Turbulence Closure, Scales, and Similarity Theory
Review Terms

- Statically stable
- Dynamically unstable
- Isotropic / anisotropic
- Turbulence tends to homogenize
“Unstable” profiles

• How does one even get an unstable potential temperature profile?
  • Because of outer space
    • Radiative – Convective (Dis)Equilibrium

• What profiles of wind velocity (shear) produce turbulence?
  • On the board

• Richardson # : A balance between mechanical production and buoyant dissipation of turbulence
\[ Ri = \frac{-B}{M} \approx \frac{g}{\theta_v} \frac{\partial \theta_v}{\partial z} \left( \frac{\partial \bar{u}}{\partial z} \right)^2 + \left( \frac{\partial \bar{v}}{\partial z} \right)^2 \]

For \( Ri < 0.25 \), laminar flow becomes turbulent.
For \( Ri > 1 \), turbulent flow becomes laminar.
For \( 0.25 < Ri < 1 \), the existence of turbulence depends on the flow’s history.
9.1.4 Turbulent transport and fluxes

• Kinematic heat flux: \( w' \theta' \)

• For a layer of air:

\[
\frac{\partial \theta}{\partial t} = - \frac{\partial w' \theta'}{\partial z} + \ldots
\]
9.1.4 Turbulent transport and fluxes

• For a layer of air:

\[
\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial \bar{w}' \bar{\theta}'}{\partial z} + \ldots
\]

• The turbulent flux of the heat flux:

\[
\frac{\partial \bar{w}' \bar{\theta}'}{\partial t} = -\frac{\partial \bar{w}' \bar{w}' \bar{\theta}'}{\partial z} + \ldots
\]
9.1.4 Turbulent transport and fluxes

- This is the turbulence closure problem
  - Make a “closure assumption”
  - Can such a parameterization ever be perfect?
9.1.5 Turbulence Closure

- Statistical Order and Non-localness
- An example:
- “Gradient-transfer theory” or “K-theory” or “Eddy-diffusivity theory”

\[ F_H = \overline{w' \theta'} = -K \frac{\partial \overline{\theta}}{\partial z} \]
9.1.5 Turbulence Closure

- This is a first-order closure
- K is an “Eddy diffusivity” – work out units on board

\[ F_H = \overline{w'\theta'} = -K \frac{\partial \overline{\theta}}{\partial z} \]
Turbulence Closure

• A local closure for $K$

• Relate to local TKE and a turbulence length scale:

\[ K = le, \]

where

\[ e = \left( \sigma_u^2 + \sigma_v^2 + \sigma_w^2 \right)^{1/2} \]
9.1.5 Turbulence Closure

- A local closure for $K$
  - Related to the local wind shear and a mixing length “$l$”

- $L$ is an average eddy size
  - At very low levels (in the surface layer) $L$ can be approximated by $kz$ where $k=0.4$

\[
K = l^2 \left| \frac{\partial V}{\partial z} \right|
\]

\[
l = \left( \frac{z^2}{2} \right)^{1/2}
\]

\[
l \approx kz
\]
9.1.5 Turbulence Closure

• What would a non-local closure for $K$ be?

\[ F_H = \overline{\omega' \theta'} = -K \frac{\partial \overline{\theta}}{\partial z} \]
9.1.6 Scales, scales, scales

• Velocity Scales

- Friction velocity:

\[
\begin{align*}
  u_* &= \left[ \frac{u' w'^2 + v' w'^2}{4} \right]^{1/4} \\
  u_*^2 &= \left( u' w' \right)
\end{align*}
\]

For one-dimensional case

- Convective velocity scale (Deardorff velocity):

\[
\begin{align*}
  w_* &= \left[ \frac{g \cdot z_i}{T_v} \left( \frac{w'}{\theta_S} \right) \right]^{1/3}
\end{align*}
\]

Deardorff velocity relevant to unstably stratified boundary layer

- \( z_i \) – height of capping inversion (PBL height)
- \( T_v \) – virtual temperature

Friction velocity relevant to statically neutral conditions in the surface layer
9.1.6 Scales, scales, scales

• Length Scales
  • $z_i$: the altitude of the inversion. Relevant length scale for statically unstable and neutral conditions
  • $z_0$: the roughness length
  • The Obukhov length $L$
    • For statically non-neutral conditions: This is the height in the surface layer below which mechanical production of turbulence dominates

\[
L = \frac{-u_*^3}{k \frac{g}{T_v} \left( \overline{w' \theta'} \right)_s}
\]
9.1.6 Scales, scales, scales

- Length Scales
  - $z_0$: the roughness length

\[
U(z) = \frac{u_\ast}{k} \log\left(\frac{z}{z_0}\right) \quad \text{Logarithmic wind profile valid for neutral conditions}
\]
HW 4 Problem 2 ($u_z=0.5$)

Neutral
Stable ($L=100$)
Unstable ($L=-10$)
9.1.6 Scales, scales, scales

• Time Scales
  • For convective boundary layer (turnover time for largest eddies)

    \[ t^* = \frac{z_i}{w^*} \]

• For neutral surface layer: (faster times possible)

    \[ t^* SL = \frac{z}{u^*} \]
9.1.6 Scales, scales, scales

• Summary:
  • For the convective boundary layer
    • $w^*$ and $z_i$
  • For the surface layer
    • If neutral: $u^*$ and $z_0$
    • If non-neutral: $u^*$ and $z_0$ and $L$
9.1.6 Similarity

- Example: Through the depth of a convective boundary layer:

\[
\frac{\overline{w'^2}}{w_*^2} = a \left( \frac{z}{z_i} \right)^b \left( 1 - c \frac{z}{z_i} \right)^d
\]

- Example: For a statically stable surface layer (compare with neutral log wind profile)

\[
\frac{V}{u_*} = 2.5 \left[ \ln \left( \frac{z}{z_0} \right) + 8.1 \frac{z}{L} \right]
\]