Atmospheric Sciences 5700

Exercise #6

This exercise is about numerical integration of the parcel model.

Each of the accompanying programs (in Matlab and Python) on the class web page numerically solves (integrates in time) the set of differential equations which governs parcel motions in an atmosphere given a profile of buoyancy $B(z)$:

$$\frac{dw}{dt} = g\frac{T - \bar{T}}{\bar{T}} \equiv B(z), \tag{1}$$

$$\frac{dz}{dt} = w. \tag{2}$$

These are the same equations as (8) and (9) in the notes on the numerical solution of parcel model equations. The default buoyancy profile is shown in Figure 1.

Each program contains three user-specified parameters:

dt Time step (s).

alt_buoy A logical switch: If **alt_buoy**=0, use default $B(z)$. If **alt_buoy**=1, use the alternate $B(z)$. The alternate $B(z)$ changes the buoyancy profile for the parcel below its EL after the parcel first reaches its maximum height. The alternate buoyancy profile is similar to that for a parcel undergoing dry adiabatic descent (or ascent).

Heun A logical switch: If **Heun**=0, use the Euler Forward time scheme. If **Heun**=1, use the Heun (or Euler Trapezoidal) time scheme.

Problems

1. Set **alt_buoy**=0 and **Heun**=0. Examine the convergence properties of the Euler Forward time scheme. Plot $w(t)$ vs $z(t)$ on the x and y axes for each **dt**.

    (a) Set **dt**=0.001 s. In this case, the solution has *converged*, which means it no longer changes if **dt** is decreased further. Note how much calculation time this takes. In Matlab, this will be printed as **Elapsed time is 0.569818 seconds** for example.

    (b) Increase the time step (**dt**) from 0.01 s to 10 s by factors of 10 (i.e, use 0.01, 0.1, etc), and note the changes in the plot and in the elapsed time.

(c) Use your w vs z plot for the converged case (dt=0.001 s) to answer these questions:

    i. What is the maximum height reached by the parcel?

    ii. What is the parcel's level of free convection (LFC)?

    iii. What is the parcel's maximum (positive) vertical velocity?

    iv. What is the height for the parcel's vertical velocity?

    v. What is the parcel's neutral buoyancy level (NBL)?

2. Set alt_buoy=1 and Heun=0. Examine the convergence properties of the Euler Forward time scheme for the alternate buoyancy profile. Plot $w(t)$ vs $z(t)$ for each dt.

    (a) Set dt=0.001 s. In this case, the solution has *converged*. Note how much calculation time this takes.

    (b) Increase the time step (dt) from 0.01 s to 10 s by factors of 10 (i.e, use 0.01, 0.1, etc), and note the changes in the plot and in the elapsed time.

    (c) What is the largest dt for which the result is converged (i.e., is unchanged from solution for dt=0.001 s)?

3. Set alt_buoy=1 and Heun=1. Examine the convergence properties of the Heun time scheme for the alternate buoyancy profile. Plot $w(t)$ vs $z(t)$ for each dt.

    (a) Set dt=0.001 s. In this case, the solution has *converged*. Note how much time this takes.

    (b) Increase the time step (dt) from 0.01 s to 10 s by factors of 10 (i.e, use 0.01, 0.1, etc), and note the changes in the plot and in the elapsed time.

    (c) What is the largest dt for which the Heun scheme result is converged?

    (d) Compare the elapsed time for this converged Heun scheme solution to that for the converged Euler Forward scheme (i.e., with dt=0.001 s) in Problem 2.
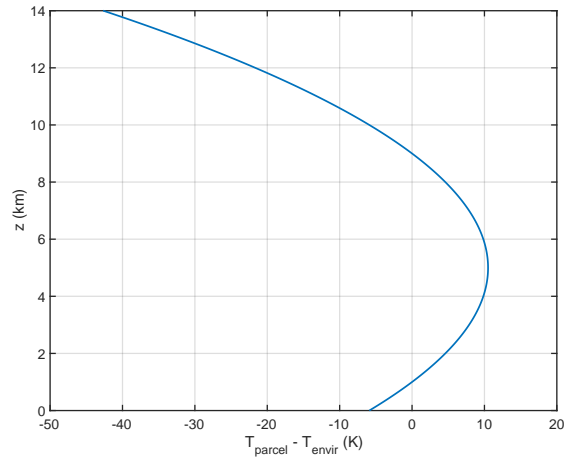
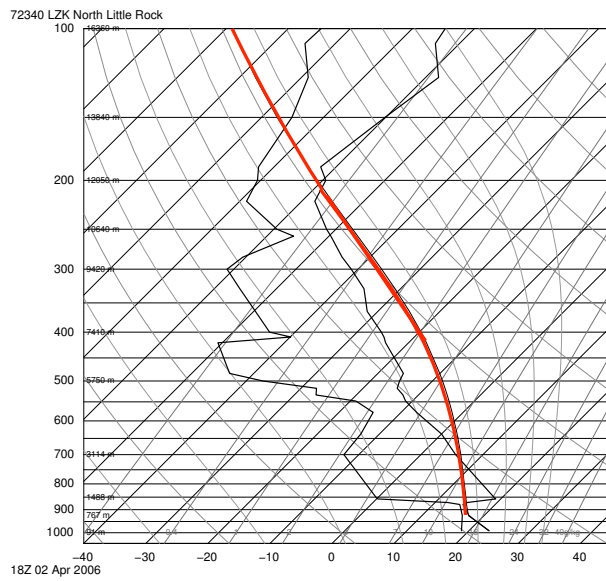Figure 1: $T_{\text{parcel}} - T_{\text{env}} = B(z) \times (\theta_0/g) = B(z) \times 28 \text{ K}/(\text{J kg}^{-1})$.

Figure 2: Red is ascending parcel temperature.

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