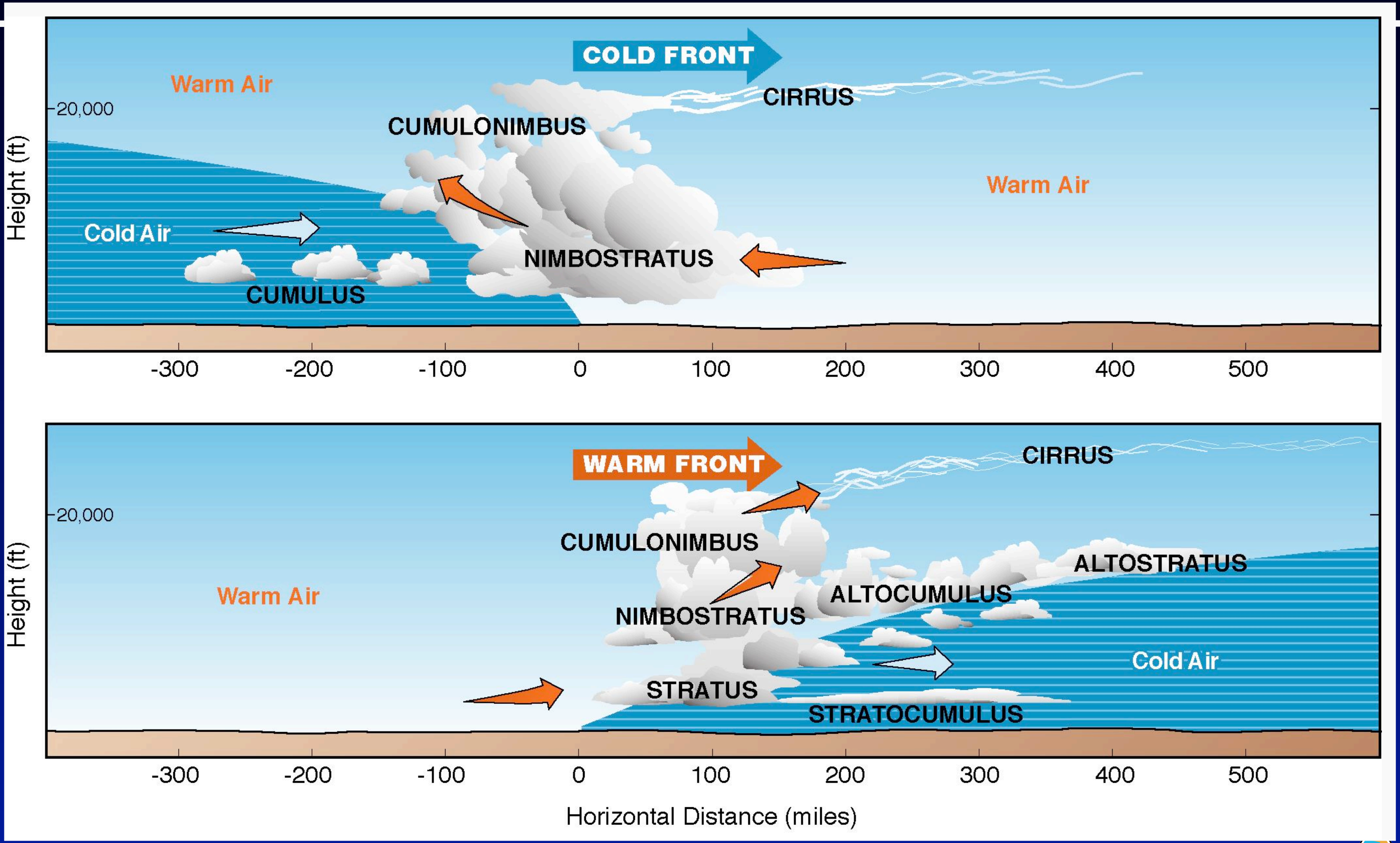
Clouds associated with cold and warm fronts



Whiteman (20

METEOROLOGY

Dalton’s law of partial pressures

- The total pressure exerted by a mixture of gases equals the sum of the partial pressure of the gases

$$p = p_{0_2} + p_{N_2} + e$$

$$e = p_{H_2 0} \ (vapor\_pressure)$$

- Partial pressure – pressure a gas would exert if it alone occupied the volume the entire mixture occupies
- Meteorologists differentiate between “dry” gas partial pressure and water vapor partial pressure (vapor pressure)

$$p = p_d + e$$

Mixing ratio

- Measure of the amount of water vapor in the air
- Ratio of mass of water vapor to the mass of dry air in a volume of air

$$w \equiv m_v / m_d$$

- Units g/kg (use g/g or kg/kg in calculations)
- Typical values
  - Midlatitude winter = 1-5 g/kg
  - Midlatitude summer = 5-15 g/kg
  - Tropics = 15-20 g/kg
- Conserved following parcel motion if there is no net condensation/evaporation

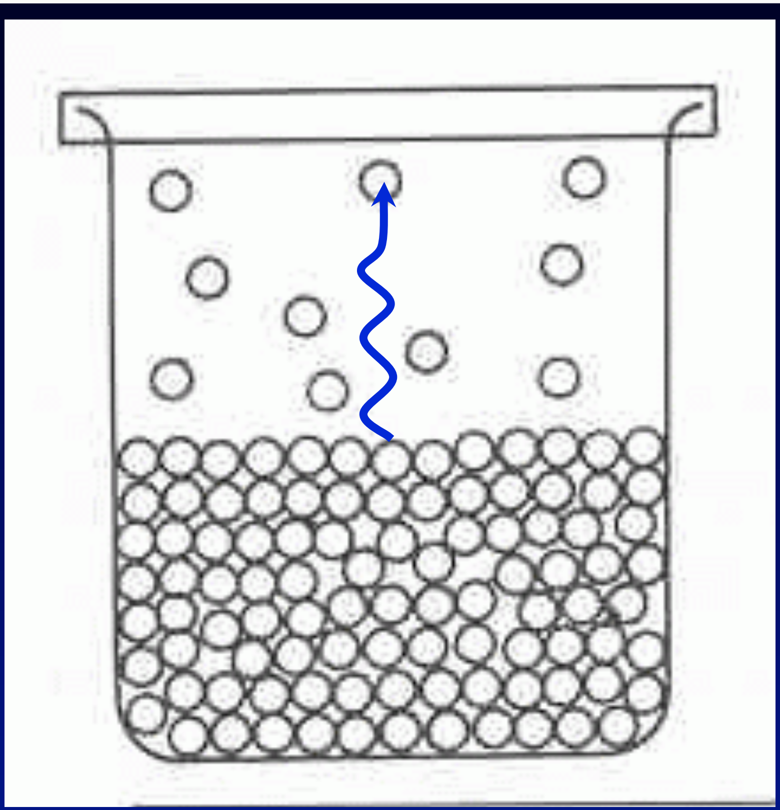
Mixing ratio/vapor pressure relationship

- Relationship between mixing ratio and vapor pressure

$$w = 0.622 \frac{e}{p - e} \qquad e = \frac{w}{w + 0.622} p$$

- Assuming mean sea-level pressure (1013 mb)
  - Midlatitude winter = 1-5 g/kg ~ 1.5-8 mb
  - Midlatitude summer = 5-15 g/kg ~ 8-24 mb
  - Tropics = 15-20 g/kg ~ 24-32 mb
- Thus, e<<p

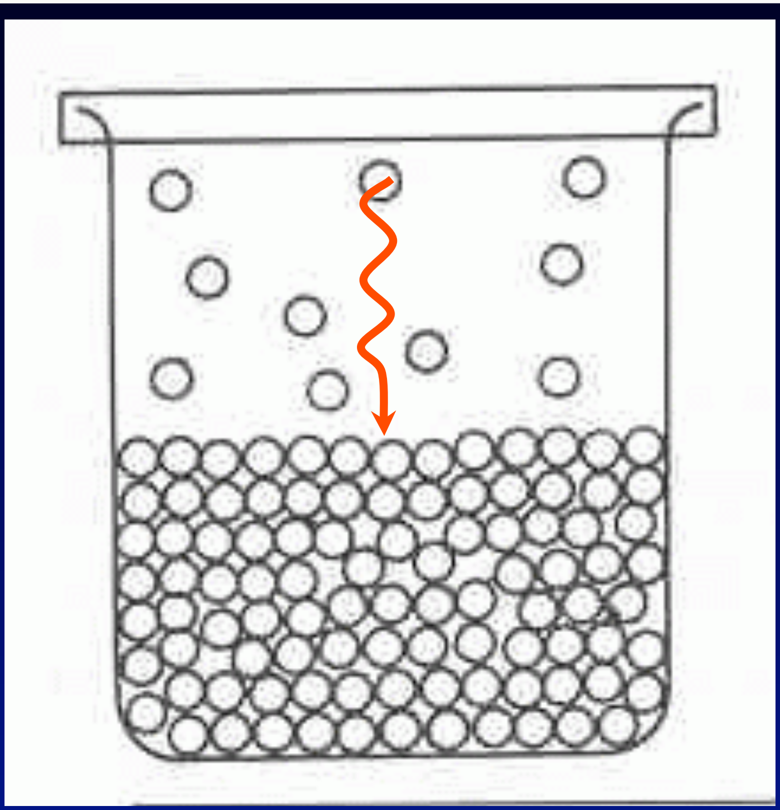
Evaporation



Bohren (1987)

- Water molecules moving from liquid to vapor phase
- Accompanied by latent cooling
- Always occurring

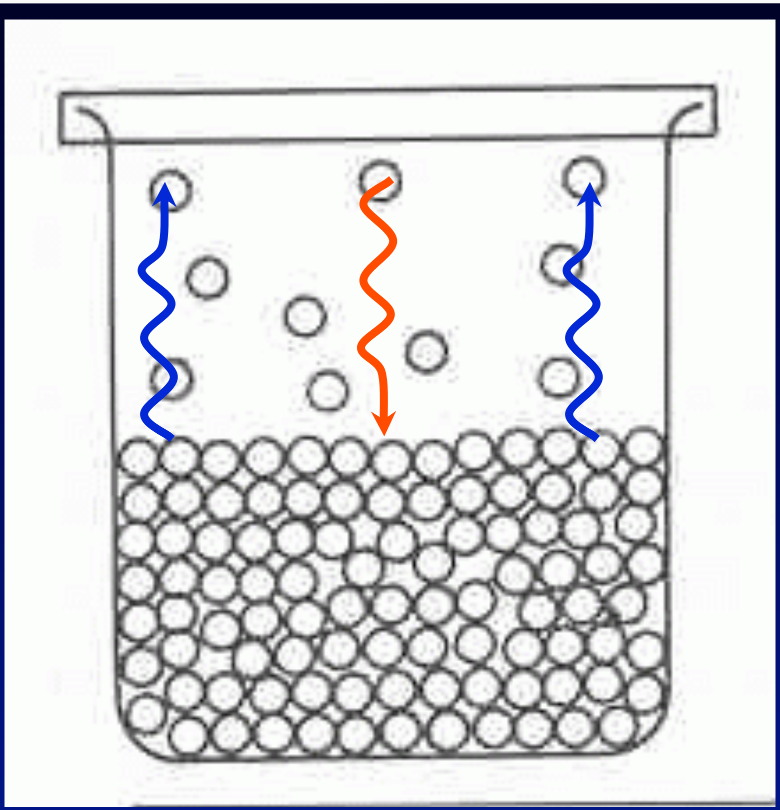
Condensation



Bohren (1987)

- Water molecules moving from vapor to liquid phase
- Accompanied by latent heating
- Always occurring

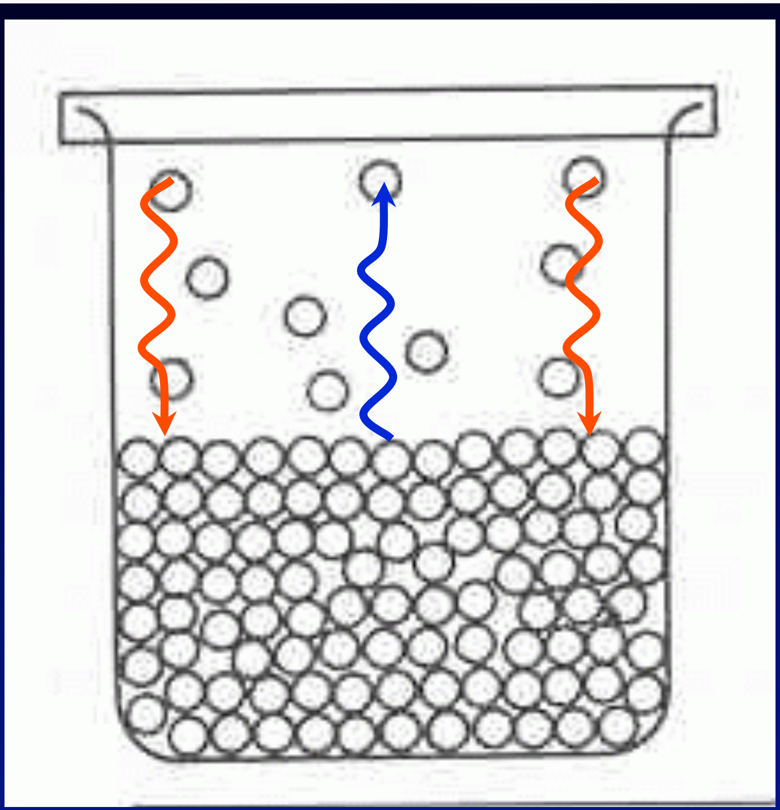
Net evaporation



Bohren (1987)

- What we commonly refer to as evaporation is when the rate of evaporation exceeds the rate of condensation

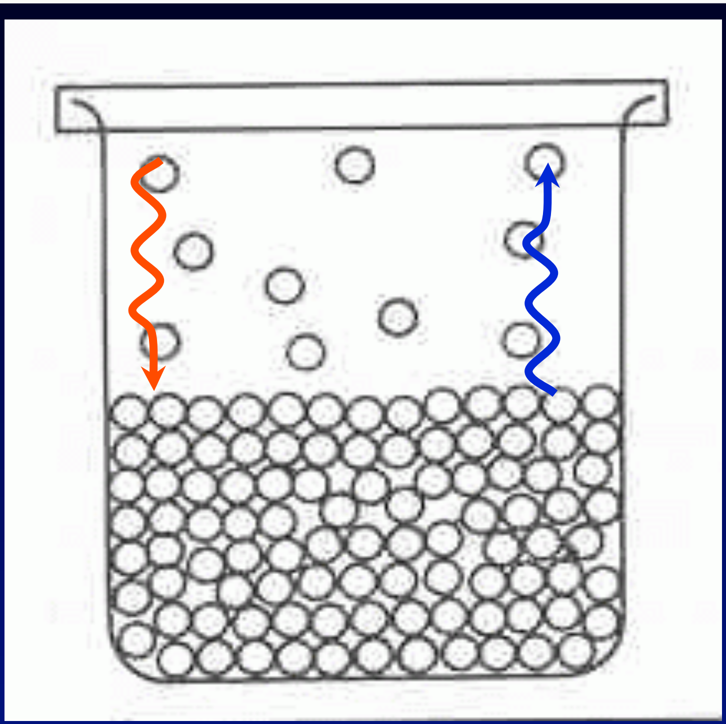
Net condensation



Bohren (1987)

- What we commonly refer to as condensation is when the rate of condensation exceeds the rate of evaporation

Equilibrium and saturation vapor pressure



Bohren (1987)

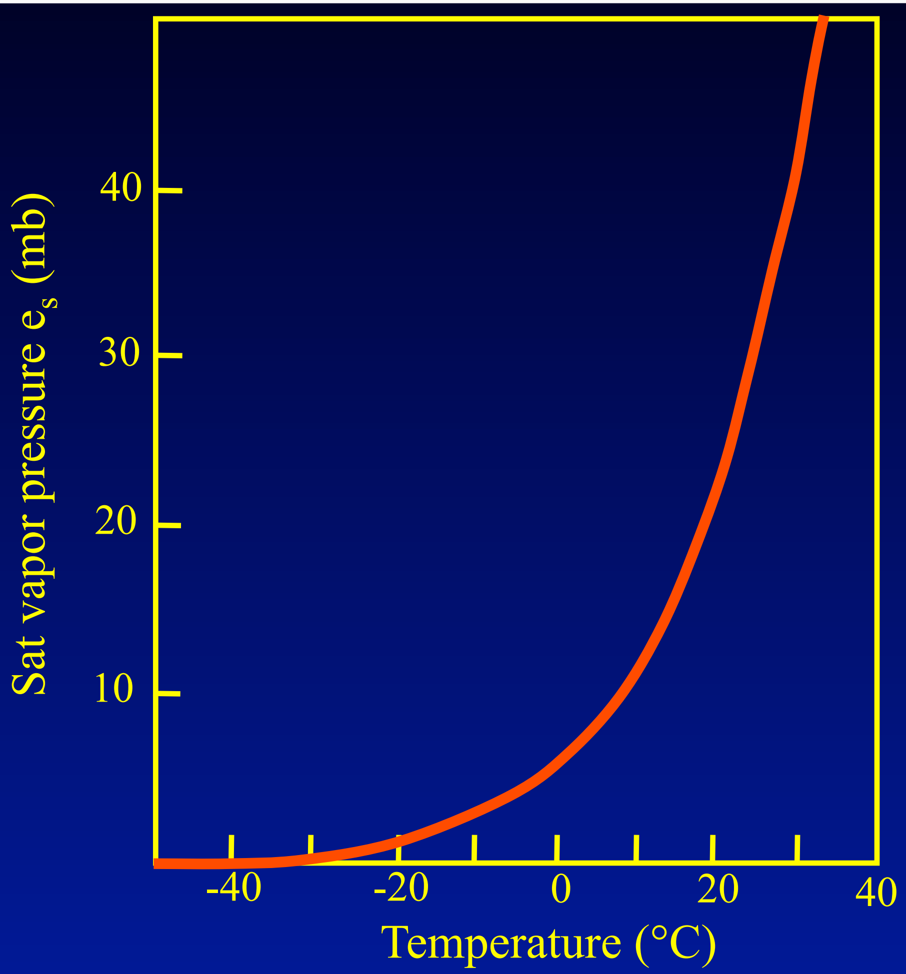
- Equilibrium vapor pressure – evaporation and condensation are occurring, but are in equilibrium
- Saturation vapor pressure – equilibrium vapor pressure for a plane surface of pure water
- For solutions and cloud droplets, equilibrium vapor pressure does not necessarily equal the saturation vapor pressure

Saturation vapor pressure

- Varies with temperature

$$e_s \cong 6.11\exp[\frac{L}{R_v}(\frac{1}{273}-\frac{1}{T})]$$

- L=latent heat of condensation (2.5 x 10^6 J/kg)
- R_v = gas constant for water vapor (461.5 J/kg/K)



Wallace and Hobbs (1977)

Saturation mixing ratio

- The ratio of the mass (m_{vs}) of water vapor in a given volume that is saturated with respect to a plane surface of pure water to the mass (m_d) of dry air in the same volume

$$w_s = \frac{m_{vs}}{m_d}$$

- Relationship to saturation vapor pressure

$$w_s = .622\left(\frac{e_s}{p - e_s}\right) \cong .622\frac{e_s}{p}$$

Relative humidity, dewpoint, and supersaturation

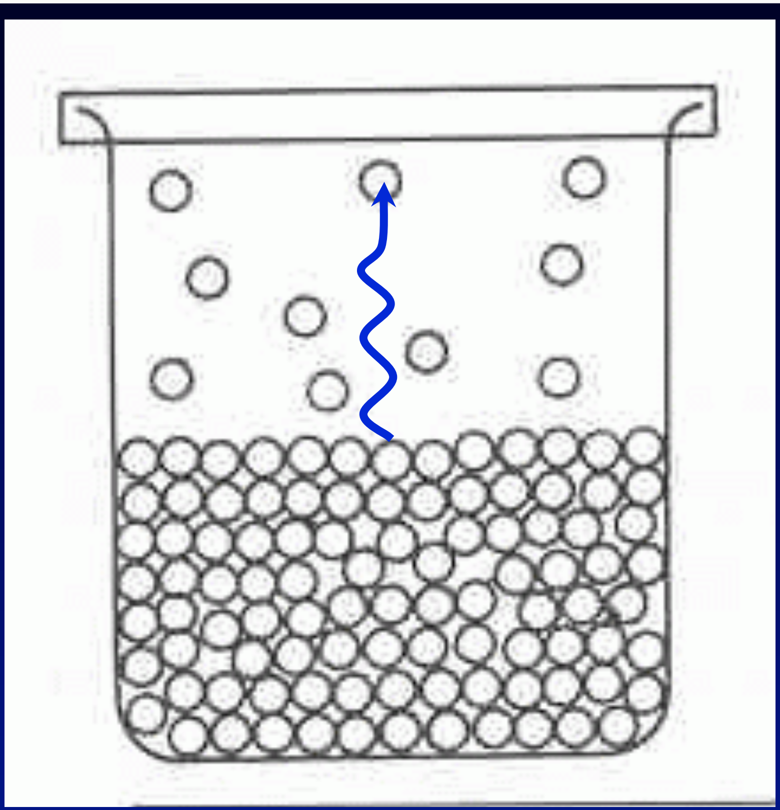
- Relative humidity (with respect to water) - the ratio of the actual vapor pressure to the saturation vapor pressure at the same temperature

$$r = \frac{e}{e_s(T)} \approx \frac{w}{w_s(T,p)}$$

- Dewpoint - the temperature to which air must be cooled at constant pressure for it to become saturated with respect to a plane surface of pure water

- Supersaturation = r - 1

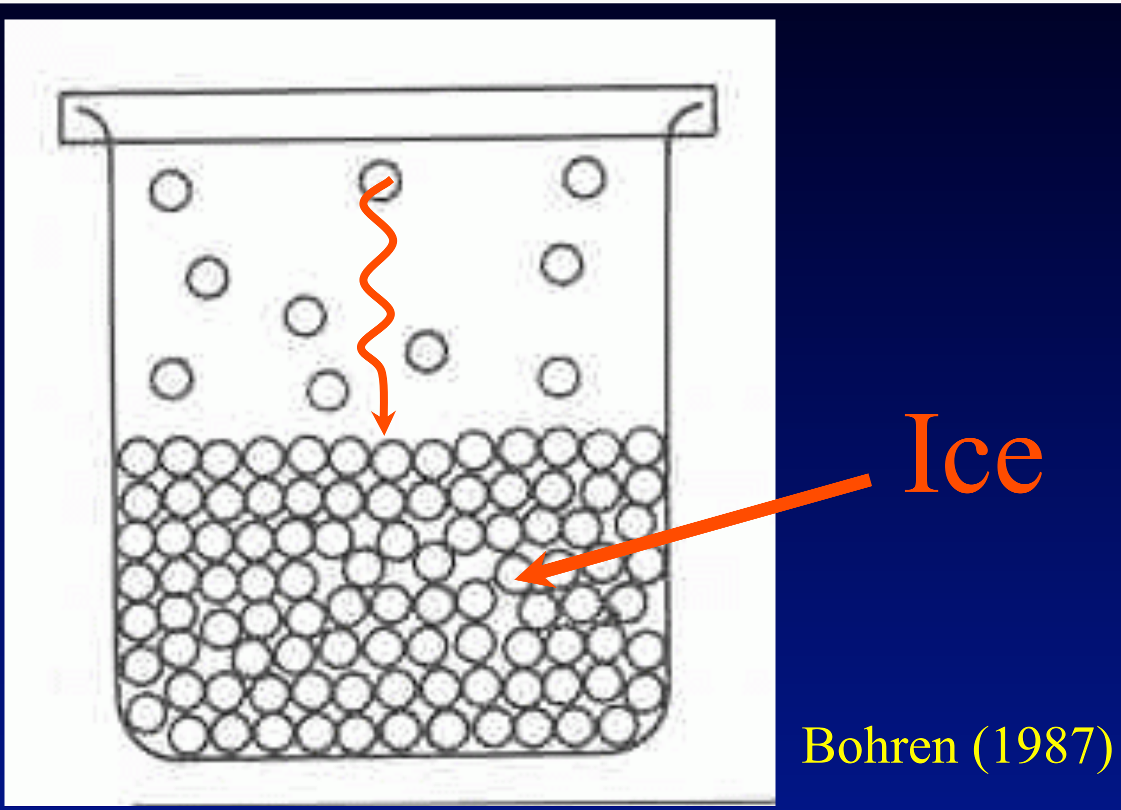
Evaporation of ice (sublimation)



Bohren (1987)

- **Sublimation** occurs when water molecules move directly from ice to vapor phase (no liquid phase)
- Accompanied by latent cooling
- Always occurring

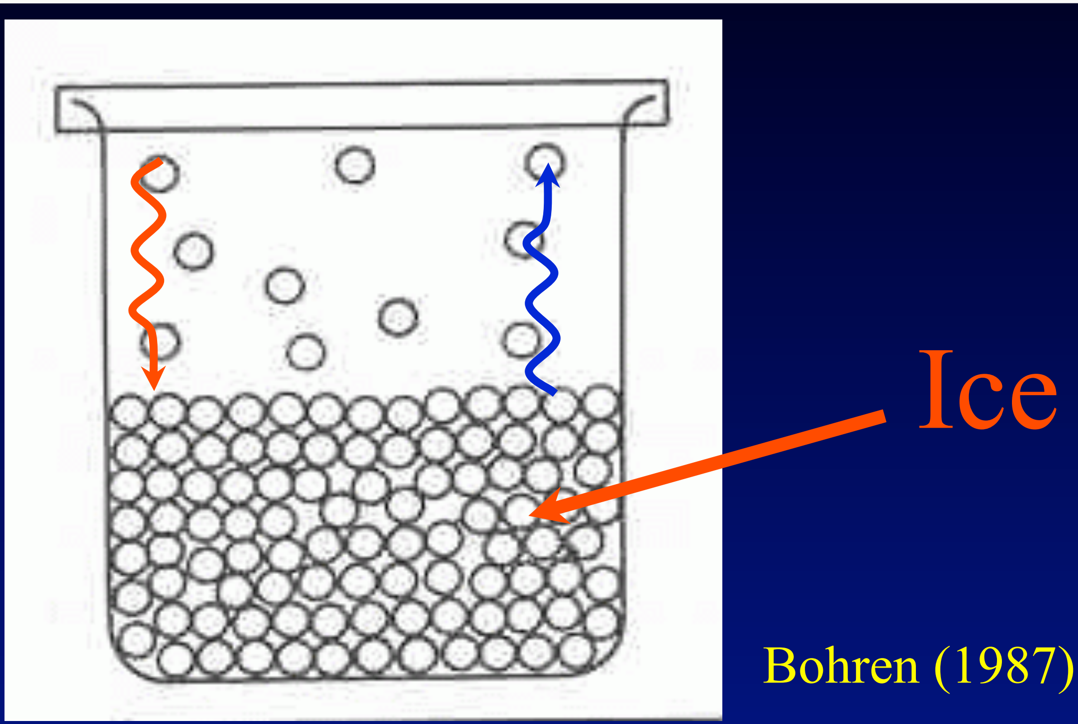
Vapor deposition



Bohren (1987)

- Water molecules move directly from vapor to ice phase
- Accompanied by latent heating
- Always occurring
- Also called deposition or vapor deposition

Equilibrium and saturation vapor pressure for ice

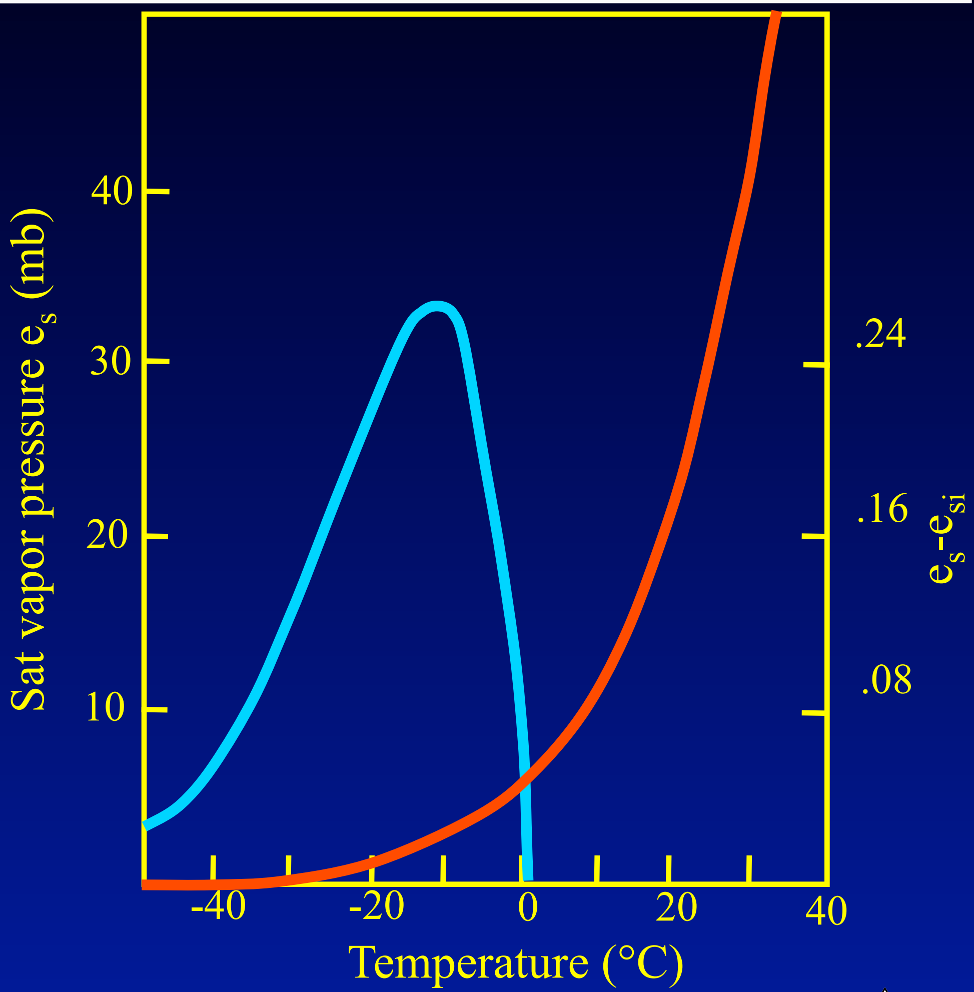


Bohren (1987)

- Equilibrium vapor pressure for ice – sublimation and deposition are occurring, but are in equilibrium
- Saturation vapor pressure for ice – Equilibrium vapor pressure for a plane surface of pure ice
- For solutions and ice particles, equilibrium vapor pressure for ice does not necessarily equal the saturation vapor pressure for ice

Saturation vapor pressure for ice

- The saturation vapor pressure for ice is ≤ that for water
  - $e_{si}=e_s$ at 0°C
  - Otherwise $e_{si}<e_s$
  - e_s-e_{si} is largest at –10°C to –15°C



Wallace and Hobbs (1977)

Saturation mixing ratio with respect to ice

- The ratio of the mass (m_{vs}) of water vapor in a given volume that is saturated with respect to a plane surface of pure ice to the mass (m_d) of dry air in the same volume

$$\boxed{w_{si} = \frac{m_{vsi}}{m_d}}$$

- Always less than or equal to the saturation mixing ratio with respect to water
    - $w_{si}=w_s$ at 0°C
    - $w_{si} \leq w_s$ if $T<0°C$

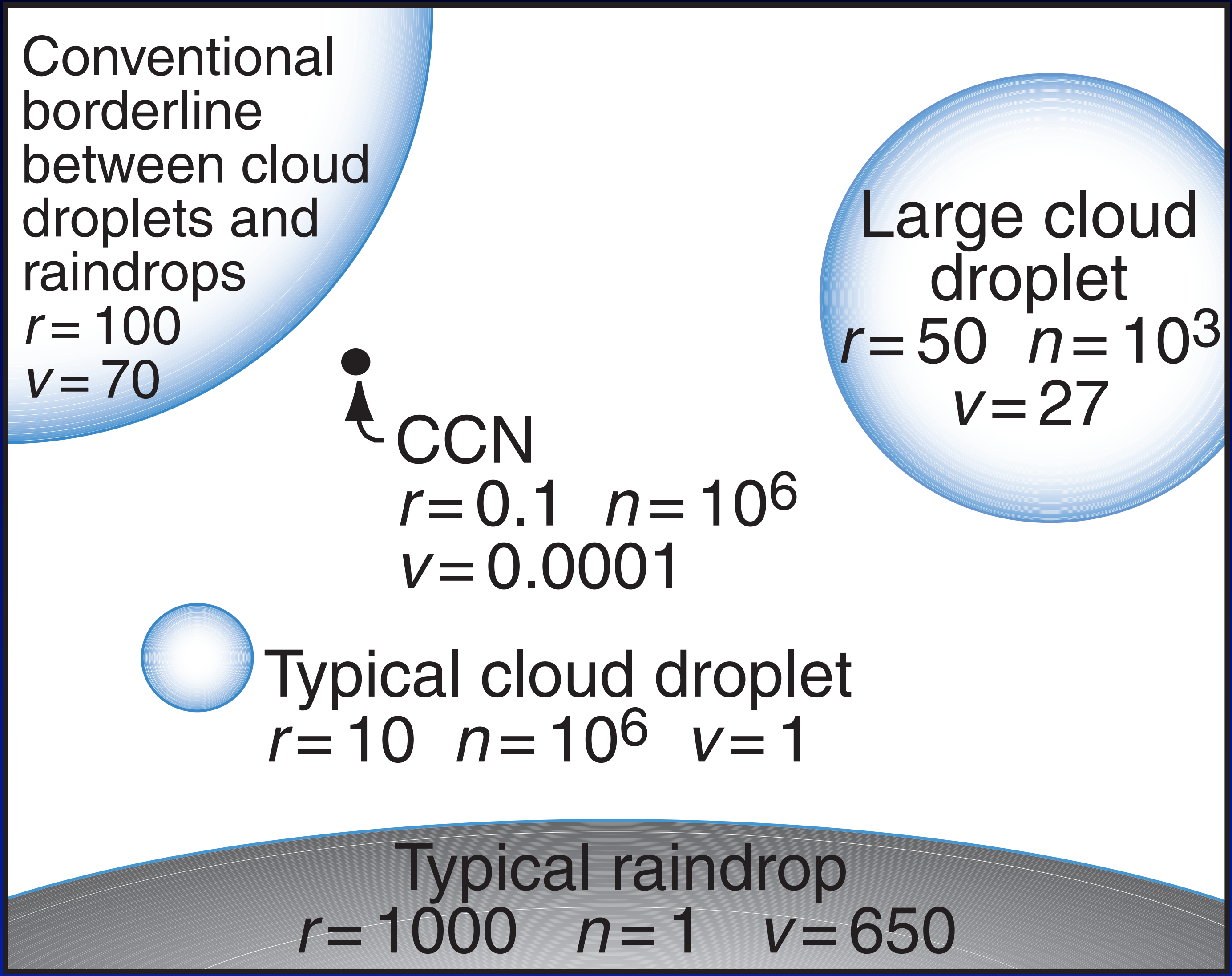
Relative humidity with respect to ice

- Relative humidity (with respect to ice) - the ratio of the actual vapor pressure to the saturation vapor pressure at the same temperature
  - RH wrt ice > RH wtr water (equal at 0°C)

$$r = \frac{e}{e_{si}(T)} \approx \frac{w}{w_{si}(T, p)}$$

- Frost point - the temperature to which air must be cooled at constant pressure for it to become saturated with respect to a plane surface of pure ice
  - Frost point > dewpoint (equal at 0°C

Growth of Cloud Droplets in Warm Clouds



Conventional borderline between cloud droplets and raindrops
$r = 100$
$v = 70$

Large cloud droplet
$r = 50$ $n = 10^3$
$v = 27$

CCN
$r = 0.1$ $n = 10^6$
$v = 0.0001$

Typical cloud droplet
$r = 10$ $n = 10^6$ $v = 1$

Typical raindrop
$r = 1000$ $n = 1$ $v = 650$

Formation of cloud droplets

- Clouds can form when the air becomes supersaturated with respect to water (or ice)

- Usually occurs due to adiabatic cooling produced by ascent

- Can also occur due to
  - Radiational cooling (e.g., radiation fogs)
  - Sensible cooling (e.g., advection fogs)
  - Mixing (e.g., contrails)
  - Other processes that cool or moisten parcels

- The formation of a cloud droplet is called nucleation

Homogeneous nucleation

- Homogeneous nucleation: Formation of a pure water droplet by condensation without the aid of a particle suspended in the air

- Growth of a cloud droplet represents a battle between:
  - Work required to create more droplet surface area (called the interfacial energy or surface energy)
    - Proportional to R^2, where R is the droplet radius
  - Energy provided to the system by condensation (change in Gibbs free energy)
    - Proportional to R^3
- This battle means that the saturation vapor pressure is a function of droplet radius (known as the Kelvin effect)

METEOROLOGY

Homogeneous nucleation (WH 6.1.1)

- Growth of a cloud droplet involves a net change in the energy of the system due to the formation of the droplet:

$$\Delta E = A\sigma - nV(\mu_v - \mu_l)$$

where ΔE is the net increase in the energy of the system, A is the droplet surface area, V is the droplet volume, σ is the work required to to create a unit area of vapor-liquid interface, n is the number of water molecules per unit volume of liquid, and μ_v and μ_l are the Gibbs free energies per molecule in the vapor and liquid phases.

Homogeneous nucleation

It can be shown that

$$\mu_v - \mu_l = kT \log \frac{e}{e_s}$$

where e and T are the vapor pressure and temperature of the system, e_s is the saturation vapor pressure *over a plane surface of water* at temperature T, and k is Boltzmann's constant.

Therefore,

$$\Delta E = A\sigma - nVkT \log \frac{e}{e_s}$$

Homogeneous nucleation

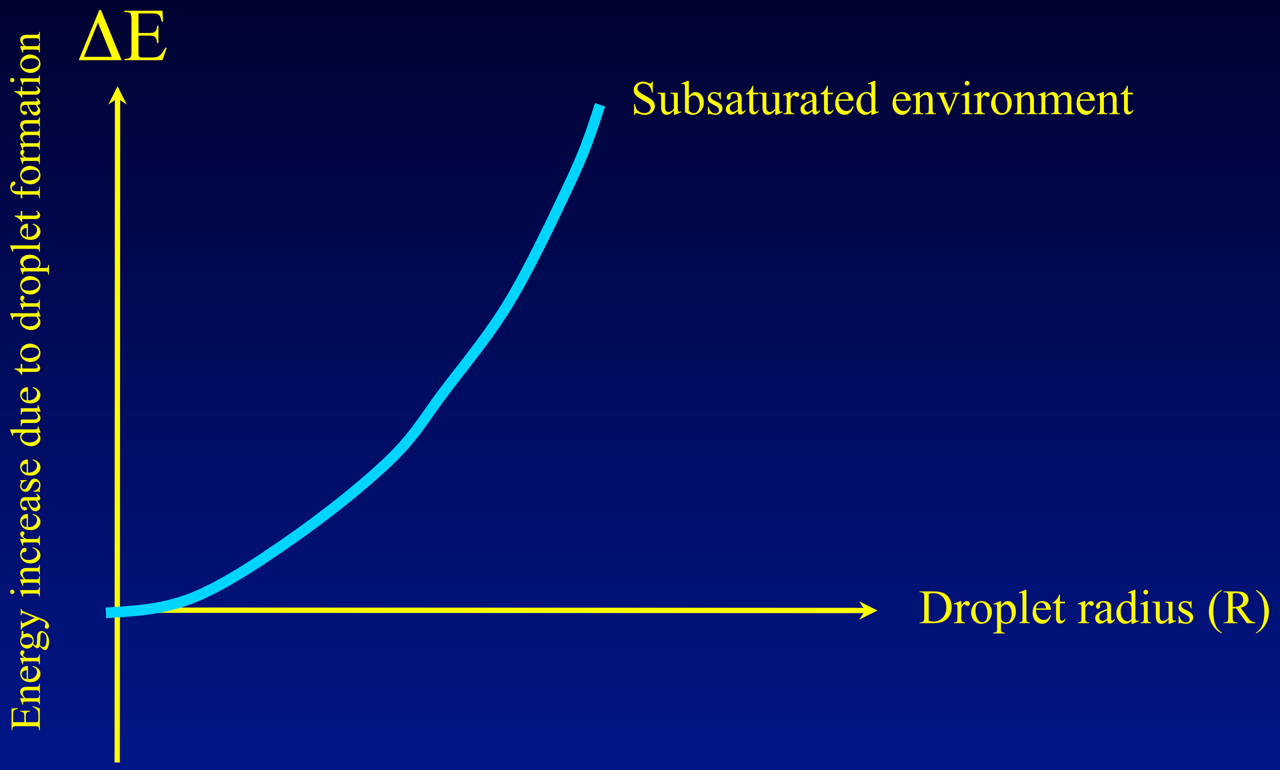
For a droplet of radius R,

$$\boxed{\Delta E = 4\pi R^2 \sigma - \frac{4}{3}\pi R^3 nkT \log \frac{e}{e_s}}$$

If subsaturated, $e < e_s$ so ΔE is always > 0, and the formation of droplets is not favored.

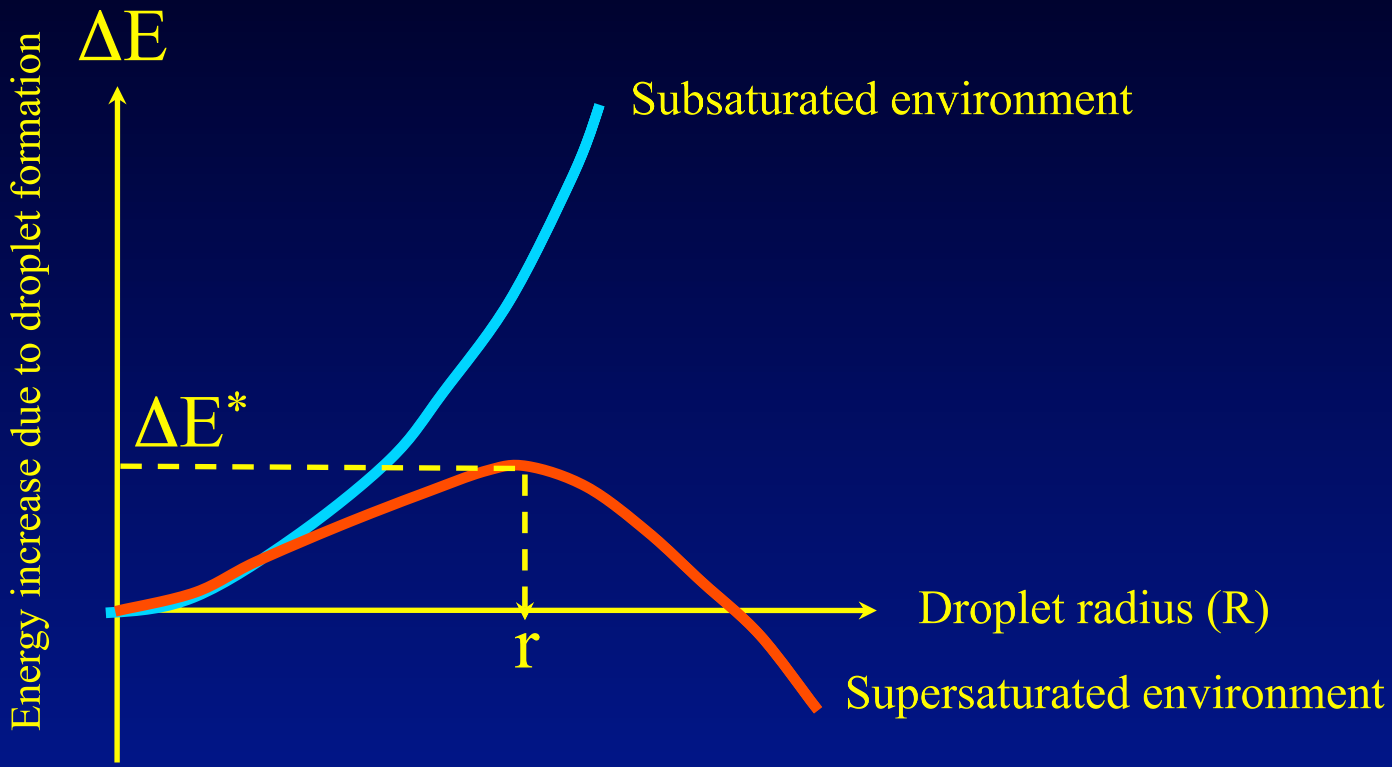
If saturated, $e > e_s$ so ΔE may be > 0 or < 0 depending on R.

Homogeneous nucleation



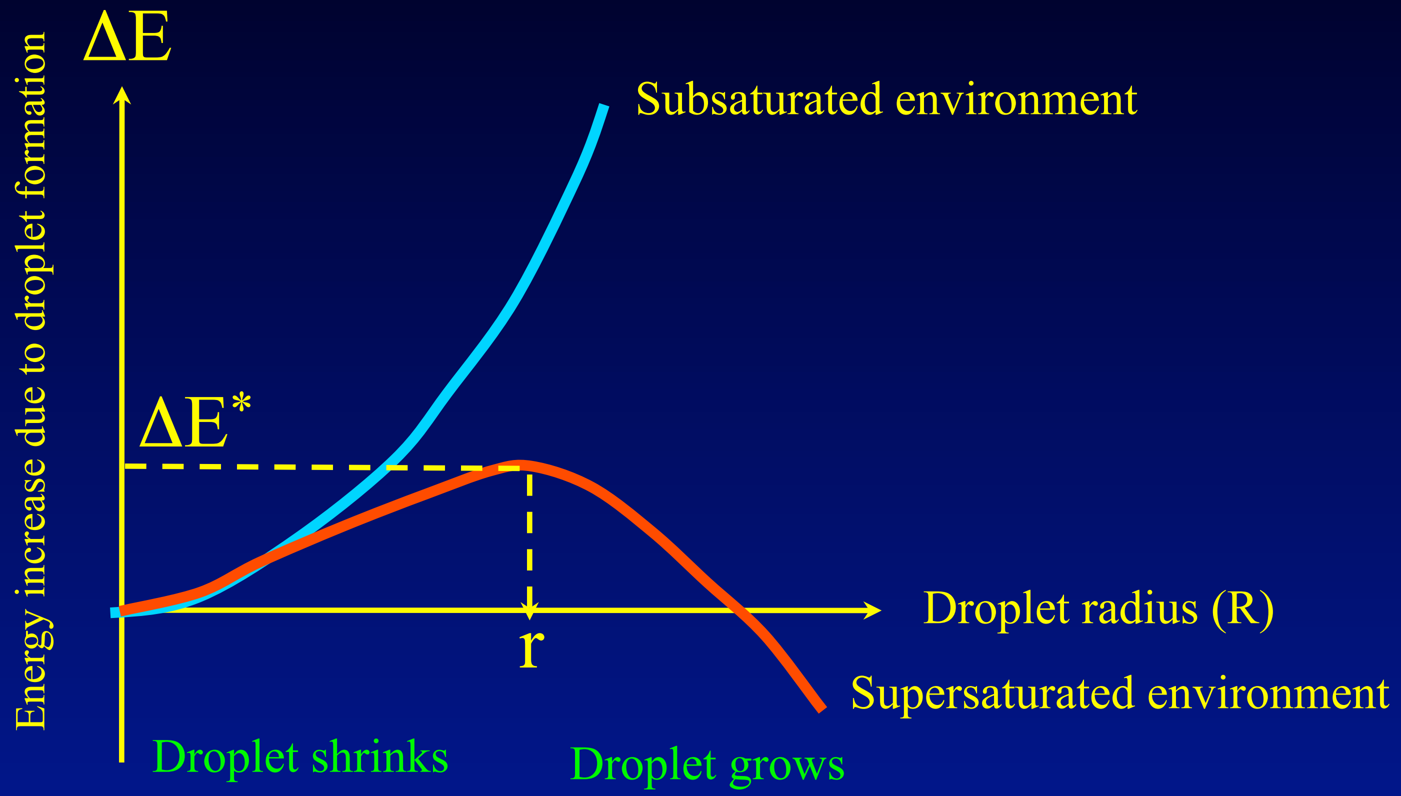
- In subsaturated air, the energy needed to increase droplet surface area is too big for energy released by condensation to overcome
- Droplets form through molecular collisions, but quickly evaporate

Homogeneous nucleation



- In supersaturated air, the energy required to increase the surface area ($\propto R^2$) exceeds that released by condensation ($\propto R^3$) for small droplets
- Beyond a critical radius, r, the energy released by condensation exceeds that needed to increase surface area and droplets spontaneously grow

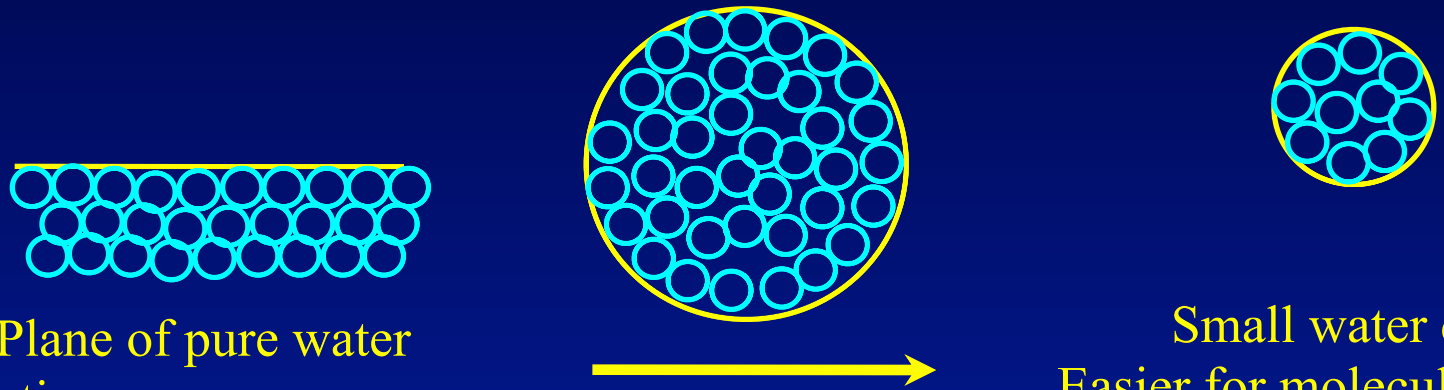
Homogeneous nucleation



- At the critical radius
  - The droplet is at its equilibrium vapor pressure (evaporation = condensation)
  - It will either shrink or grow spontaneously if R < r or R > r.

Homogeneous nucleation

- Why? It's easier for water molecules to "escape" if the radius is small (more surface area/molecule)
- Evaporation rate for small droplets is greater than for large droplets or a plane surface of pure water



Plane of pure water
Saturation vapor pressure = e_s

Small water droplet
Easier for molecules to escape
$e_s > e_s$ for plane of pure water

- Equilibrium vapor pressure is larger for small droplets
- Need large supersaturation for a pure cloud droplet to grow

Homogeneous nucleation

For a droplet of radius R,

$$\boxed{\Delta E = 4\pi R^2\sigma - \frac{4}{3}\pi R^3 nkT\log\frac{e}{e_s}}$$

To get equilibrium radius r, set $d(\Delta E)/dR = 0$:

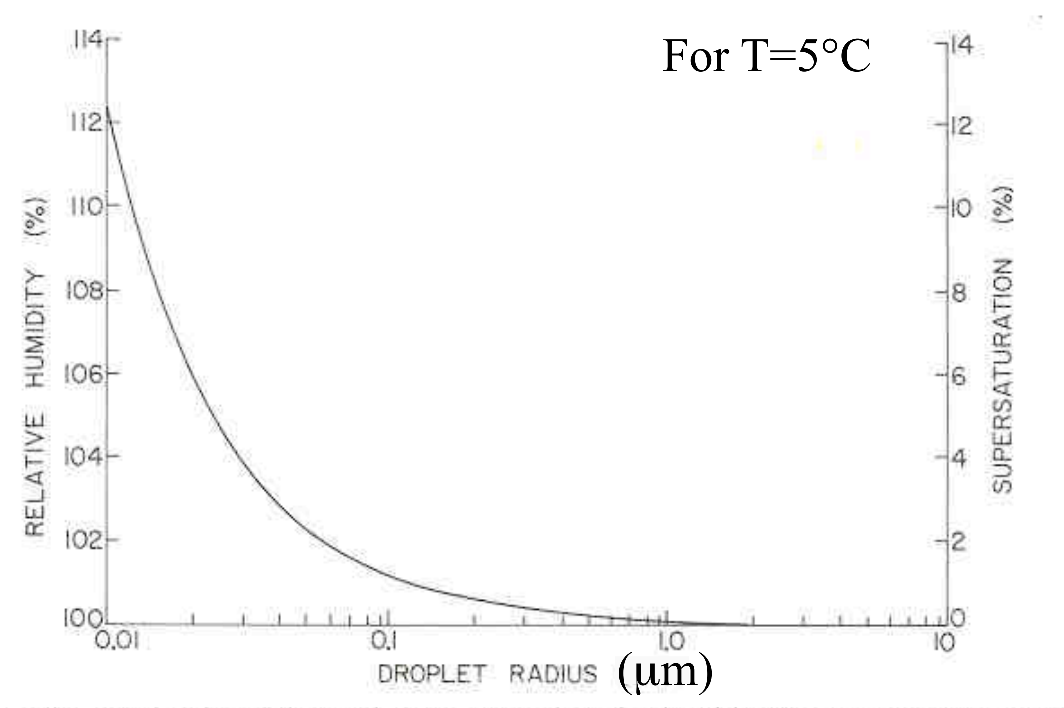
$$\boxed{r = \frac{2\sigma}{nkT\log\frac{e}{e_s}}}$$

Invert to obtain equilibrium saturation vapor pressure:

$$\boxed{\frac{e}{e_s} = \exp\frac{2\sigma}{nkTr}}$$

Homogeneous nucleation

- For a given temperature, equilibrium vapor pressure (and RH) increases with decreasing droplet radius
- If r=.01 μm, equilibrium RH is 112.5% (for T=5 C)
- RH>103% is rarely observed
- It’s very difficult for homogeneous nucleation to occur in the atmosphere



Heterogeneous nucleation (WH 6.1.1)

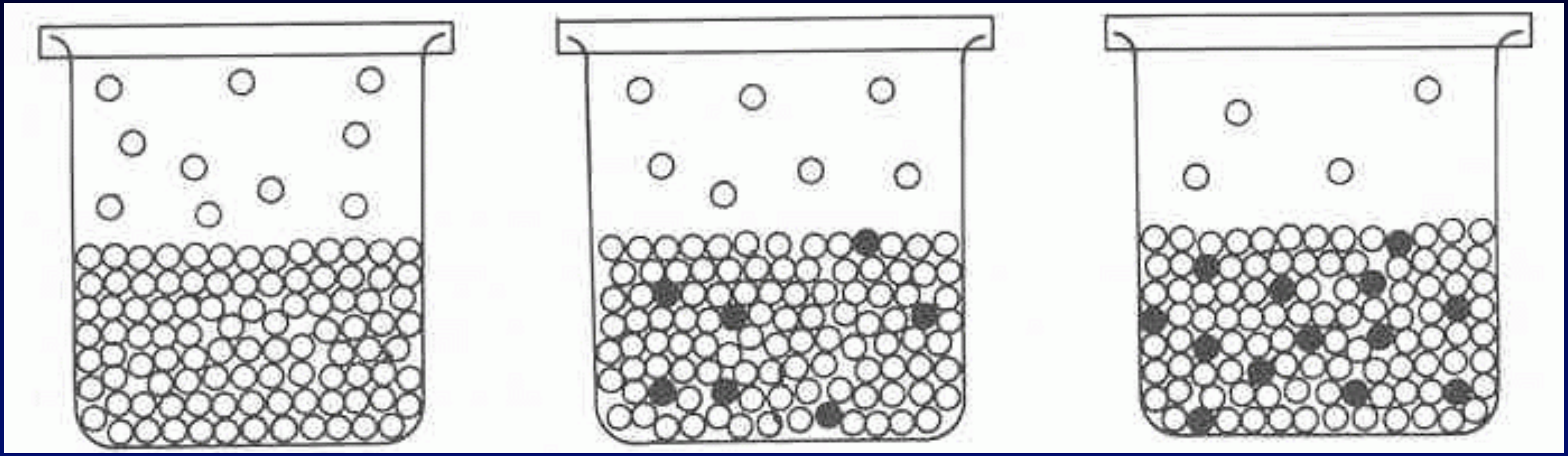
- How do clouds form?
  - They get help: Heterogeneous nucleation

- Heterogeneous nucleation: Formation of a cloud droplet on an atmospheric aerosol

- Atmospheric aerosols that are soluble in water dissolve when water begins to condense on them

- The solution lowers the equilibrium vapor pressure & thus creates more favorable conditions for droplet growth

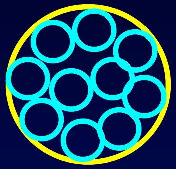
Heterogeneous nucleation



- In a solution, there are fewer water molecules on the water surface available for evaporation
- Evaporation rate is lower than for pure water
- Equil. saturation vapor pressure is lower than for pure water

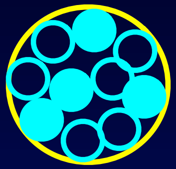
METEOROLOGY

Heterogeneous nucleation



Small pure water droplet:

- Surface is all water molecules.
- Largest possible evaporation rate.
- Maximum equilibrium vapor pressure.



Small solution droplet:

- Surface has fewer water molecules.
- Less evaporation.
- Smaller equilibrium vapor pressure.

Heterogeneous nucleation

The reduction in saturation vapor pressure adjacent to a solution droplet is given by *Raoult's law*:

$$\frac{e'}{e} = f$$

where e' is the saturation vapor pressure adjacent to a solution droplet that contains a mole fraction f of pure water, and e is the saturation vapor pressure adjacent to a pure water droplet of the same size and temperature.

METEOROLOGY

Heterogeneous nucleation

The *mole fraction of pure water* is the number of moles of pure water in the solution divided by the total number of moles (pure water plus solute):

$$f = \frac{N_w}{N_w + N_s}$$

If the solution droplet of radius r contains a mass m of solute of molecular weight M_s that dissociates into i ions per molecule, then

$$N_s = im/M_s$$

If the solution density is ρ', molecular weight of water is M_w, then

$$N_w = (\frac{4}{3}\pi r^3 \rho' - m)/M_w$$

Heterogeneous nucleation

The *mole fraction of pure water* is the number of moles of pure water in the solution divided by the total number of moles (pure water plus solute):

$$f = \frac{(\frac{4}{3}\pi r^3 \rho' - m)/M_w}{(\frac{4}{3}\pi r^3 \rho' - m)/M_w + im/M_s}$$

$$= \left[1 + \frac{imM_w}{M_s(\frac{4}{3}\pi r^3 \rho' - m)}\right]^{-1}$$

Heterogeneous nucleation

The reduction of saturation vapor pressure e' adjacent to a solution droplet of radius r relative to that over a plane surface of pure water e_s is

$$\frac{e'}{e_s} = \left[\exp\frac{2\sigma'}{n'kTr}\right]\left[1 + \frac{imM_w}{M_s(\frac{4}{3}\pi r^3\rho' - m)}\right]^{-1}$$

where σ' and n' indicate the surface energy and number concentration of water molecules for the solution.

Heterogeneous nucleation (RY Chapter 6)

For a weak solution droplet ($m \ll \frac{4}{3}\pi r^3 \rho'$), and for r not too small,

$$\frac{e}{e_s} \approx 1 + \frac{a}{r} - \frac{b}{r^3}$$

where

$$a = \frac{2\sigma'}{n'kT}$$

$$b = \frac{imM_w}{\frac{4}{3}M_s\pi\rho'}$$

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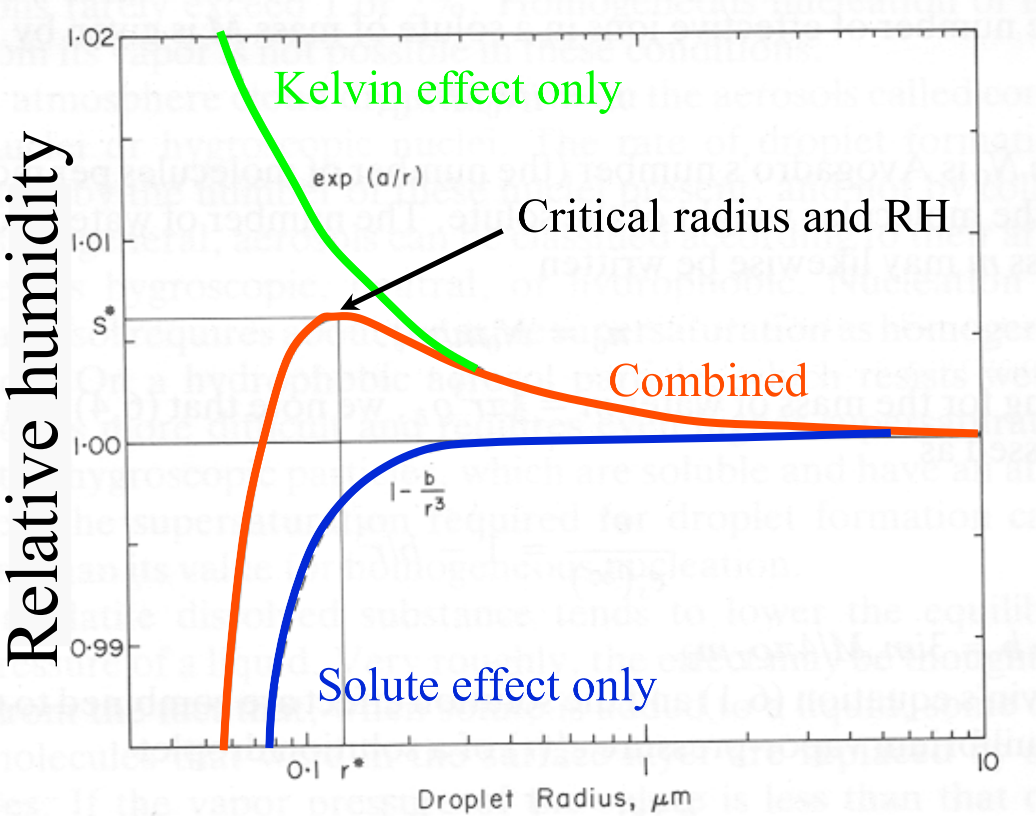
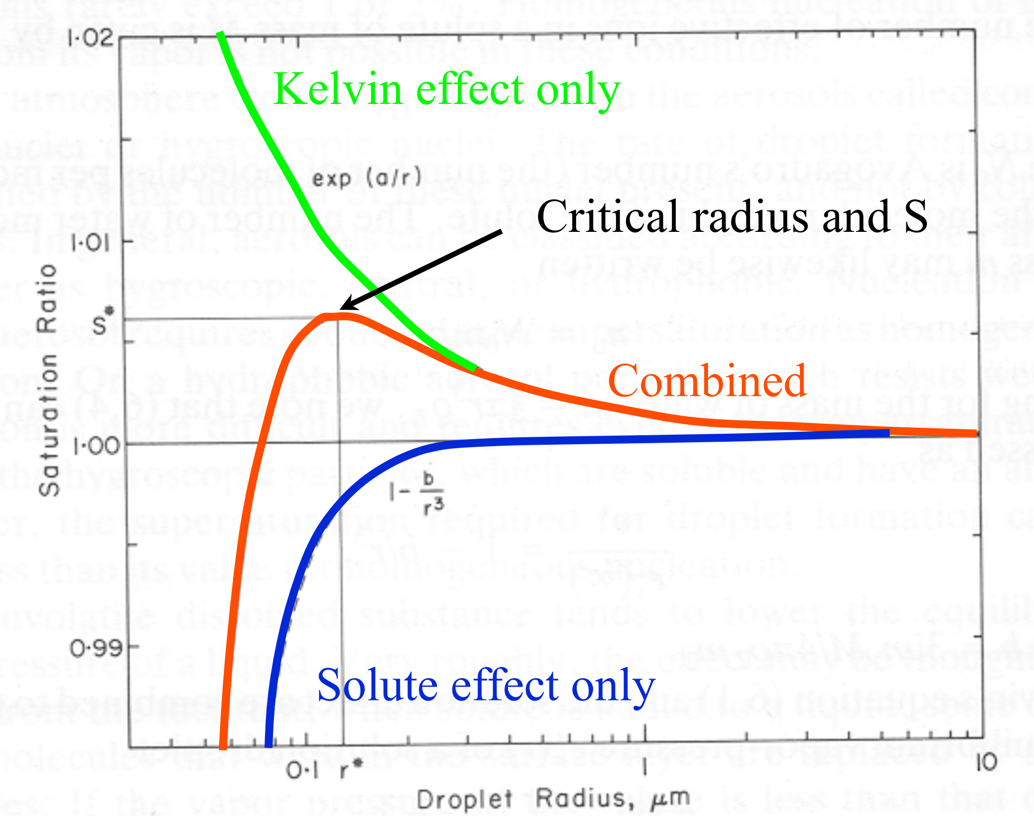
Heterogeneous nucleation



Rogers and Yau (1989)
Kohler curve for droplet
formed on 10^{-16} g
ammonium sulfate particle

- Kohler curve: Combines solute and Kelvin effects
- Equilibrium RH lowered for small droplets by solute effect
- Kelvin effect does result in a critical RH for nucleation of > 100% (in this case 100.6%)

Heterogeneous nucleation



- Very small solution droplets are in equilibrium at S < 1.
- If S increases, droplet grows to a new equilibrium radius.
- This process can continue until the critical S, S*, and critical radius, r*, are reached.
- If S > S* so that r > r*, then the equilibrium S < S*, and the droplet will continue to grow without any change in S.

- For r < r* or r=r*, droplet is in stable equilibrium:
  r adjusts to its equilibrium size for a given S.
- For r > r*, droplet is in unstable equilibrium:
  r increases as long as S > S_{eq}

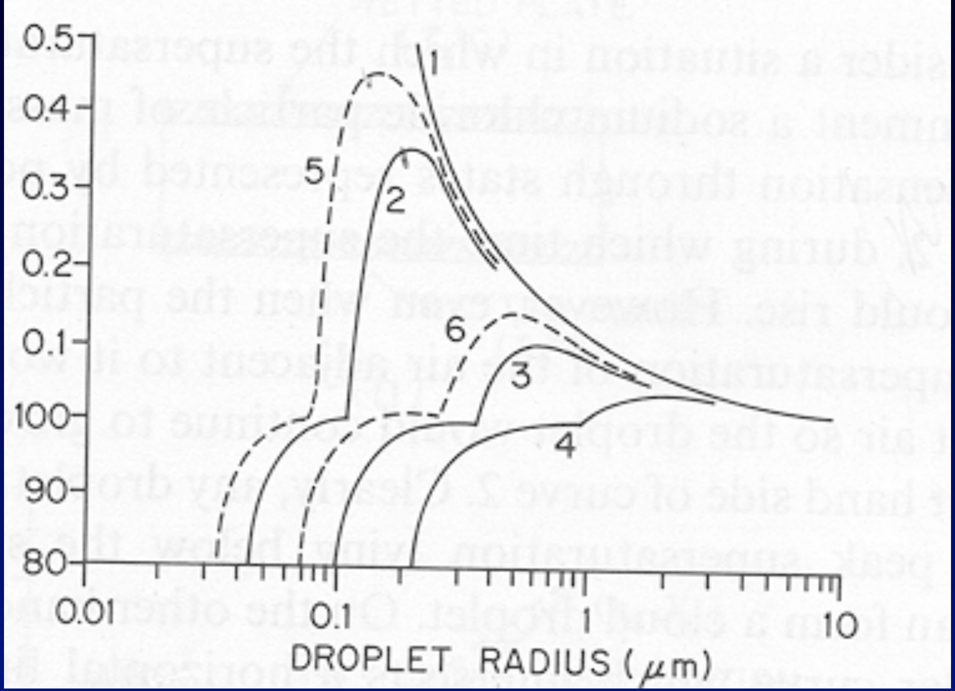
Heterogeneous nucleation

For a weak solution droplet ($m \ll \frac{4}{3}\pi r^3 \rho'$), and for r not too small, the peak in the Köhler curve occurs at

$$r = \left(\frac{3b}{a}\right)^{1/2}$$

$$\frac{e'}{e_s} = 1 + \left(\frac{4a^3}{27b}\right)^{1/2}$$

Heterogeneous nucleation

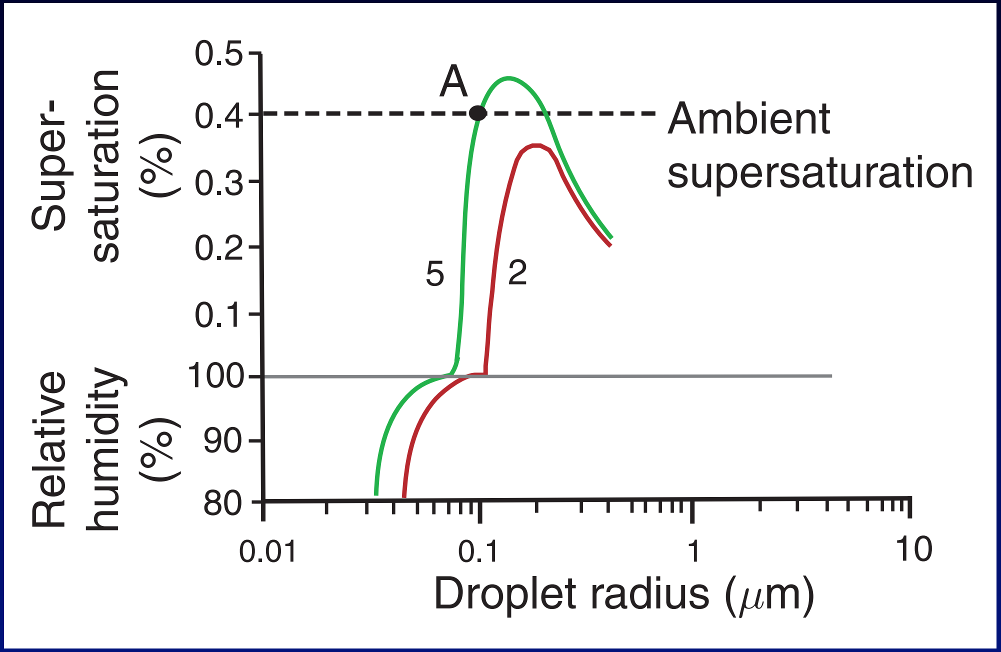


(1) pure water
(2) 10^{-19} kg of NaCl
(3) 10^{-18} kg of NaCl
(4) 10^{-17} kg of NaCl
(5) 10^{-19} kg of $(NH_4)_2SO_4$
(6) 10^{-18} kg of $(NH_4)_2SO_4$

Wallace and Hobbs (1977)

- Kohler curves, critical RH, and critical radius vary with type and concentration of solution
- A droplet that has passed over its Kohler curve peak is called activated
- Not all droplets are activated – there are winners and losers!

Heterogeneous nucleation



(2) 10^{-19} kg of NaCl

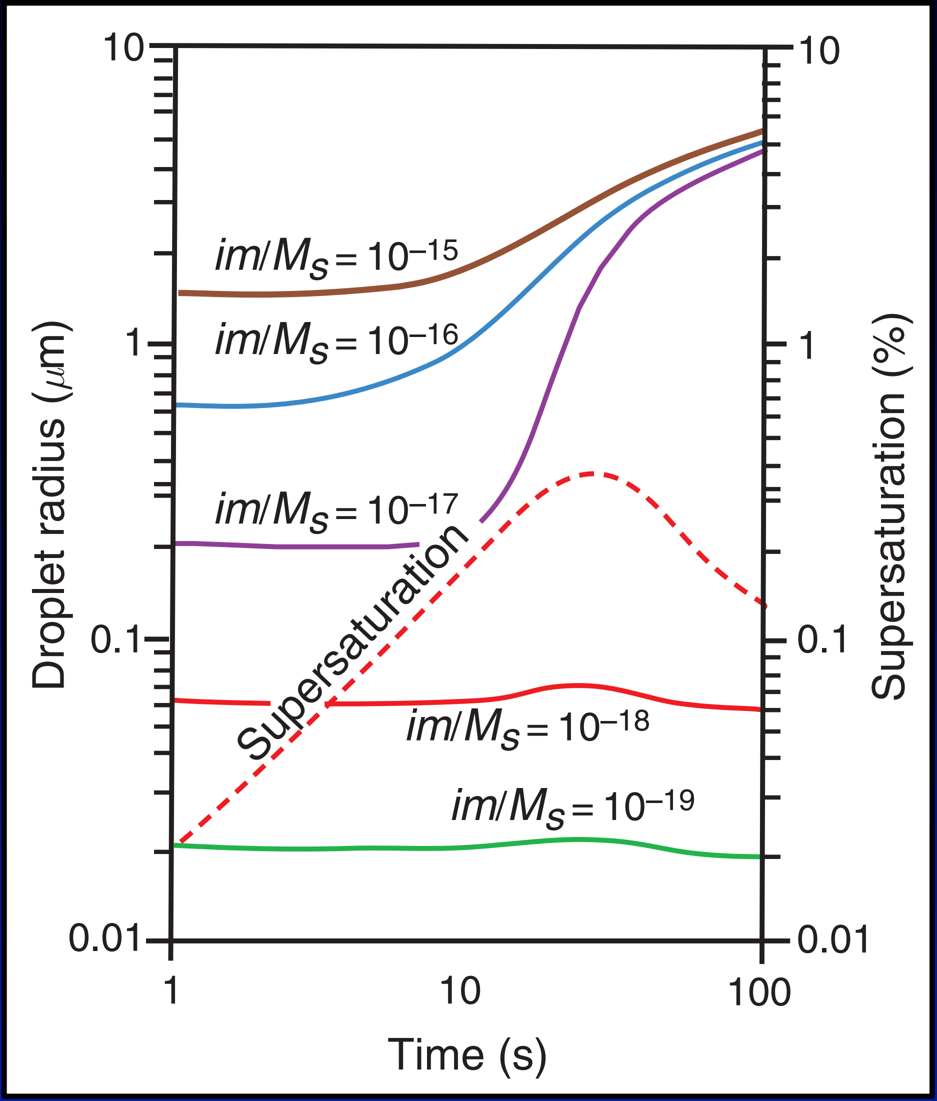
(5) 10^{-19} kg of $(NH_4)_2SO_4$

Wallace and Hobbs (2006)

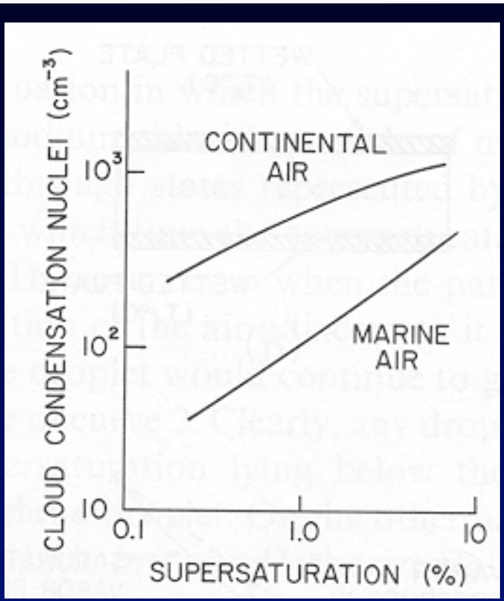
- Droplet 2 is activated in a supersaturation of 0.4 %.
- Droplet 5 will grow only up to A in a supersaturation of 0.4 %, and is unactivated.

Condensation

- Calculation of the growth of CCN (500/cm^3) by condensation in an updraft of 60 cm/s.
- Activated droplets are monodisperse by 100 s.



Cloud condensation nuclei



Wallace and Hobbs (1977)

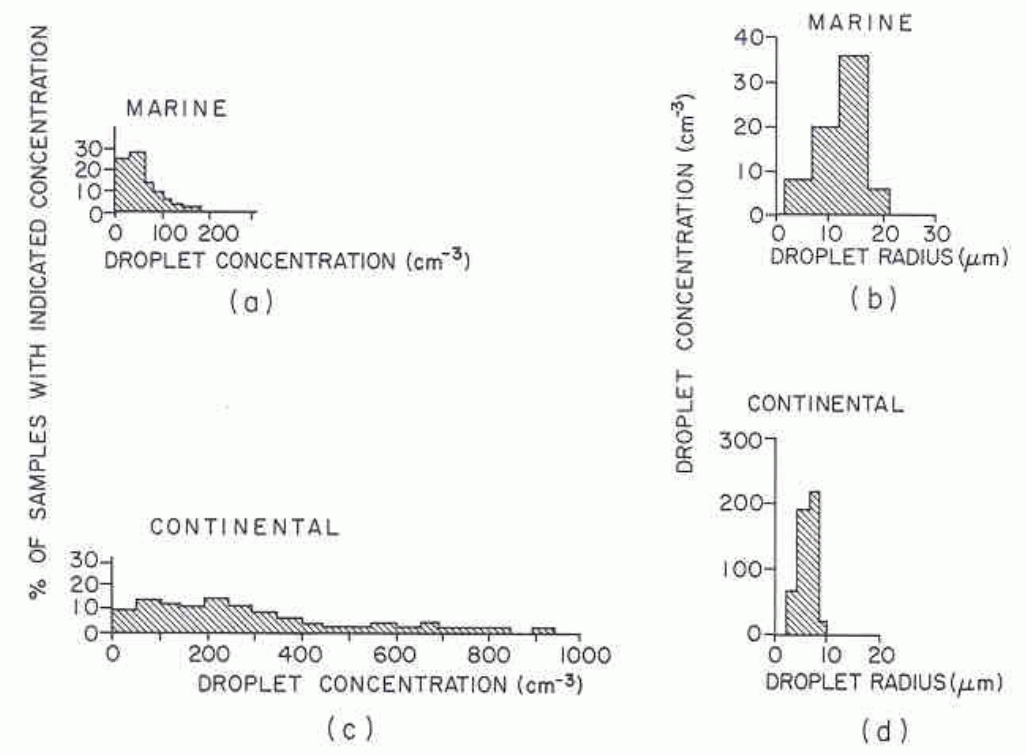
- Cloud condensation nuclei (CCN) – Aerosol particles that serve as nuclei for water vapor condensation
- The larger and more soluble the aerosol, the lower the supersaturation needed for activation
- There are an order of magnitude more CCN in continental air than maritime air

METEOROLOGY

Cloud condensation nuclei

Maritime

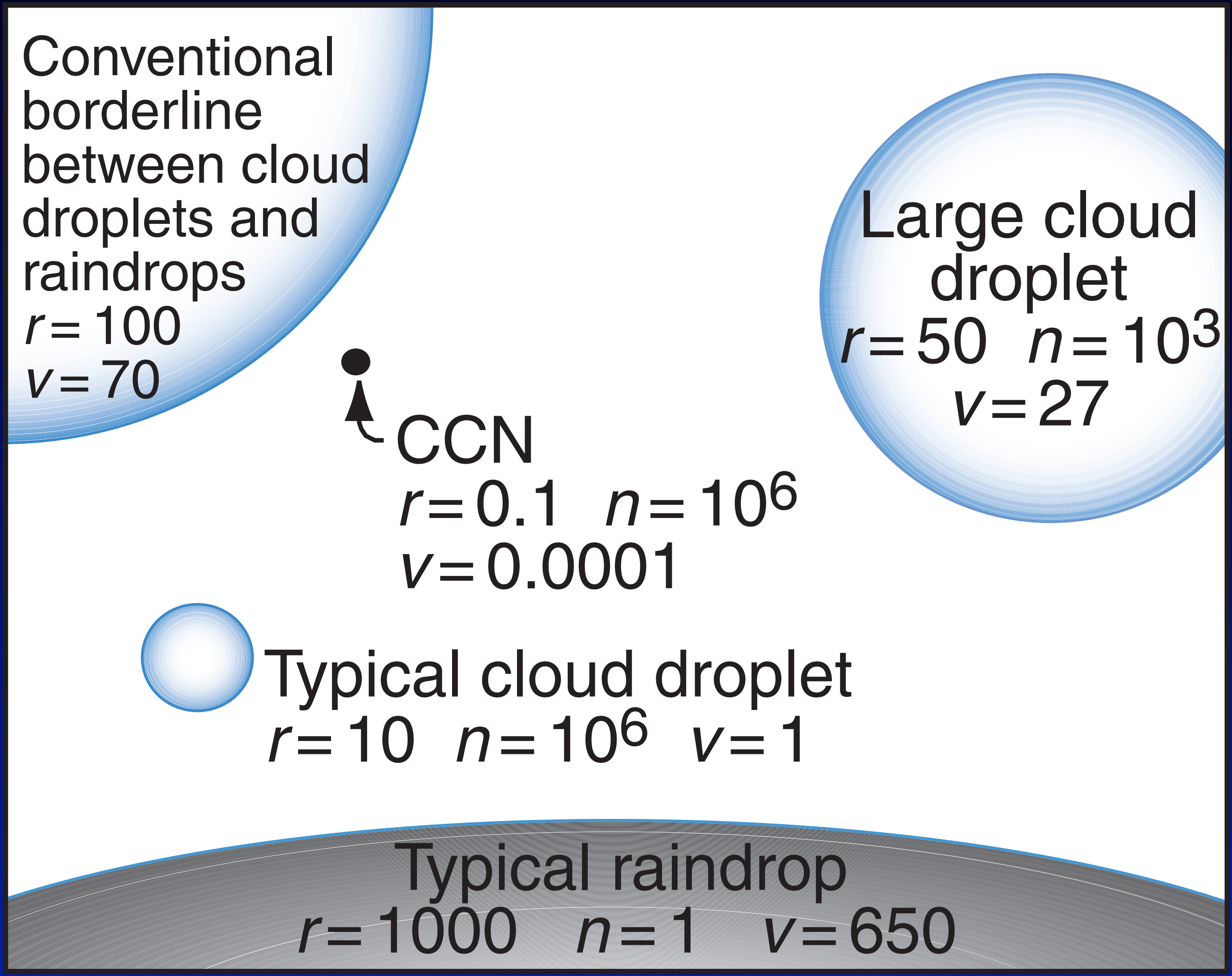
Continental



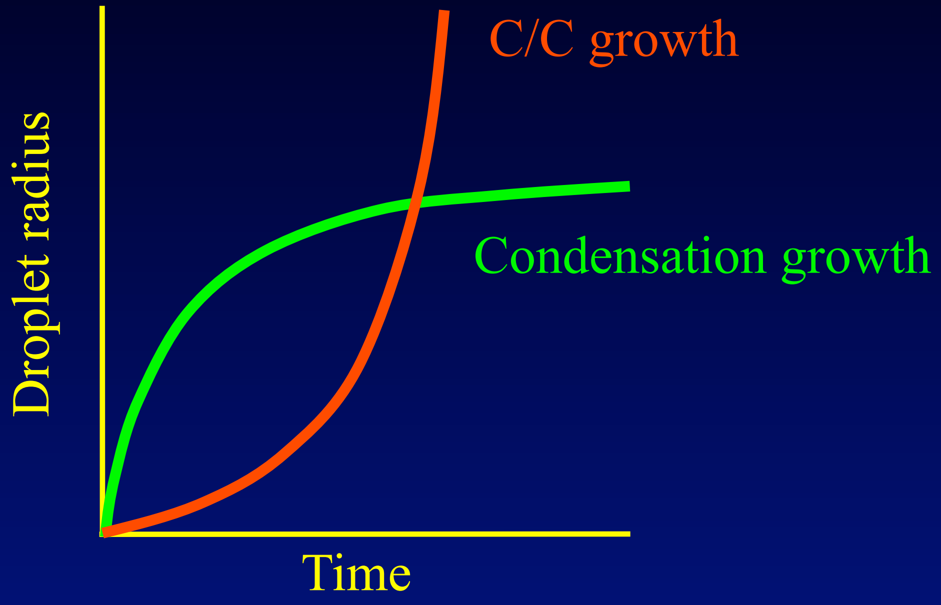
Wallace and Hobbs (1977)

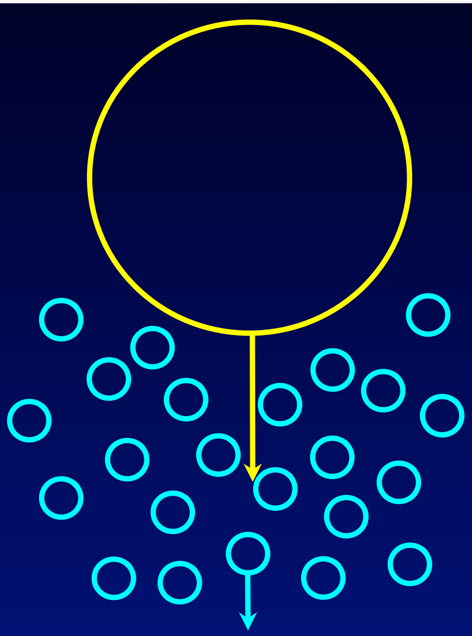
- Continental clouds feature
  - Large cloud droplet number concentrations
  - Smaller cloud droplets
- Maritime clouds feature
  - Smaller cloud droplet number concentrations
  - Larger cloud droplets

Growth of Cloud Droplets in Warm Clouds



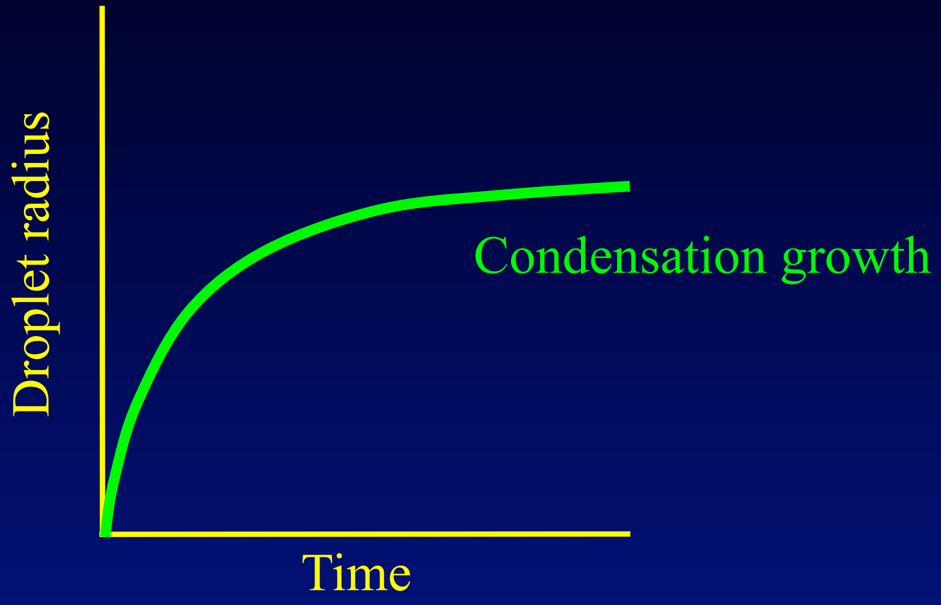
The warm cloud rain process





- Cloud droplet growth initially dominated by condensation
- Growth into raindrops dominated by collision-coalescence
- Most effective in maritime clouds
  - small concentrations of large cloud droplets (due to fewer CCN)

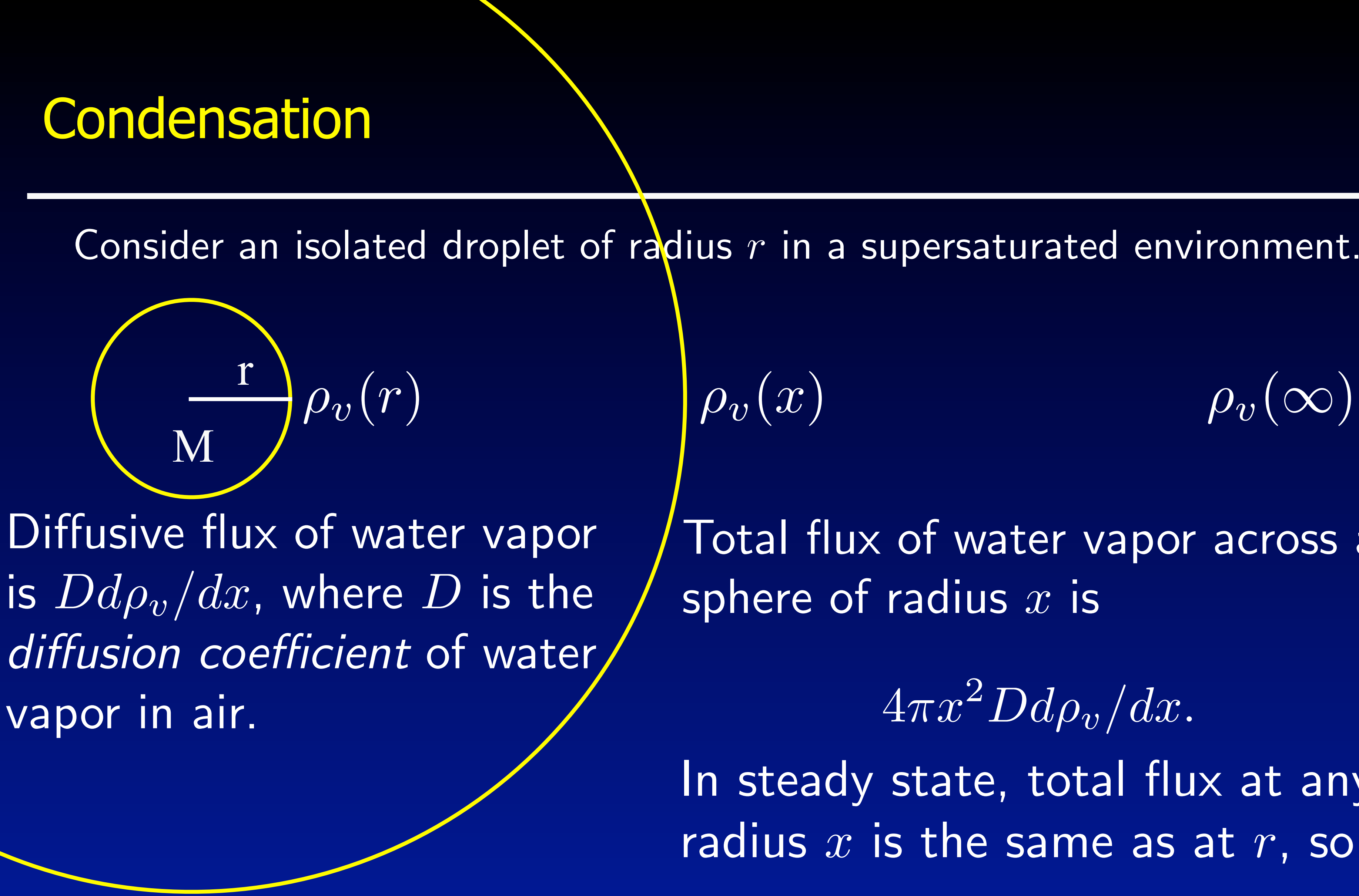
Condensation



- Increase of droplet radius by condensation is initially rapid, but diminishes as droplet grows.
- Condensational growth by itself cannot produce raindrops. (Why not?)

Condensation

Consider an isolated droplet of radius r in a supersaturated environment.



Diffusive flux of water vapor is $D d\rho_v/dx$, where D is the *diffusion coefficient* of water vapor in air.

Total flux of water vapor across a sphere of radius x is

$$4\pi x^2 D d\rho_v/dx.$$

In steady state, total flux at any radius x is the same as at r, so

$$M = \frac{4}{3}\pi r^3 \rho_l$$

$$\frac{dM}{dt} = 4\pi x^2 D \frac{d\rho_v}{dx}.$$

Condensation

Write

$$\frac{dM}{dt} = 4\pi x^2 D \frac{d\rho_v}{dx}$$

as

$$\frac{dM}{dt}\frac{dx}{x^2} = 4\pi D d\rho_v$$

then integrate from $x = r$ to $x = \infty$:

$$\frac{dM}{dt}\int_{x=r}^{x=\infty}\frac{dx}{x^2} = 4\pi D\int_{\rho_v(r)}^{\rho_v(\infty)} d\rho_v$$

to get

$$\frac{dM}{dt} = 4\pi r D[\rho_v(\infty) - \rho_v(r)].$$

Condensation

Use

$$\rho_v = \frac{e}{R_v T} \quad \text{(EOS)}$$

$$M = \frac{4}{3}\pi r^3 \rho_l \quad \text{(droplet mass)}$$

in

$$\frac{dM}{dt} = 4\pi r D[\rho_v(\infty) - \rho_v(r)]$$

to get

$$r\frac{dr}{dt} = \frac{D e_s(T)}{\rho_l R_v T}\,\frac{e(\infty) - e(r)}{e_s(T)}.$$

Condensation

$$r\frac{dr}{dt} = \frac{De_s(T)}{\rho_l R_v T}\frac{e(\infty) - e(r)}{e_s(T)}$$

can be written as

$$r\frac{dr}{dt} = \frac{De_s(T)}{\rho_l R_v T}S,$$

where

$$S \equiv \frac{e(\infty) - e_s}{e_s} \approx \frac{e(\infty) - e(r)}{e_s(T)},$$

and $e_s(T)$ is the saturation vapor pressure over a plane surface of water.

Condensation

Our equation for droplet growth

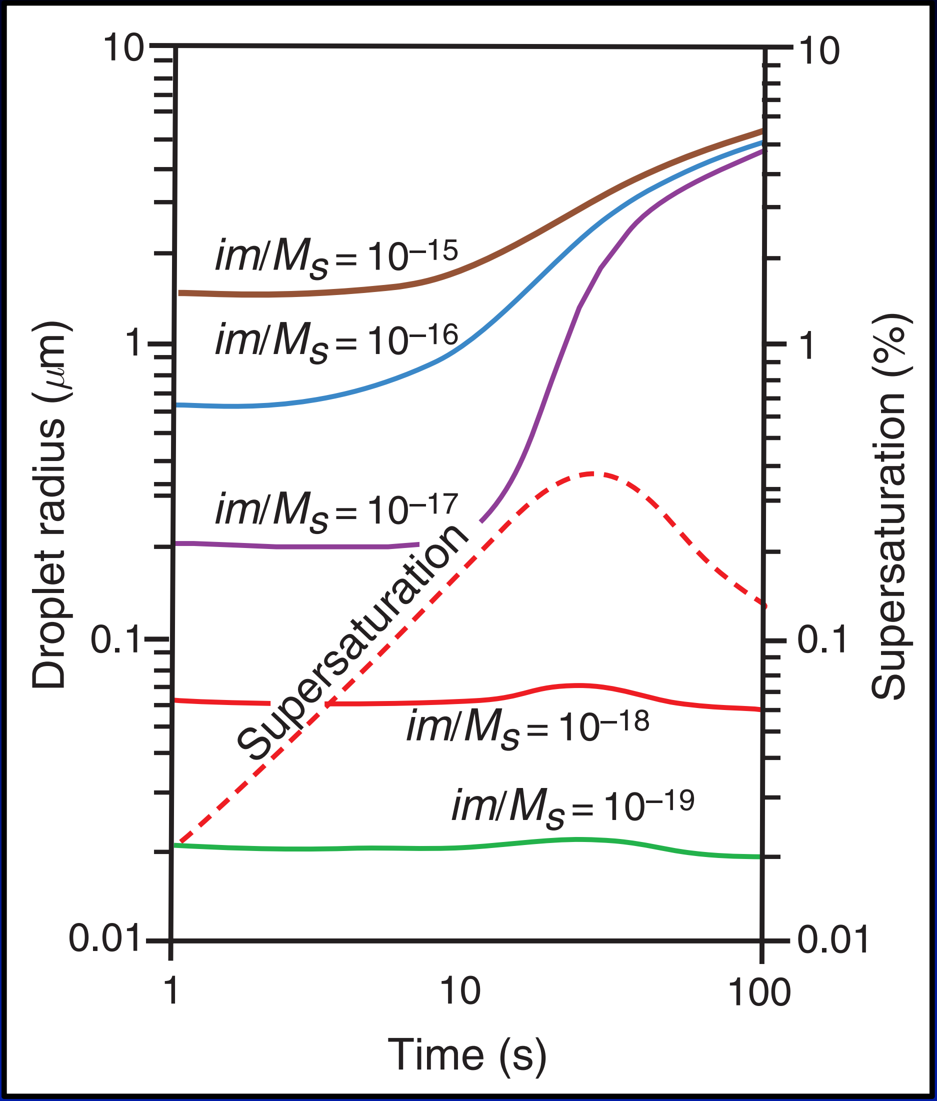
$$r\frac{dr}{dt} = \frac{De_s(T)}{\rho_l R_v T} S$$

neglects the solute and Kelvin curvature effects. These are not important for droplets with $r > 1\mu$m.

It also neglects the diffusion of heat away from the droplet as the droplet is warmed by condensation.

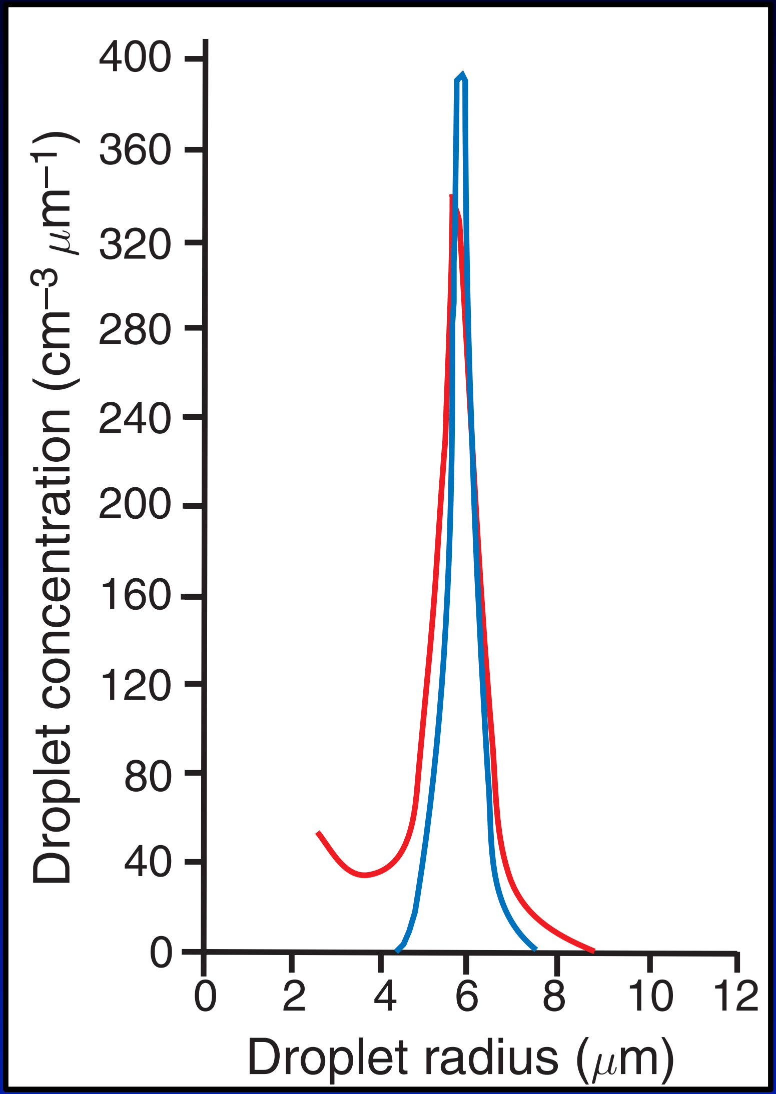
Condensation

- Calculation of the growth of CCN (500/cm^3) by condensation in an updraft of 60 cm/s.
- Activated droplets are monodisperse by 100 s.



Condensation

◆ Cloud droplet size distribution measured 244 m above cloud base (red), and corresponding DSD calculated with only condensation (blue).



Condensation

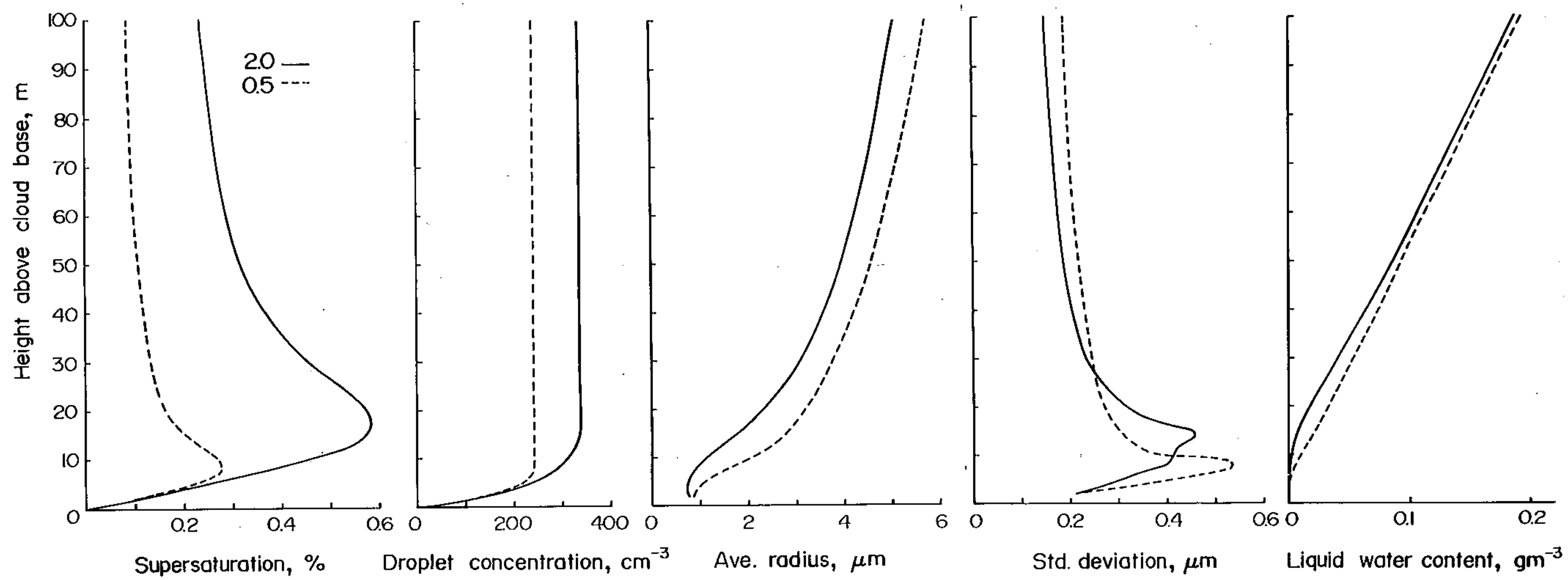
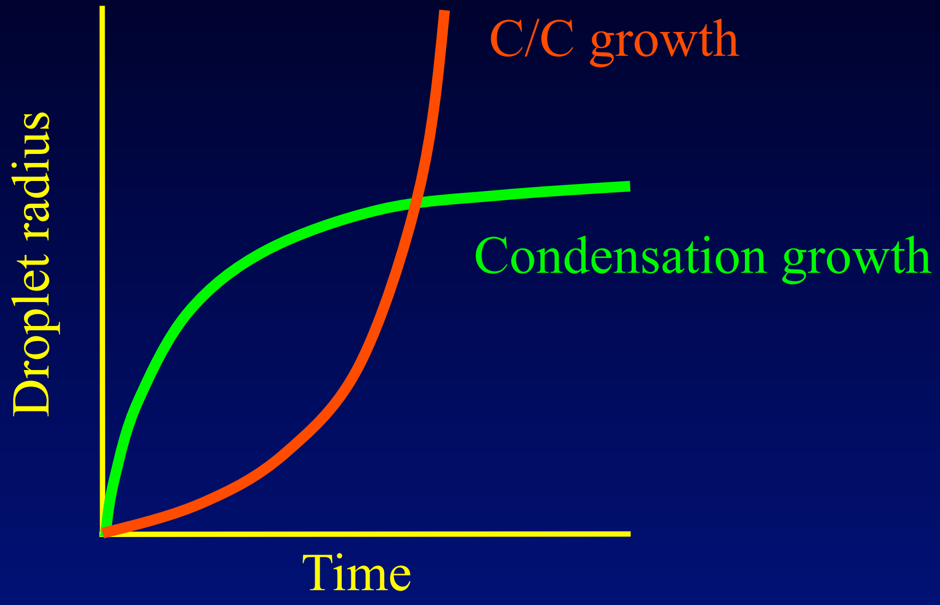
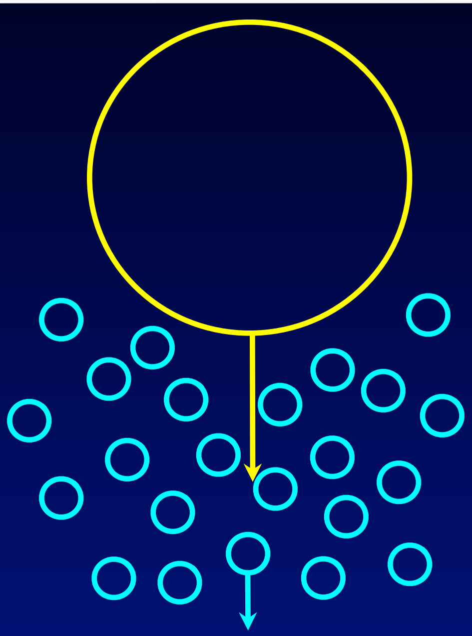


FIG. 7.4. Early development of cloud properties in air ascending at constant velocity of 0.5 m/s or 2 m/s.

Collision-coalescence





- Growth of droplets into raindrops is achieved by collision-coalescence.
- Fall velocity of a droplet increases with size.
- Larger drops collect smaller cloud droplets and grow.

Collision-coalescence

Terminal fall speed of a droplet in air is due to a balance between gravity and drag:

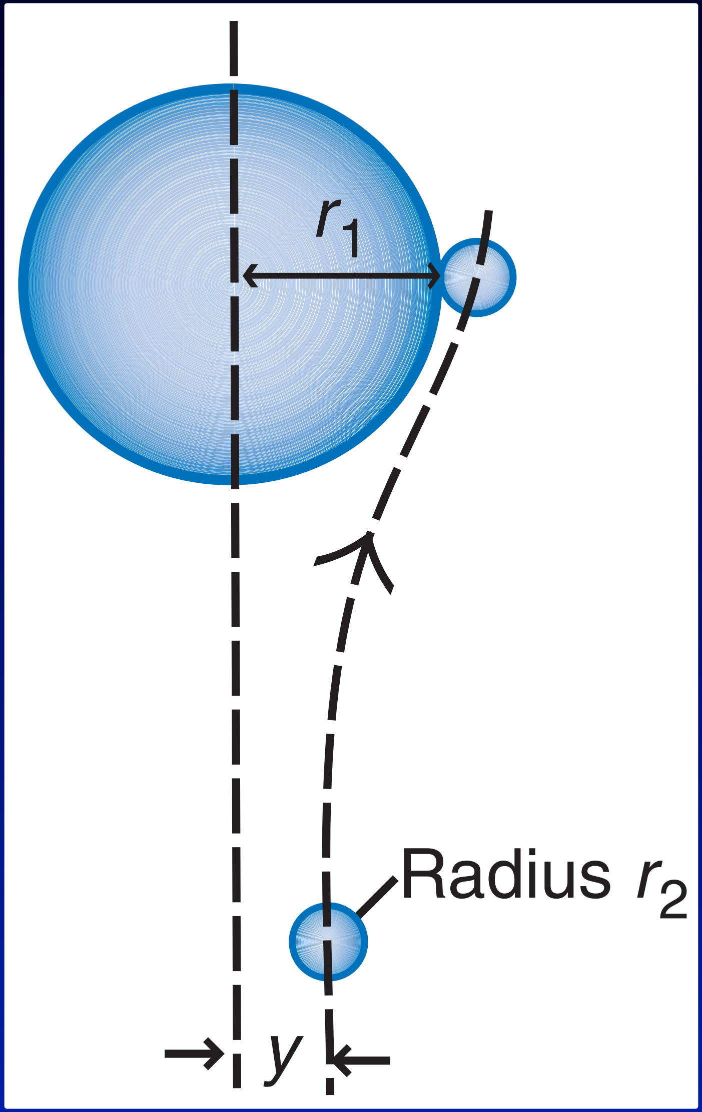
$$\frac{4}{3}\pi r^3 g\rho_l = F_{\text{drag}} = 6\pi\eta r v \ (r \leq 30\mu\text{m})$$

where v is the terminal velocity and η is the dynamic viscosity.

Solve for v:

$$v = \frac{2}{9}\frac{g\rho_l}{\eta}r^2 = k_1 r^2$$

with $k_1 = 1.19 \times 10^6$ cm^{-1} s^{-1}.



Collision-coalescence

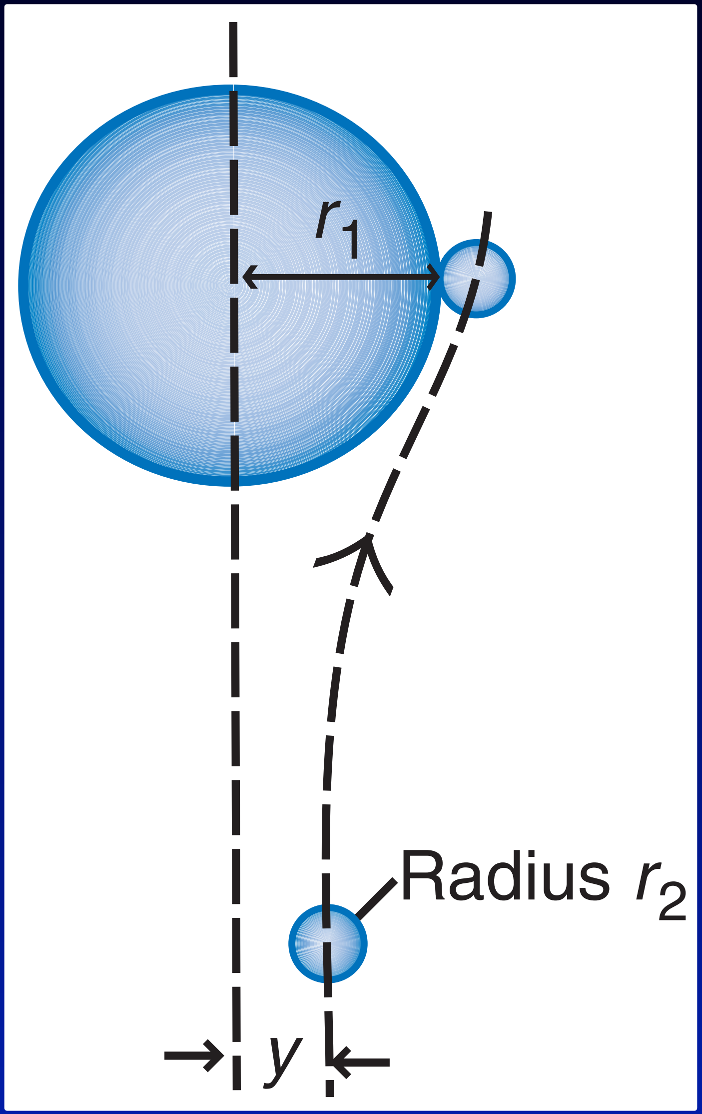
Relative motion of a droplet with respect to a collector drop. At the radius y the two make a grazing collision.

The *collision efficiency* is

$$E = \frac{\text{effective collision cross section}}{\text{geometrical collision cross section}}$$

therefore

$$E = \frac{y^2}{(r_1 + r_2)^2}$$



Collision-coalescence

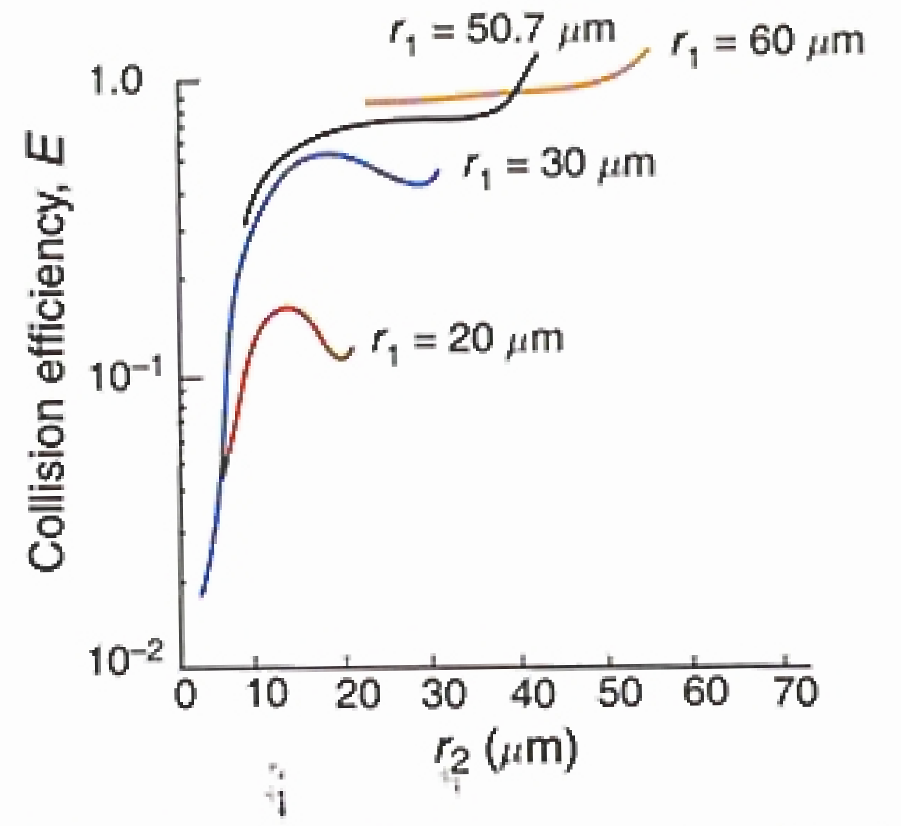
Relative motion of a droplet with respect to a collector drop. At the radius y the two make a grazing collision.

The *collision efficiency* is

$$E = \frac{\text{effective collision cross section}}{\text{geometrical collision cross section}}$$

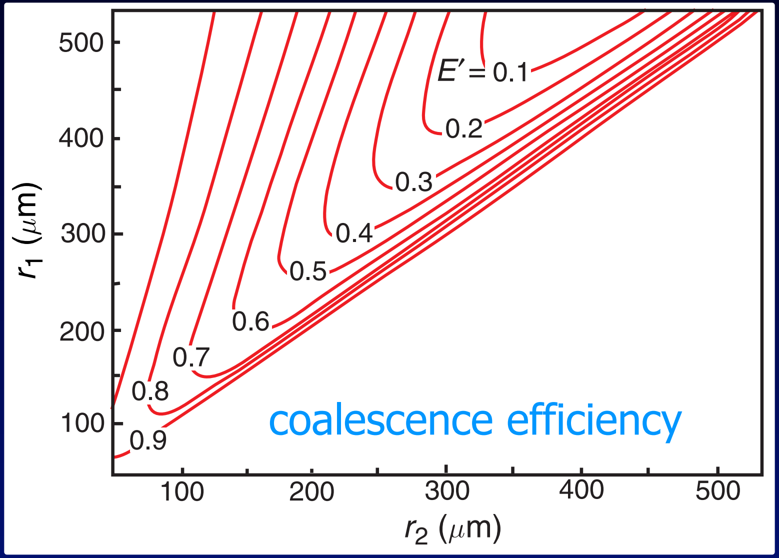
therefore

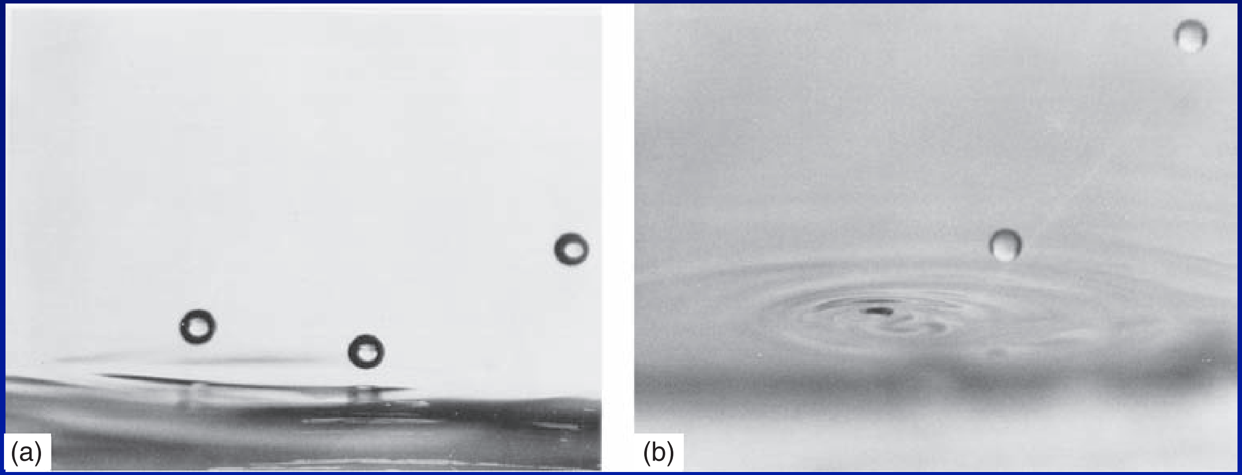
$$E = \frac{y^2}{(r_1 + r_2)^2}$$



Collision-coalescence

- Collection efficiency = collision efficiency X coalescence efficiency





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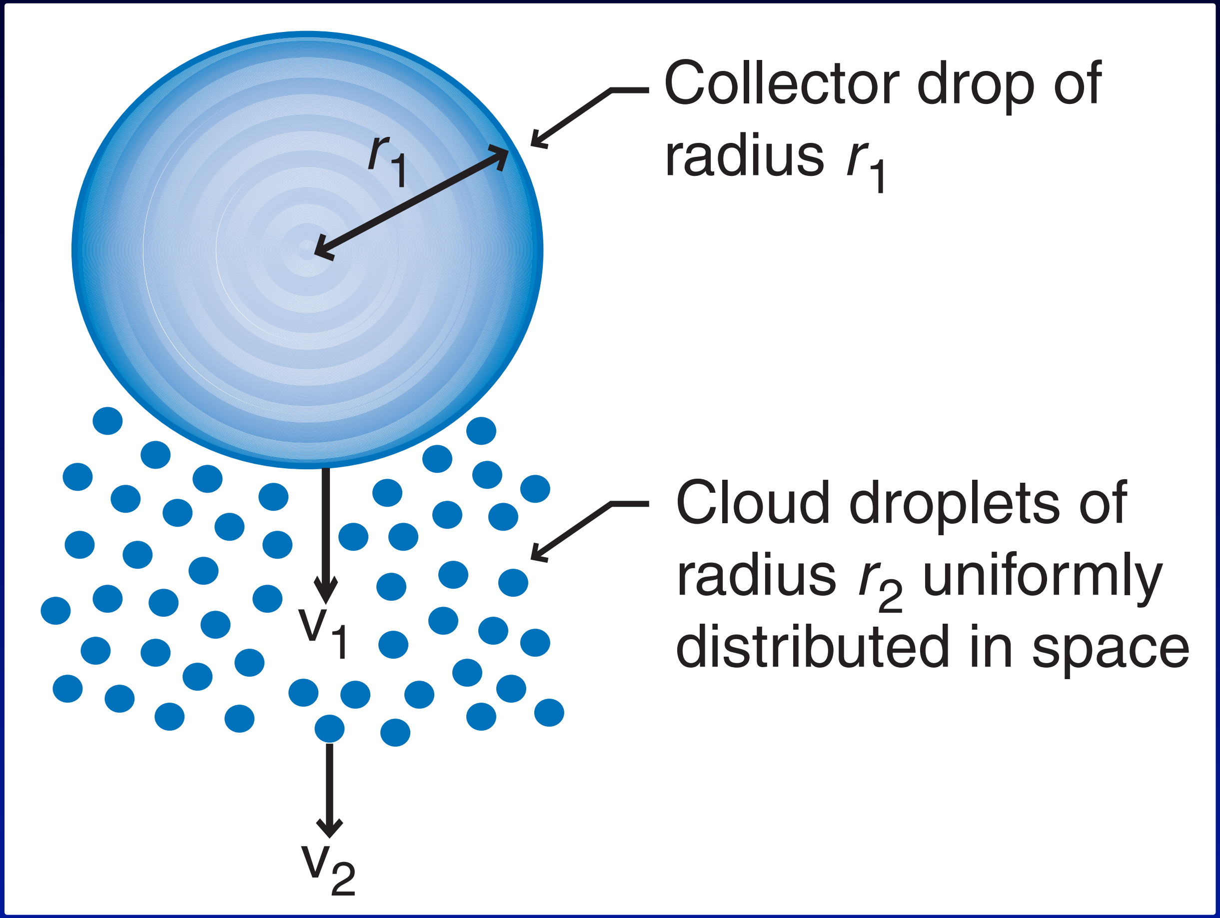
Collision-coalescence



Collector drop of radius r_1

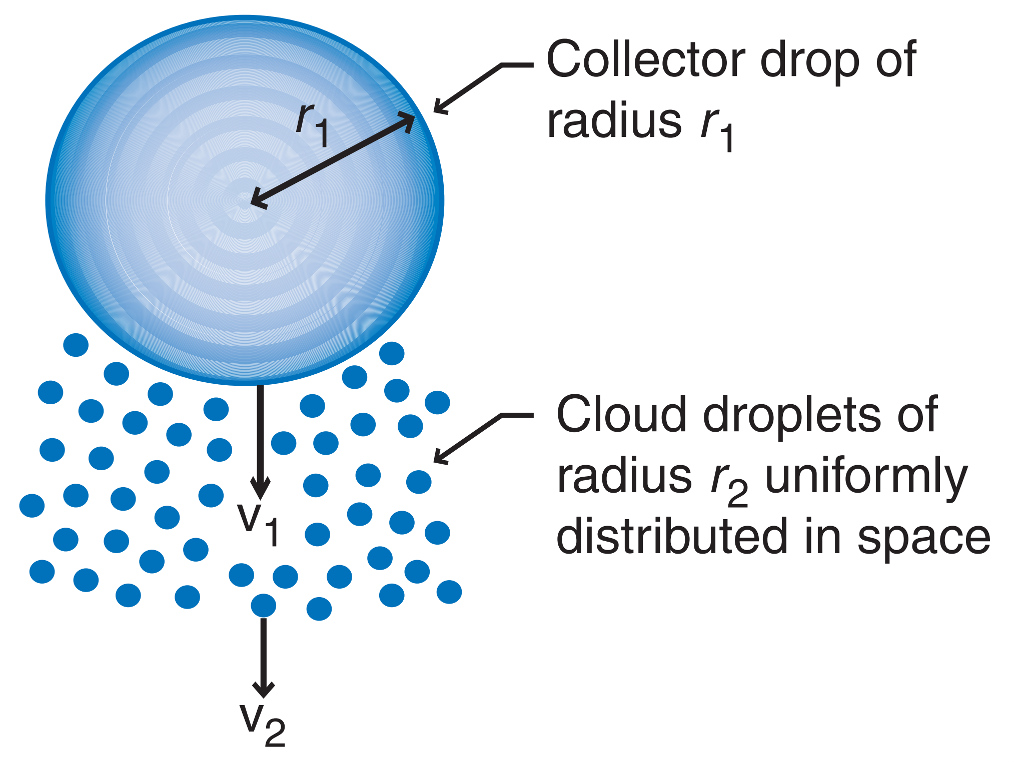
Cloud droplets of radius r_2 uniformly distributed in space

Collision-coalescence

According to the *continuous collection model*, the rate of increase of the collector drop's mass M due to collisions is the volume of the cylinder swept out per unit time by the collector drop moving at the relative velocity $v_1 - v_2$ × LWC × collection efficiency:

$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_l E_c$$

where w_l is the LWC of the cloud droplets of radius r_2.



Collision-coalescence

Substitute $M = 4/3\pi r_1^3 \rho_l$ into

$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_l E_c$$

to get

$$\frac{dr_1}{dt} = \frac{(v_1 - v_2) w_l E_c}{4\rho_l}$$

If $v_1 \gg v_2$ and $E_c = E$, then we get

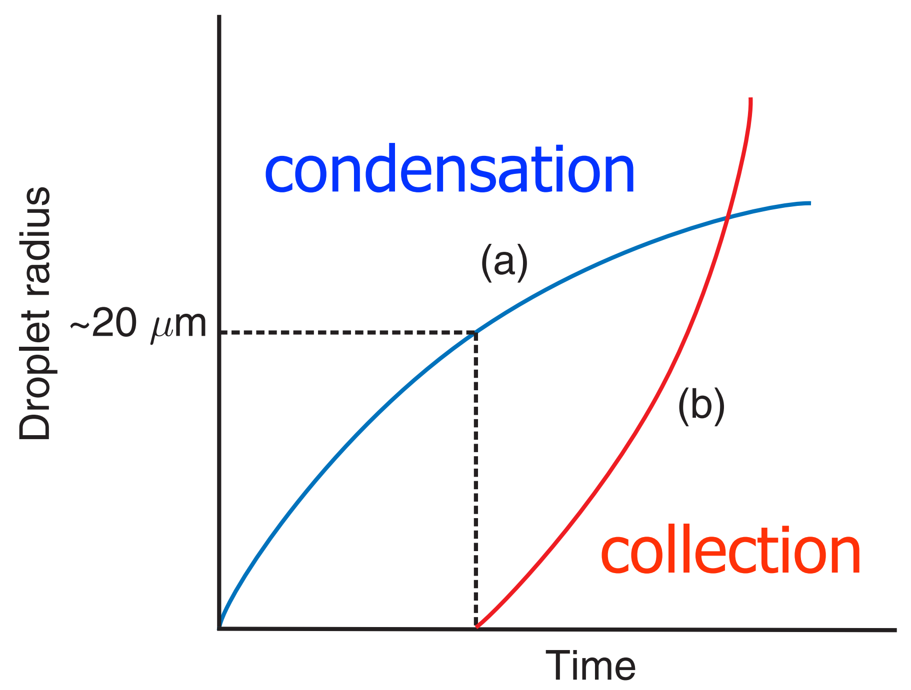
$$\frac{dr_1}{dt} = \frac{v_1 w_l E}{4\rho_l}$$

Note that for collection,

$$\frac{dr}{dt} \sim v(r) \sim r^2$$

while for condensation,

$$\frac{dr}{dt} \sim \frac{1}{r}$$



Collision-coalescence

If there is a steady updraft of speed w, the speed of a collector drop with respect to the ground is $w - v_1$, so that its motion is

$$\frac{dh}{dt} = w - v_1$$

where h is the height of the drop. Combine this with

$$\frac{dr_1}{dt} = \frac{v_1 w_l E}{4\rho_l}$$

to get

$$\frac{dr_1}{dh} = \frac{v_1 w_l E}{4\rho_l (w - v_1)}$$

Collision-coalescence

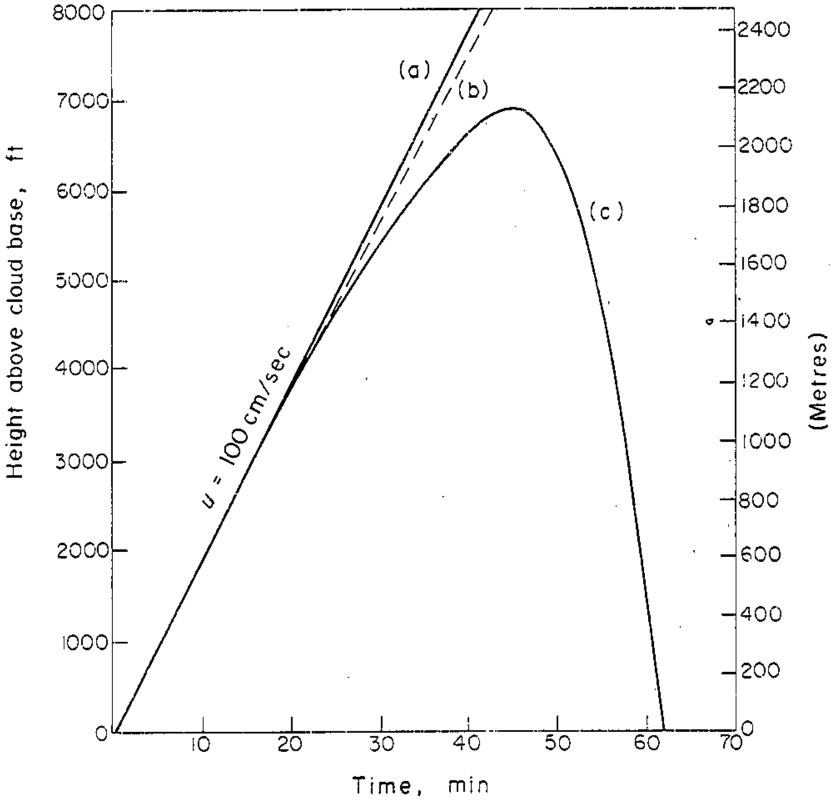


FIG. 8.4. Bowen's calculated trajectories of (a) the air, (b) cloud droplets, initially 10 μm in radius, and (c) drops which have initially twice the mass of the cloud droplets. Updraft speed 1 m/sec, cloud water content $M = 1$ g/m^3. (From Fletcher, 1962.)

Collision-coalescence

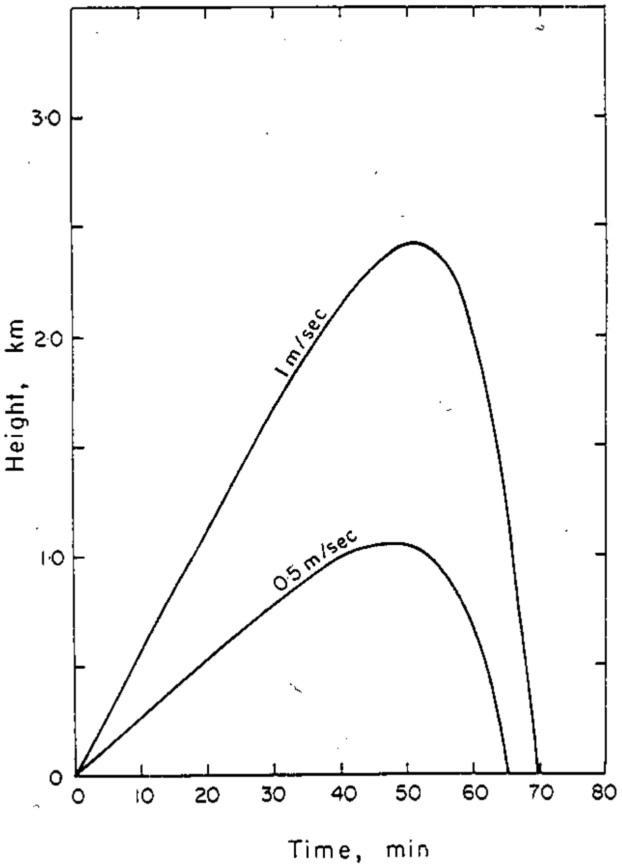


FIG. 8.5. Drop trajectories calculated for the collision efficiencies of Table 8.2 and Fig. 8.3, assuming a coalescence efficiency of unity. Initial drop radius 20 μm. Cloud water content 1 g/m^3; all cloud droplets of 10 μm radius.

Collision-coalescence

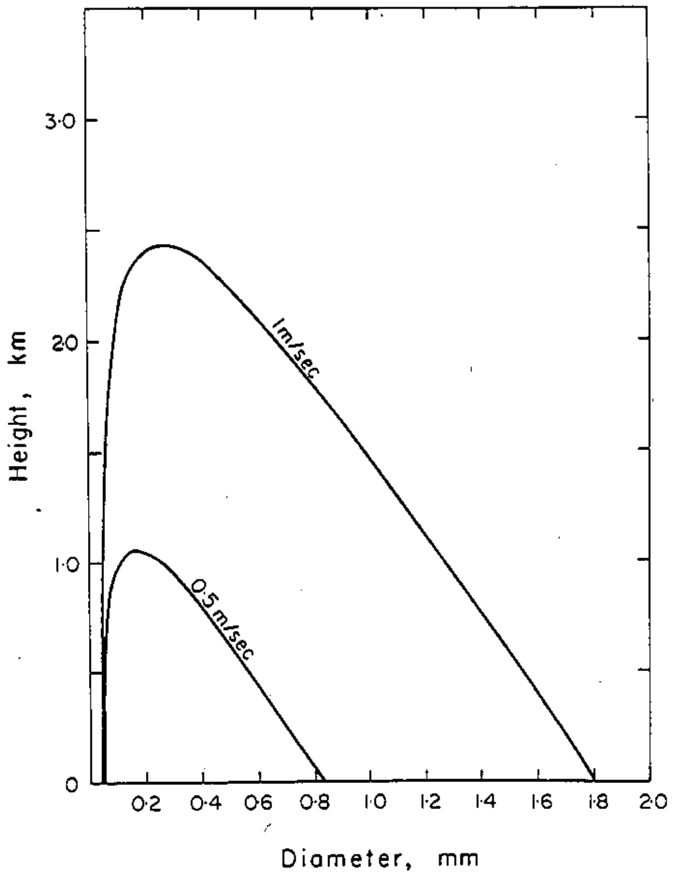


FIG. 8.6. Drop diameters for the trajectories of Fig. 8.5.

Collision-coalescence

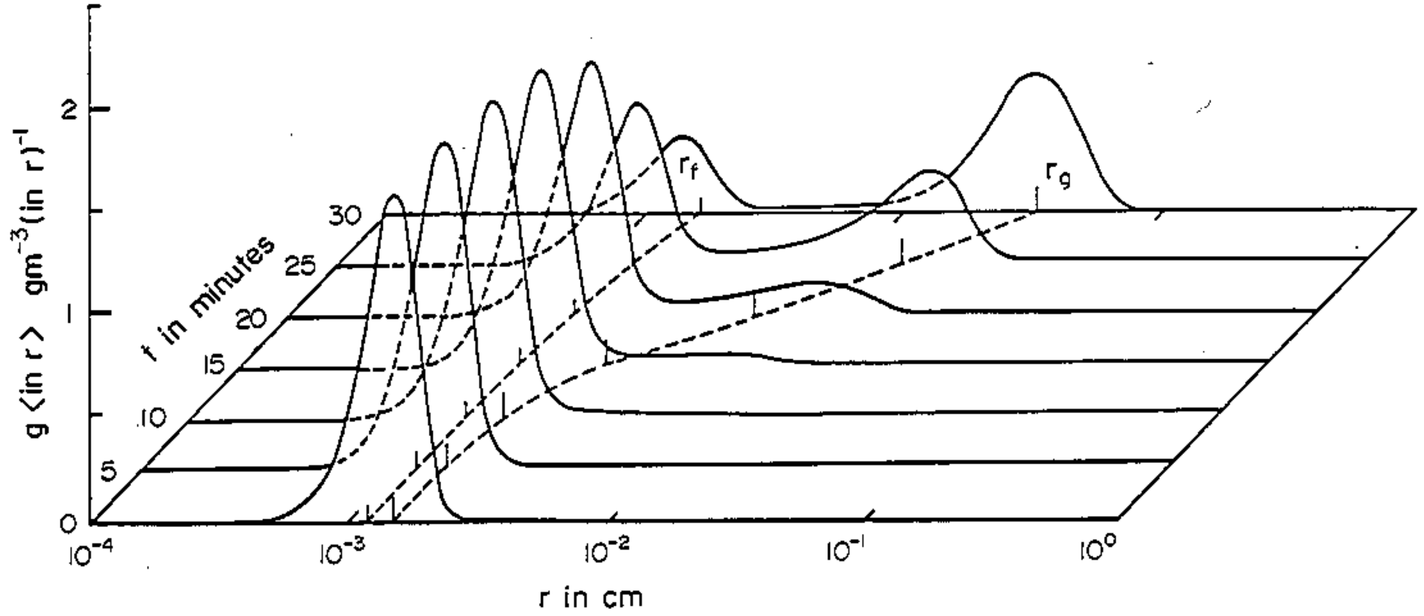


FIG. 8.10. Example of the development of a droplet spectrum by stochastic coalescence. (From Berry and Reinhardt, 1974b.)

Collision-coalescence

Rearrange this

$$\frac{dr_1}{dh} = \frac{v_1 w_l E}{4\rho_l (w - v_1)}$$

into

$$4\rho_l \frac{(w - v_1)}{v_1 E}\, dr_1 = w_l\, dh$$

then integrate

$$4\rho_l \int_{r_0}^{r_H} \frac{(w - v_1)}{v_1 E}\, dr_1 = \int_0^H w_l\, dh = w_l H$$

Collision-coalescence

$$H = \frac{4\rho_l}{w_l} \int_{r_0}^{r_H} \frac{(w - v_1)}{v_1 E}\, dr_1$$

A drop starts at cloud base with radius r_0, goes up and down in a cloud with constant E, then reaches the base again with radius R. Relate R to r_0 and the (constant) updraft speed w. Set $H = 0$ and $r_H = R$ to get

$$0 = \int_{r_0}^{R} \frac{(w - v_1)}{v_1}\, dr_1 = \int_{r_0}^{R} \frac{w}{v_1}\, dr_1 - \int_{r_0}^{R} dr_1$$

or

$$R = r_0 + w \int_{r_0}^{R} \frac{dr_1}{v_1}$$