Mountain waves (Holton §9.4)

Relax assumptions used previously of periodic ridges, constant static stability and basic flow, and linear dynamics.

Flow over Isolated Ridges (§9.4.1)

- Can be approx. by the sum of a series of Fourier components (see §7.2.1).

- Represent topography as Fourier series

$$h_M(x) = \sum_{s=1}^{\infty} \mathrm{Re}\left[h_s \exp(i k_s x)\right] \qquad (9.29)$$

$k_s = \dfrac{2\pi s}{L}$, s = integer, L = distance around latitude circle,

h_s = amplitude of sth Fourier component.

- Then solution to wave eq. (7.46) is also a sum of Fourier components. Each w' component satisfies the b.c. due to a topography component h_s, just as (7.48) satisfies the b.c. due to a single Fourier component. Thus,

$$w'(x,z) = \sum_{s=1}^{\infty} \mathrm{Re}\left\{W_s \exp\left[i(k_s x + m_s z)\right]\right\} \quad (9.30)$$

where $W_s = i k_s \bar{u} h_s$ and $m_s^2 = N^2/\bar{u}^2 - k_s^2$, as req'd by the dispersion relationship (7.47).

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Ex. What is component s of w', if $m_s^2 > 0$?

$$\mathrm{Re}\{W_s \exp[i(k_s x + m_s z)]\}$$

$$= \mathrm{Re}\{i k_s \bar{u} h_s \exp[i(k_s x + m_s z)]\}$$

$$= \mathrm{Re}\{i k_s \bar{u} h_s [\cos(k_s x + m_s z) + i \sin(k_s x + m_s z)]\}$$

$$= -k_s \bar{u} h_s \sin(k_s x + m_s z).$$

This is the same form as (7.48) for $\bar{u}k < N$.

Each component or mode will be vertically propagating or decaying according to sign of m_s^2 (m_s real or imaginary) which depends on $k_s^2 \bar{u}^2$ compared to N^2.

Narrow ridge: Components with $k_s \bar{u} > N$ dominate topography (9.29), so wave decays.

Wide ridge: $k_s \bar{u} < N$ modes dominate topography, so wave vert. propagates.

Limiting case of wide mtn occurs when $m_s^2 \approx N^2/\bar{u}^2$ (i.e., $k_s \ll m_s$); waves periodic in z, with wavelength $2\pi/m_s$; phase lines tilt upstream as shown in Fig. 9.7.

Lee Waves (§9.4.2)

- If N and $\bar{u}$ vary with ht., then (7.46) is replaced by

$$\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2} + \ell^2 w' = 0 \qquad (9.31)$$

where the Scorer parameter ℓ is

$$\boxed{\ell^2 = \frac{N^2}{\bar{u}^2} - \frac{1}{\bar{u}}\frac{d^2\bar{u}}{dz^2}}$$

and the dispersion relationship becomes

$$m_s^2 = \ell^2 - k_s^2$$

(ℓ^2 replaces $N^2/\bar{u}^2$).

- Vertical propagation ($m_s^2 > 0$) now occurs for $k_s^2 < \ell^2$ instead of for $k_s^2 < N^2/\bar{u}^2$. If $d\bar{u}/dz = 0$, we recover previous condition.

- ℓ can vary with ht. if either N or $\bar{u}$ does.

  Example: $\bar{u}$ increases rapidly with ht., or N decreases rapidly with ht., so ℓ decreases rapidly.

  Lower layer: $\ell^2 > k_s^2$ vert. prop.

  Upper layer: $\ell^2 < k_s^2$ decaying.

Waves are reflected at interface, and may be "trapped" as shown in Fig. 9.8.

Downslope Windstorms (§9.4.3)

- Nonlinear processes are essential.

- Assume troposphere has a stable lower layer below a weakly stable upper layer, and that lower layer behaves like a barotropic fluid with a free surface $h(x,t)$.

- The motion of the lower layer may be described by the shallow water eqs. (§7.3.2) over topography $h_M(x)$.

- Characteristics of flow depend on Froude number $Fr \equiv \bar{u}^2/c^2$, where c is speed of shallow water gravity waves, $\sqrt{g(h-h_M)}$.

  Fr < 1 : subcritical flow; $\bar{u} < c$.
  Over mt, $h' < 0$, $u' > 0$, as shown in Fig. 9.9a. This has same structure as a surface gravity wave (Fig. 7.6) and is indeed such a wave stationary w.r.t. ground.

  Fr > 1 : supercritical flow ; $\bar{u} > c$.
  Gravity waves cannot play a role in establishing steady-state h, u over mt, since they are swept downstream.

Fluid thickens ($h' > 0$) and slows ($u' < 0$) over mtn. (Fig. 9.9b).

- Traffic flow analogy -
- From the nonlinear shallow water eqs.,

$$u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = 0 \qquad \text{mom. eq.} \quad (9.35)$$

$$\frac{\partial}{\partial x}\left[u(h - h_m)\right] = 0 \qquad \text{cont. eq.} \quad (9.36)$$

- we see that KE + PE is constant from (9.35):

$$\frac{\partial}{\partial x}\left(\frac{u^2}{2} + gh\right) = 0 \quad \text{so} \quad \frac{u^2}{2} + gh = \text{const.}$$

- Also, mass flux $u(h - h_m)$ is constant.

- $u \times$ (9.35), eliminate $\partial h/\partial x$ using (9.36):

$$\left(1 - Fr^2\right) \frac{\partial u}{\partial x} = \frac{ug}{c^2} \frac{\partial h_m}{\partial x} \qquad (9.37)$$

This shows that for

$$\left.\begin{array}{ll} Fr < 1 : & \partial u/\partial x > 0 \\ Fr > 1 : & \partial u/\partial x < 0 \end{array}\right\} \text{ for } \partial h_m/\partial x > 0 \text{ (upslope)}$$

- As subcritical flow ascends, Fr increases $\left(Fr = \frac{u^2}{c^2} = \frac{u^2}{g(h-h_m)}\right)$ since u increases and $h - h_m$ decreases.

If $Fr = 1$ at crest, then $\partial u/\partial x$ will continue to increase as $\partial h_m/\partial x < 0$ [see (9.37)] and flow descends with $Fr > 1$ until it adjusts back to ambient conditions in a hydraulic jump (Fig. 9.9c).

- Very high speeds can occur along lee slope since PE → KE during entire traverse over mt.

- Num. simuls. have demonstrated that the hydraulic jump model captures the essential dynamics involved in downslope windstorms.

Analogies

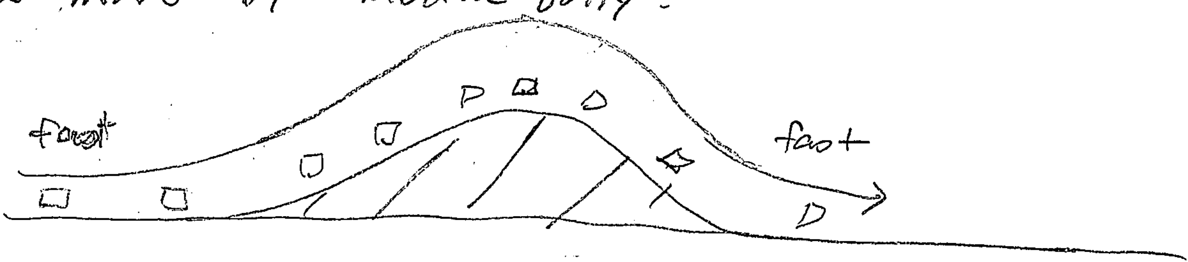
Traffic flow:

Can get same number of cars per hour (analogous to mass flux) in two flow regimes:

(1) Fast-moving, widely spaced cars

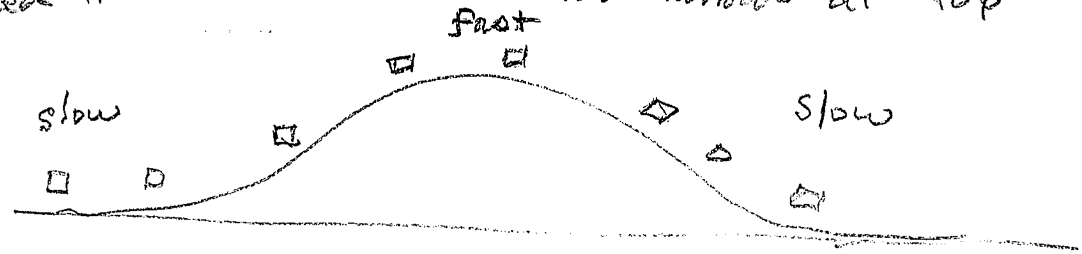
(2) Slow-moving, closely spaced cars

Cars move by inertia only.



"Supercritical"

Speed limit increases as road narrows at top



"Subcritical"

speed limit increases: [illegible] hill