor and Testa (1969). Data sources in (b) as shown.
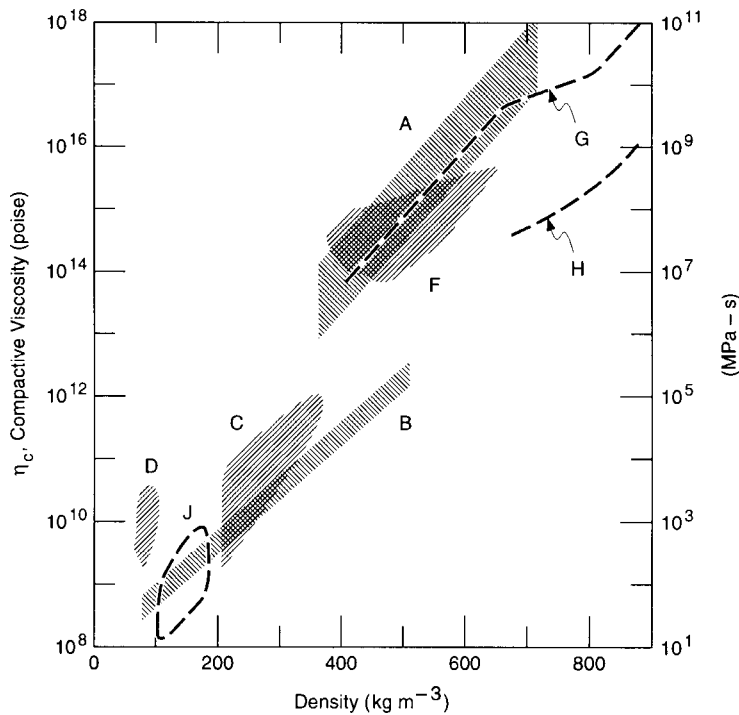

Figure B4. Compactive viscosity vs. density, modified from Mellor (1975, Fig. 11) by Ambach and Eisner (1985). Data from (A) Greenland and Antarctica at –20° to –50°C (Bader 1962b); (B) Seasonal snow in Japan at 0° to –10°C (Kojima 1967); (C) Alps and Rocky Mountains (Keeler 1969a); (D) Uniaxial-strain creep tests at –6° to –8°C (Keeler 1969a); (F) Uniaxial strain creep tests at –23° to –48°C (Mellor and Hendrickson 1965); (G) Dorr and Jessberger (1983) and Ambach and Eisner (1985); (H) Seasonal snow in Japan at 0° to –6°C (Endo et al. 1990).
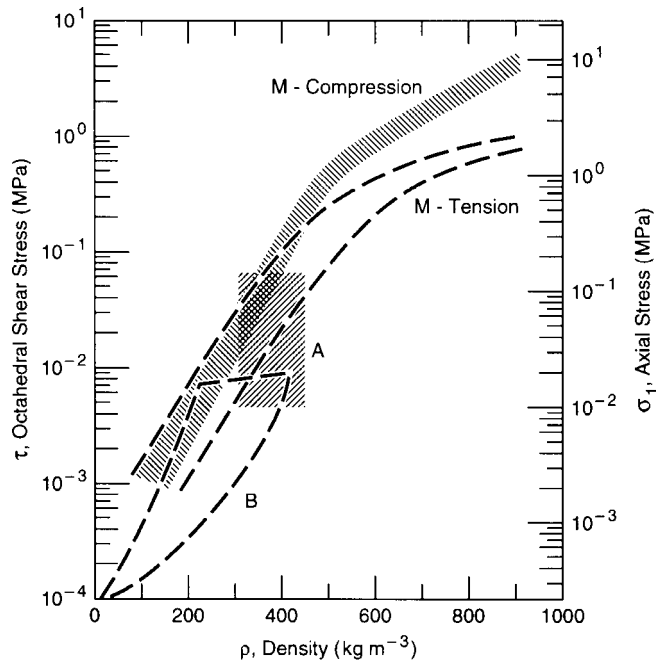
Figure B5. Uniaxial compressive and tensile strengths of snow under rapid loading rates from Mellor (1975). Original data (M-compression and M-tension) from Bucher (1948), Butkovich (1956), Haefeli (in Bader et al. 1939), Hawkes and Mellor (1972), Keeler (1969a), Keeler and Weeks (1967), Kovacs et al. (1969), Mellor and Smith (1965), Ramseier (1963), and Smith (1963, 1965). Additional data are (A) for strain rates greater than $5 \times 10^{-4} s^{-1}$ from Narita (1980) and (B) the compilation of in situ tensile strength data in Jamison and Johnston (1990).

updated that figure to include field data developed by them, as well as the results of earlier work by Dorr and Jessberger (1983). That figure is shown here in Figure B4 to which we have added data from Endo et al. (1990).

STRENGTH

In his discussion of failure, Mellor (1975) discussed the ambiguity of the term and adopted the definition that failure is "related to the maximum deviatoric stress that can be reached at a given strain rate...". He used this definition because it does not distinguish between brittle and ductile cases, and thus applies across the spectrum of possible failure modes for snow. However, he presented data only for the brittle regime (i.e., at high rates of loading) in uniaxial tension and compression, and some data for shear from various experiments and from estimates based on the analysis of avalanche fractures. The data from Mellor (1975) are shown in Figures B5 and B6 to which we have added data from later studies by McClung (1977) and Narita (1980). Note that a compilation of in situ measurements of tensile strength of snow has recently been prepared by Jamieson and Johnston (1990) and has been added to Figure B5.
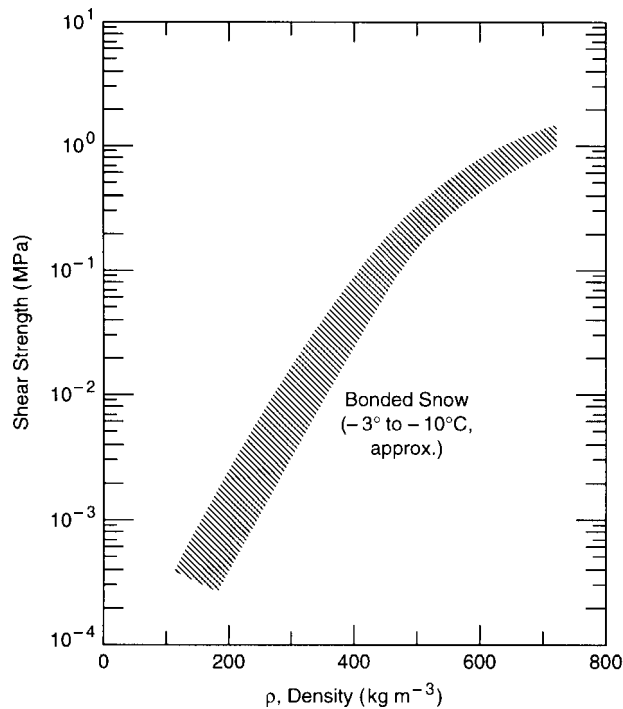

Figure B6. Shear strength of snow from Mellor (1975). Original data from Ballard and McGaw (1965), Butkovich (1956), Haefeli (in Bader et al. 1939), Keeler (1969a), Keeler and Weeks (1967).

CONFINED COMPRESSION (UNIAXIAL DEFORMATION) AND BEARING STRENGTH TESTS

The relationship between the major principal stress and the density in uniaxial strain has been of concern in the subject of snow mechanics because it simulates, to some extent, the process of densification of natural snowpacks and is important in considerations of the bearing strength of snowpacks. Bader (1962a) included a discussion of the data from plate bearing tests by Landauer and Royse (1956), Yosida et al. (1957) and Bucher and Roch (1946) as they relate to this subject. Mellor (1964) discussed some of the material on plate indentation, and treated the combined subject thoroughly in Mellor (1975). Experimental work has since been done on the stress vs. density relationship by Abele and Gow (1975, 1976) (reviewed in Mellor 1977) and Yong and Metaxas (1985).

Mellor (1975) produced a plot of maximum principal stress vs. density for the range of densities from less than 100 kg/m³ to 900 kg/m³ (the comparable stress range is from about 0.1 kPa to 1 MPa) by combining 1) densification curves from numerous unspecified locations, 2) miscellaneous data on the compression of ice and firn, 3) the data which had appeared in Bader (1962a), 4) shock experiments and 5) new data by Kinosita (1967). Data from static loading experiments cover a significant range of the density, which was expanded further in the work of Abele and Gow (1975, 1976). Those authors included the effects of initial density, temperature, strain rate and aging. A plot from the latter paper which shows the present state of the data is given in Figure B7. In a later work, Yong and Metaxas (1985) studied the effect of strain rate and aging on the deformation of snow in confined compression and direct shear. The range of strain rates they used (about 3×10^{-3} to 1×10^{-2} s^{-1}) overlapped part of the range of rates used by Abele and Gow (1975), so the results may be comparable. The data in Yong and Metaxas (1985) showed the effect of strain rate, with the stress at a strain of 35% elevated at the lower rates; however, the differences may not be significant for strain less than about 20–25%. In addition, the influence of aging of the samples was clearly shown by the increase in the stress at any strain as the samples aged. Abele and Gow (1975, 1976) did not find any influence of strain rate in the range from about 2×10^{-3} s^{-1} to 11 s^{-1}, but their data do show the same trend of the influence of aging as those of Yong and Metaxas (1985).
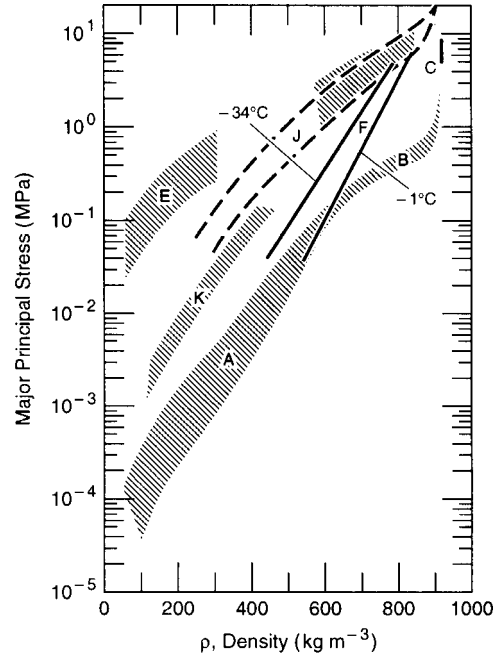

Figure B7. Stress vs. snow density data from various sources from Mellor (1975) as modified by Abele and Gow (1976). Data fields are (A) natural densification of snow at –1° to –48°C, (B) Slow natural compression of dense firn and porous ice (from depth/density curves for polar ice caps), (C) Slow compression of solid ice, (E) Calculated values for plane wave impact at 20–40 m s⁻¹. (F) Hugoniot data for explosively generated shock waves (impact velocities 1 to 12 m s⁻¹) at –7° to –18°C; (J) Compression at approximately constant strain rate e ≈ 10⁻⁴ s⁻¹ at –7° to –18°C (Kinosita 1967); (K) Compression in uniaxial strain with incremental loading to collapse, –2° to –3°C (Bucher and Roch 1946). Heavy lines labeled –1°C and –34°C are the boundaries of the data of Abele and Gow (1975, 1976).

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APPENDIX C: DETERMINATION OF MICROSTRUCTURAL VARIABLES BY PLANE SECTION STEREOLOGY

Independent variables measured for snow undergoing mechanical testing should represent fundamental states or conditions of the snow, including relevant macroscopic and microscopic features. Examples of such state variables might be temperature, density, mean grain size (and or grain size distribution), mean number and radial cross-sectional area of grain bonds per grain, the mean number and length of unsupported chains of grains, the mean distance between grains, and other physically measurable characteristics. While determining macroscopic state variables can be done reliably, measuring microscopic variables and determining their significance is problematic.

Attempts to define and determine state variables using plane section and thick section stereology have demonstrated the importance of snow microstructure on the response of snow to an applied load (Kry 1975a,b; Gubler 1978a,b; Alley 1986; Good 1987; Dozier et al. 1987; Hansen and Brown 1987; Hansen 1988; Edens and Brown 1991; Brown and Edens 1991). However, there are two difficulties with using stereological methods that need to be taken into account when evaluating their usefulness. First, the accuracy with which stereological methods can be used to determine many of the parameters they are intended to represent is, in general, unknown. While bulk density and parameters related to the two-dimensional plane section can be determined, extending these measures to a three-dimensional representation of microstructural measures requires assumptions about the geometry of the material which cannot be rigorously tested (Alley 1986). The accuracy of the determination of the average number of grains per unit volume (N_v) strongly depends on the grain shape. Errors of more than an order of magnitude in the value of N_v can result from only small variations in grain shape (Dehoff and Rhines 1961), and seasonal snow consists of grains of complex and variable shapes. Thus, estimates of N_v, and variables that depend upon values of N_v from plane section stereology should be considered inaccurate (Alley 1986). Ex-

amples of such variables are measures relating to chains of grains, the number of bonds per grain and others.

Other problems arise because the assumptions used to convert measurements from plain section or thick section images into the desired microstructural parameters are often overly simplified and may contain free variables (Gubler 1978a,b; Alley 1986). In addition, the lack of objective methods for identifying many structural features in snow using plane section and thick section stereology has been well recognized (Kry 1975a, Alley 1986, Dozier et al. 1987).

Stereological methods require relatively large investments in time and effort to construct plane section or thick section samples, obtain good quality images, and analyze the results. Because of these difficulties few experimental studies relating stereological parameters to the deformational behavior of snow have been published since the work of Voitkovsky et al. (1975) and Kry (1975a,b). Voitkovsky et al. (1975) presented a relationship between cohesive force and the number of intergrain contacts per unit volume, based on about 50 tests in five distinct density categories while Kry's (1975b) results were based on repeated experiments on only five snow samples. Gubler (1978a,b) presented results for the tensile strength in terms of microstructural variables from fewer than twenty tests, and changes in microstructure under large deformations have been examined in two studies (Edens and Brown 1991, Brown and Edens 1991).

We consider it unlikely that microstructural measures truly represent those features that directly control deformational behavior. They may eventually be used to derive indexes of deformational behavior, in the same manner as the index parameters discussed in the main text in *Establishing Independent Variables or Index Properties for Snow Microstructure*, but the effort required to measure microstructural variables suggests that other more easily measured properties should be sought.

ELECTRICAL PROPERTIES

Kuroiwa (1962) described an experiment in which the components of the complex dielectric constants and the AC conductivity of a snow sample were measured several times over a period of 143 hours while the snow sintered at a temperature of –3°C. Over the frequency range from less than 10^3 to about 10^4 Hz the constants varied significantly and the conductivity increased by a factor of 10 during the experiment. The changes were interpreted as due to the shortening of the electrical paths because of bond growth (Fig. D1), suggesting that the AC electrical conductivity is sensitive to the bonding and could be a useful index property. In addition, since it is a directional property, the AC conductivity may also give information about the degree of anisotropy of the snow. In other work, Keeler (1969a) found no relationship between the dielectric properties and the structure of the snow he studied. Denoth (1985) found that the static dielectric constants depended strongly on porosity grain shape

and liquid water content; the effects of sintering were not directly observed.

The DC conductivity also shows some possibility of being useful as an index property, although it is more sensitive than the properties noted above to the presence of impurities in the snow (Mellor 1977).

ELASTIC WAVE VELOCITY AND DYNAMIC ELASTIC MODULI

Nakaya (1961) showed that the dynamic Young's modulus of processed snow was related to the bond structure of the snow, and Voitkovsky et al. (1975) found that the wave propagation velocity of snow samples undergoing creep deformation increased with time as the snow density and grain contact area increased (Fig. D2). These
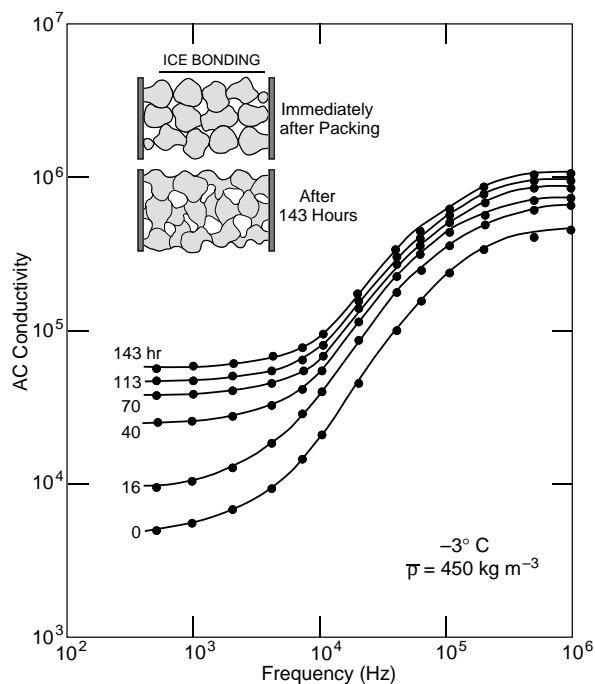

Figure D1. AC conductivity of an aging snow sample as a function of frequency. Measurements were made at the indicated elapsed times and the change in structure of the sample over 143 hours is indicated schematically by the accompanying sketch (from Kuroiwa, in Bader 1962a).
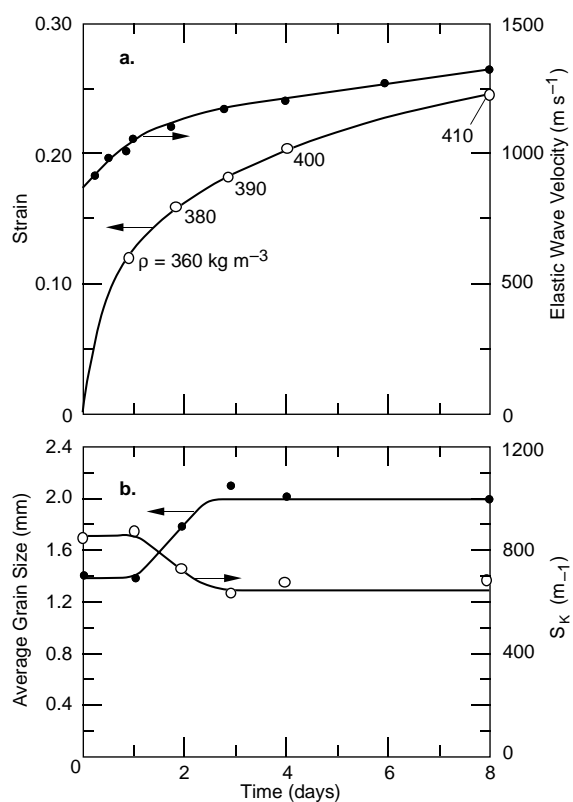

Figure D2. (A) Creep strain at constant stress and elastic wave velocity (V) with changing density (ρ), (B) average grain size (d) and specific grain contact surface (S_k) as a function of time (t) in days. Measurements of ρ, V, d and S_k were made periodically as a series of samples deformed at constant stress (from Voitkovsky 1975).

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results indicate that seismic velocity (or the dynamic elastic moduli calculated from the velocity data and the density) is a potential index property.

SNOW HARDNESS

Hardness, as indicated by a penetrating cone (e.g., Swiss Rammsonde, Russian AARI penetrometer), flat plate penetrometer (Kragelski 1949) or by the hand-hardness scale (de Quervain 1950), has been used for many years as an index parameter in a semi-quantitative sense. Quantitative interpretations have been hampered by difficulties in interpreting the results of measurements with these devices because the results depend on the shape of the penetrometer, the size of penetrometers of the same shape (Kragelski 1949) and other variables. Abele (1963) attempted to derive a quantitative correlation between Rammsonde hardness and the uniaxial compressive strength of snow but was not successful. Later, Waterhouse (1967) reanalyzed Abele's data using alternative approaches but was still not able to establish a reliable correlation. He concluded that his inability to account for the structure of the snow was a major reason for the lack of success. In fact, Gubler (1975) concluded that it is not possible to define a unit of hardness that is independent of the instrument used to measure it. However, if it is possible to establish a relationship between the nature of the bonding in a range of snow samples, and the hardness of those samples as measured by any particular instrument, then the hardness values as determined by that instrument could be useful as an index parameter.

Snow hardness is relatively easy to measure in the field and for that reason it is worth seeking a measurement of that type that can be quantitatively related to the snow structure. A test that may be useful for the purpose is the measurement of the blade penetration force as suggested by Fukue (1979). He measured the force required to push a thick blade into a snow sample at a constant rate and then showed that the results were related to both the bonding between grains and the uniaxial compressive strength (Fig. D3). Similarly, Kovacs (1976 and pers. comm.*) noted that the resistance to driving hollow piles in polar snow was closely correlated with the uniaxial compressive strength of the snow. This suggests, by

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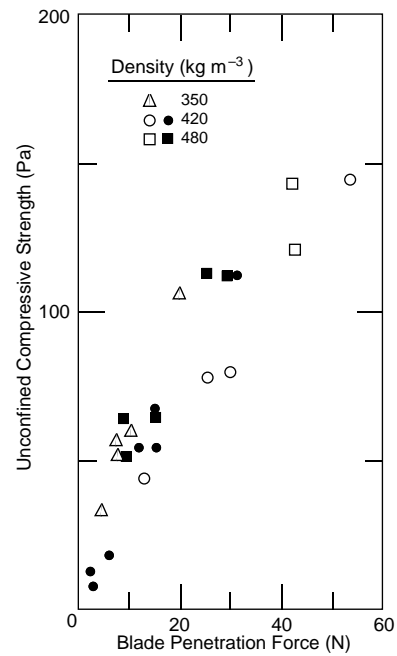*A. Kovacs, CRREL, personal communication 1992.

Figure D3. Uniaxial compressive strength of snow samples vs. blade penetrating force (from Fukue 1979). The linear relationship indicates the potential value of the blade penetrating force as an index property.

analogy, that it might be useful to experiment with penetrometers in the form of thin-walled hollow tubes.

Kuvaeva et al. (1967) described another penetration technique but, unfortunately, without giving the dimensions of the equipment. They pushed a small spherical penetrometer into snow samples up to some value of a parameter they define as the hardness, H (see Fig. D4). They then allowed the penetrometer to settle further under its own weight and calculated H at a series of later times, using the weight and dimensions of the penetrometer and assumptions about the manner in which the displacement was partitioned between different mechanisms. Eventually, the hardness became asymptotic to some value, designated as H_∞, which was presented in a table along with the tensile and shear strengths of the samples and information on grain sizes and bonding. The samples were of different densities and grain sizes, and the temperatures varied through the tests. However, the plot of H_∞ against the values for the strength measurements (Fig. D4) shows that the relationships may be linear, which suggests that this technique should be investigated further as a possible index test.
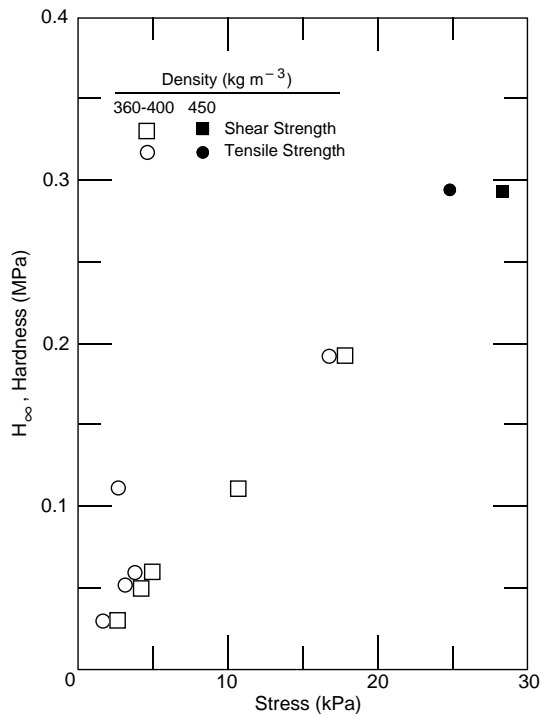
Figure D4. Snow hardness, H_∞ *(see text for definition) vs. shear strength and tensile strength at densities indicated. Data from Kuvaeva et al. (1967).*

PERMEABILITY

Permeability has long been regarded as an important property of snow. Bader et al. (1939) showed that several types of snow fall into distinct fields on a plot of permeability vs. porosity, and permeability has also been shown to vary with grain size and morphology (Bender 1957b; Bader 1962a; Chacho and Johnson 1987). It is possible that the contact area between grains could affect the permeability, since increases in contact area can decrease the fraction of the cross section of a sample that a fluid can pass through, and possibly increase the tortuosity. However, changes in permeability might be independent of the extent of grain bonding rather than simply the contact area. Thus, two snow samples with the same density, grain size distribution and grain morphology might have the same permeability but very different mechanical properties. For these reasons, we do not consider permeability alone to have potential as an index property. However, it may be of use when interpreted in conjunction with other index properties.

ENERGY OF DISAGGREGATION

A device to measure the energy of disaggregation (in effect, the work done in separating the grains in a snow sample) was described in Mantis (1951) and was used to establish a relationship between that parameter and the uniaxial compressive strength of snow (Bender 1957a, as described in Mellor 1964). Other investigators have had less success (see discussion in Mellor 1964) and the method has received little attention since that time. However, it was suggested in both Mellor (1977) and Oakberg (1982) that the energy of disaggregation be studied again as a possible index parameter.

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For conversion of SI units to non-SI units of measurement consult ASTM Standard E380-93, *Standard Practice for Use of the International System of Units,* published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

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13. ABSTRACT (*Maximum 200 words*)

A review of snow mechanics indicates that, with the exception of avalanche studies, it is seldom used. In this report we give our interpretation of why this is the case, and suggest ways to help expand the range of problems to which snow mechanics can be applied. Until the late 1960s, most experimental work in snow mechanics was devoted to finding values of the parameters for equations of linear elasticity, viscosity, and viscoelasticity. In about 1970, work on that approach stopped and since then the emphasis has been on 1) the development of nonlinear theories to describe the deformation and fracture of snow, and 2) attempts to develop constitutive relationships based on the study of the microstructural aspects of snow deformation. We believe that the best hope of encouraging more applications for snow mechanics in the near term lies in improving and expanding the database on the response of snow to applied loads, and organizing it in a manner that makes it easy for potential users to determine the anticipated deformational behavior of snow in any particular application. To do this, we suggest developing a classification of snow based on physical properties and index parameters that give information about the bonding and microstructure. Mechanical properties, constitutive relations under various loading conditions, and other relevant information can then be associated with each class.

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