Diabatic Processes

- Diabatic processes are non-adiabatic processes such as
  - precipitation fall-out
  - entrainment and mixing
  - radiative heating or cooling
**Parcel Model**

\[
\frac{d\theta}{dt} = \frac{L}{c_p \bar{\pi}}(C - E_r) + D_\theta
\]

\[
\frac{dw}{dt} = -(C - E_r) + D_w
\]

\[
\frac{dl}{dt} = C - A_r + D_l
\]

\[
\frac{dr}{dt} = P_r + A_r - E_r + D_r
\]

\[\bar{\pi} = \left(\frac{\bar{p}}{p_0}\right)^{R/c_p},\]  
\[C\] is the net condensation rate,  
\[E_r\] is the rain evaporation rate,  
\[A_r\] is the cloud-to-rain water conversion rate,  
\[P_r\] is the convergence of rain water flux,  
and  
\[D_i\] represents the effects of entrainment and mixing.
Microphysics

- Water vapor
- Cloud droplets
- Rain drops

- Condensation
- Evaporation
- Collection

Evaporation: $E_r$
Condensation: $C$
Collection: $A_r$

Fall out: $P_r$ (precipitation)
Diabatic Processes

Process rates per unit time interval:

\[ A_r \equiv \left( \frac{dl}{dt} \right) \text{conversion to rain} = \left( \frac{dr}{dt} \right) \text{conversion from cloud water} \]

Process rates per unit pressure interval:

\[- \frac{dl}{dp} = \hat{C} - \hat{A}_r + \hat{D}_l \]

\[-\hat{A}_r \equiv \left( -\frac{dl}{dp} \right) \text{conversion to rain} = -C l, \]

for \( dp/dt < 0 \) only, with \( C = 2 \times 10^{-2} \text{ mb}^{-1} \).
Entrainment

Entrainment is the incorporation of environmental air into a parcel or cloud.

Fig. 13. Illustration of entrainment and mixing in small cumulus clouds. Key characteristics: initial entrainment and mixing near edges, simultaneous but discrete large-scale entrainment events due to cloud-scale eddies, subsequent homogenization of regions 10–100 m in length.
Evidence for Entrainment in Cu

Instruments that can reveal the fine structures of clouds (Figs. 6.10 and 6.11), indicate that adiabatic cores, if they exist at all, must be quite rare. Air entrained at the top of a cloud is distributed to lower levels as follows. When cloud water is evaporated to saturate an entrained parcel of air, the parcel is cooled. If sufficient evaporation occurs before the parcel loses its identity by mixing, the parcel will sink, mixing with more cloudy air as it does so. The sinking parcel will descend until it runs out of negative buoyancy or loses its identity. Such parcels can descend several kilometers in a cloud, even in the presence of substantial updrafts, in which case they are referred to as penetrative downdrafts. This process is responsible in part for the "Swiss cheese" distribution of LWC in cumulus clouds (see Fig. 6.6). Patchiness in the distribution of LWC in a cloud will tend to broaden the droplet size distribution, since droplets will evaporate partially or completely in downdrafts and grow again when they enter updrafts.

Over large areas of the oceans stratocumulus clouds often form just below a strong temperature inversion at a height of $\frac{1}{H} \approx 0.5$ – 1.5 km, which marks the top of the marine boundary layer. The tops of the stratocumulus clouds are cooled by longwave radiation to space, and their bases are warmed by longwave radiation from the surface. This differential heating drives shallow convection in which cold cloudy air sinks and droplets within it tend to evaporate, while the warm cloudy air rises and the droplets within it tend to grow. These motions are responsible in part for the cellular appearance of stratocumulus clouds (Fig. 6.13).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6_10.png}
\caption{High-resolution liquid water content (LWC) measurements (black line) derived from a horizontal pass through a small cumulus cloud. Note that a small portion of the cumulus cloud had nearly an adiabatic LWC. This feature disappears when the data are smoothed (blue line) to mimic the much lower sampling rates that were prevalent in older measurements. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 105.]
}\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6_11.png}
\caption{Blue dots are average liquid water contents (LWC) measured in traverses of 802 cumulus clouds. Squares are the largest measured LWC. Note that no adiabatic LWC was measured beyond $\frac{1}{H} \approx 900$ m above the cloud base. Cloud base temperatures varied little for all flights, which permitted this summary to be constructed with a cloud base normalized to a height of 0 m. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 106.]
}\end{figure}
Evidence for Entrainment in Cu

6.3 Cloud Liquid Water Content and Entrainment

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Over large areas of the oceans stratocumulus clouds often form just below a strong temperature inversion at a height of \(0.5 - 1.5\) km, which marks the top of the marine boundary layer. The tops of the stratocumulus clouds are cooled by longwave radiation to space, and their bases are warmed by long-wave radiation from the surface. This differential heating drives shallow convection in which cold cloudy air sinks and droplets within it tend to evaporate, while the warm cloudy air rises and the droplets within it tend to grow. These motions are responsible in part for the cellular appearance of stratocumulus clouds (Fig. 6.13).

Fig. 6.10: High-resolution liquid water content (LWC) measurements (black line) derived from a horizontal pass through a small cumulus cloud. Note that a small portion of the cumulus cloud had nearly an adiabatic LWC. This feature disappears when the data are smoothed (blue line) to mimic the much lower sampling rates that were prevalent in older measurements. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 105.]

Fig. 6.11: Blue dots are average liquid water contents (LWC) measured in traverses of 802 cumulus clouds. Squares are the largest measured LWC. Note that no adiabatic LWC was measured beyond \(900\) m above the cloud base. Cloud base temperatures varied little for all flights, which permitted this summary to be constructed with a cloud base normalized to a height of 0 m. [Adapted from Proc. 13th Intern. Conf. on Clouds and Precipitation, Reno, NV, 2000, p. 106.]

Fig. 6.12: Schematic of entrainment of ambient air into a small cumulus cloud. The thermal (shaded violet region) has ascended from cloud base. [Adapted from J. Atmos. Sci. 45, 3957 (1988).]
Entrainment in Stratocumulus

Cloud Microphysics

Entrainment of warm, dry air from the free troposphere into the cool, moist boundary layer air below plays an important role in the marine stratocumulus-topped boundary layer. The rate at which this entrainment occurs increases with the vigor of the boundary layer turbulence, but it is hindered by the stability associated with the temperature inversion.

Figure 6.14, based on model simulations, indicates how a parcel of air from the free troposphere might become engulfed into the stratocumulus-topped boundary layer. As in the case of cumulus clouds, following such engulfment, cooling of entrained air parcels by the evaporation of cloud water will tend to drive the parcel downward.

Exercise 6.2

Derive an expression for the fractional change in the potential temperature of a parcel of cloudy air produced by a fractional change in the mass of the parcel due to the entrainment of mass of unsaturated ambient air.

Solution:

Let the parcel of cloudy air have temperature $T$, pressure $p$, and volume $V$, and the ambient air have temperature $T$ and mixing ratio $w$. The heat $dQ$ needed to warm the entrained air of mass $dm$ is

$$dQ = c_p (T_{9258} - T_{11032}) dm$$
Entrainment in Stratocumulus

Entrainment in a 3D high-resolution simulation of Sc.
Entrainment: Kelvin-Helmholtz Instability
Entrainment into a turbulent jet
droplet evaporation
molecular diffusion
turbulent deformation
entrainment
saturated parcel
Fractional Rate of Entrainment

\[
\frac{10 \times 100 + 0 \times 20}{100 + 20} = 8.33
\]
Fractional Rate of Entrainment

\[
\frac{(10 \times 100 + 0 \times 20)}{(100 + 20)} = 8.33
\]
Entrainment

The fractional rate of entrainment of a parcel of mass $m$ that entrains a blob of mass $dm$ while the pressure changes by $-dp$ (due to ascent) is

$$\hat{\lambda} \equiv -\frac{1}{m} \frac{dm}{dp}.$$

The rate of change of a scalar $\phi$ due to entrainment is

$$\hat{D}_\phi \equiv \left(-\frac{d\phi}{dp}\right)_{\text{entrainment}} = -\hat{\lambda}(\phi - \phi_e),$$

where $\phi_e$ is the value of $\phi$ in the entrained air.
Entrainment

We can derive this from

\[
\left( -\frac{d\phi}{dp} \right)_{\text{entrainment}} = \lim_{\Delta p \to 0} \frac{\phi_{\text{after ent}} - \phi_{\text{before ent}}}{-\Delta p}
\]

using

\[
\phi_{\text{before ent}} = \phi
\]

and

\[
\phi_{\text{after ent}} = \frac{m\phi + \Delta m \phi_e}{m + \Delta m}.
\]
Entrainment

Substitution gives

\[
\left(- \frac{d\phi}{dp}\right)_{\text{entrainment}} = \lim_{\Delta p \to 0} \frac{1}{m + \Delta m} \frac{\Delta m}{\Delta p} (\phi - \phi_e)
\]

\[
= \frac{1}{m} \frac{dm}{dp} (\phi - \phi_e)
\]

\[
= -\hat{\lambda} (\phi - \phi_e).
\]

\[
\hat{D}_\theta = -\lambda (\theta - \theta_e),
\]

\[
\hat{D}_w = -\lambda (w - w_e),
\]

\[
\hat{D}_l = -\lambda (l - l_e) = -\lambda l.
\]
Entrainment

• In cumulus clouds, the fractional rate of entrainment, \( \lambda \equiv (1/m) \, dm/dz \), ranges from about 0.1 km\(^{-1}\) to 2 km\(^{-1}\).

• Cloud-top height is largely determined by \( \lambda \): deep clouds are associated with small values, and shallow clouds with large values.

• Field studies suggest that \( \lambda \sim 0.2/R \), where \( R \) is the cloud radius.
Diabatic Processes

\[ 0 = c_p \pi \Delta \theta + L \Delta w \]

\[ w^{n+1} = w_s(\pi \theta^{n+1}, p^{n+1}) \]

which can be written as

\[ 0 = c_p \pi (\theta^{n+1} - \theta^*) + L (w^{n+1} - w^*) \]

\[ w^{n+1} = w_s(\pi \theta^{n+1}, p^{n+1}) \]