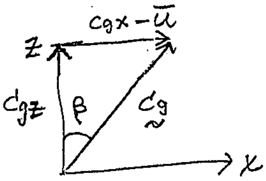
◇ We can show that $\underset{\sim}{c}_g$ is parallel to phase lines:

Let β be angle of $\underset{\sim}{c}_g$ to vertical. Then



$$\cos\beta = \frac{-|c_{gz}|}{\left[(c_{gx}-\bar{u})^2 + c_{gz}^2\right]^{1/2}}$$

$$= \frac{k}{(m^2+k^2)^{1/2}}$$

$$= \cos\alpha$$

$c_{gx}-\bar{u}$ is flow relative

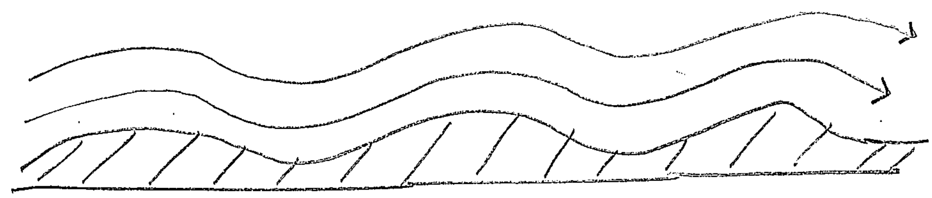
$k>0,\ m<0$

where α is angle of phase lines to vertical, derived previously.

◇ Group velocity is thus perpendicular to direction of phase propagation.

Topographic Waves (§7.4.2)

◇ Air parcels in a flow with mean speed $\bar{u}$ over a sinusoidal ridge pattern in statically stable conditions undergo buoyancy oscillations as shown in Fig. 7.10:



◇ Solutions are stationary relative to the ground (but not to the flow), so $\nu = 0$, and $\phi = kx + mz$ at all times, so

$$w' = \hat{w}\exp[i(kx+mz)].$$

Then the dispersion relationship (7.44) becomes (setting $\nu=0$)

$$-\bar{u}k = \pm Nk/(k^2+m^2)^{1/2}$$

or

$$\bar{u}^2 = N^2/(k^2+m^2)$$

or

$$\boxed{m^2 = N^2/\bar{u}^2 - k^2.} \qquad (7.47)$$

◇ For given N, k, and $\bar{u}$, (7.47) determines the vertical structure (i.e., m):

For $m^2 > 0$ (m real), $N > |\bar{u}k|$. $|\bar{u}k|$: intrinsic frequency

For $m^2 < 0$ (m imaginary), $N < |\bar{u}k|$. ~ car on washboard road

◇ $m^2 > 0$: m real, vertically propagating.

$$w' = \hat{w} \exp[i(kx + mz)]$$

Since the energy source is at the ground, the waves must transport energy upward, so vertical phase propagation is downward relative to the mean flow. (Group velocity upward ⇒ phase velocity downward)

With $k > 0$ & $\bar{u} > 0$, we have $\hat{\nu} = \nu - \bar{u}k = -\bar{u}k < 0$, so for $c_z = \frac{\hat{\nu}}{m} < 0$ (req'd for upward energy transport), $m > 0$.

Similarly, for $k > 0$ & $\bar{u} < 0$, $m < 0$. Phase lines tilt upstream.

◇ $m^2 < 0$: $m = i m_i$, vertically trapped.

$$w' = \hat{w} \exp(ikx) \exp(-m_i z), \qquad m_i \equiv (-m^2)^{1/2} > 0.$$

Choose this solution so $w' \to 0$ as $z \to \infty$.

◇ Summary:

Vertically propagating waves: $N > |\bar{u}k|$. Favored by:

small $\bar{u}$: weak flow
small k : wide ridges
large N : stable stratification

◇ Example

westerly mean flow over topography with height h_M:

$$h(x) = h_M \cos kx .$$

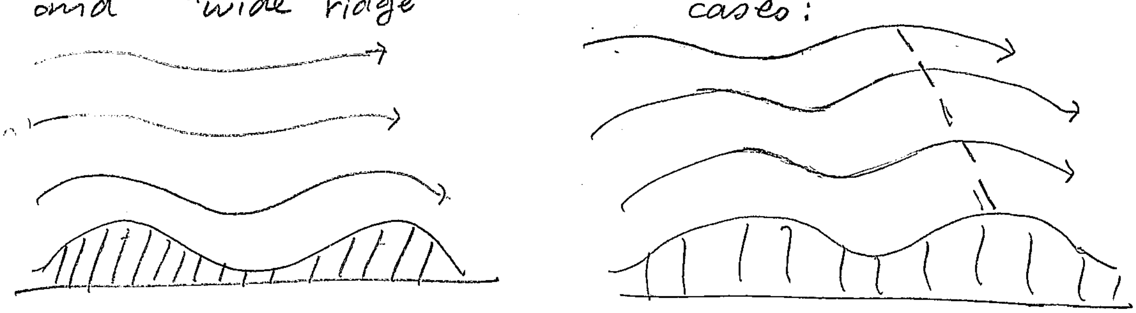
At surface

$$w'(x,0) = \left(\frac{Dh}{Dt}\right)_{z=0} = \bar{u}\frac{\partial h}{\partial x} = -\bar{u} h_M k \sin kx .$$

Solutions that satisfy this boundary condition are

$$w'(x,z) = \begin{cases} -\bar{u} h_M k \, e^{-\mu z} \sin kx & N < \bar{u}k \\ -\bar{u} h_M k \sin(kx+mz) & N > \bar{u}k \end{cases} \qquad (7.48)$$

In Fig. 7.10, these correspond to "narrow ridge" and "wide ridge" cases:



(a) "narrow ridge"
no tilt; decays upward

(b) "wide ridge"
tilts upstream;
no decay upward

◇ Limitations of this analysis

Both $\bar{u}$ and N usually vary with height;
ridges are usually isolated, not periodic.

Under certain conditions, large amplitude waves can form and may produce downslope windstorms. These are discussed in §9.4.

◇ Homework Problems due : 7.12