WH 9.1.6 Turbulence Scales & Similarity Theory

Velocity scales

Friction velocity:

$$u_* = \left[\overline{u'w'}^2 + \overline{v'w'}^2 \right]^{1/4} = \left| \frac{\tau_s}{\rho} \right|^{1/2} \qquad (9.14)$$

τ_s : stress at surface
(drag force per unit surface area)

Deardorff (or convective) velocity scale:

$$w_* = \left[\frac{g z_i (\overline{w'\theta_v'})_s}{\theta_v} \right]^{1/3} \sim 1 \text{ m/s} \quad (9.13)$$

z_i : BL depth
(See derivation from TKE eq.)

Length scales

Height of capping inversion : z_i
Aerodynamic roughness length : z_0
Height above surface : z
(Monin-)Obukhov length : L

$$(9.15) \qquad L \equiv \frac{-u_*^3}{k \cdot \frac{g}{\theta_v} (\overline{w'\theta_v'})_s} = \frac{-u_*^3}{w_*^3} \frac{z_i}{k}$$

(Ex. 9.1)

Mechanical production dominates below $z = L$.

Derive u_* and w_* from TKE eq. for surface layer

(Holton 5.2) (Not in Holton or WH.)

$$\frac{D\,(TKE)}{Dt} = MP + BPL + TR - \varepsilon \qquad (5.14)$$

$$BPL \equiv \frac{g}{\theta_0}\overline{w'\theta'}$$

$$MP \equiv -\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}.$$

Assume: balance between MP, BPL, ε, so

$$0 = MP + BPL - \varepsilon$$

CBL: Further simplify to $0 = BPL - \varepsilon$,

and assume $\varepsilon \sim \frac{w_*^3}{z_i}$ (dim. anal.)

Then $w_*^3 = \frac{g}{\theta_0}(\overline{w'\theta'})_s z_i$, used $\overline{w'\theta'} = (\overline{w'\theta'})_s$.

Neutral surface layer: Assume $BPL = 0$, so

$0 = MP - \varepsilon$, and $\varepsilon \sim \frac{u_*^3}{kz}$.

Also, align x-coordinate with sfc wind, so

$$0 = u_*^2 \frac{\partial \bar{u}}{\partial z} - \frac{u_*^3}{kz}, \text{ or}$$

$$\frac{d\bar{u}}{dz} = \frac{u_*}{kz}.$$

(WH 9.1.6)

Timescales

Eddy time scale for convective BL:

$$t_* = \frac{z_i}{w_*} \sim 10^3 s \sim 15 \text{ min}$$

Eddy time scale for neutral surface layer:

$$t_{*SL} = \frac{z}{u_*} \sim \frac{30 m}{0.3 m/s} \sim 10^2 s$$

Summary of scaling parameters

For CBL : w_*, z_i

For neutral surface layer : u_*, (z), z_0

Stratified surface layer: u_*, (z), z_0, L

Examples

(a) $\overline{w'^2}$ in CBL

$$\frac{\overline{w'^2}}{w_*^2} = a\left(\frac{z}{z_i}\right)^b \left(1 - c\frac{z}{z_i}\right)^d$$

a, b, c, d : constants

$a = 1.8$, $b = 2/3$, $c = 0.8$, $d = 2$ (Stull, 9.6.3c)

(b) Stable surface layer

$$\frac{V}{u_*} = \underbrace{2.5}_{\frac{1}{k}} \left(\log\left(\frac{z}{z_0}\right) + \underbrace{8.1}_{?\ 5 \text{ is usual value.}} \frac{z}{L}\right)$$

(note error in WH, p 385)

(Derived in 9.3.3)

(WH 9.1.b)

Ex. 9.1 (b) For $z_i = 1000$ m, $(\overline{w'\theta'})_s = 0.2\,K\frac{m}{s}$, $\frac{\tau_s}{\rho} = u_*^2 = 0.2\,\frac{m^2}{s^2}$, what is $\frac{L}{z_i}$?

use $\left|\frac{L}{z_i}\right| = +\frac{u_*^3}{k w_*^3} = \frac{(0.2)^{3/2}}{0.4 \times 6.5} = 0.034.$

$$w_*^3 = \frac{g}{\theta_v} z_i (\overline{w'\theta'})_s = \frac{10 \cdot 1000 \cdot 0.2}{300} \approx 6.5\ m^3 s^{-3}$$

So MP dominates ~~below~~ $z=34$ m.

WH 9.3.3 wind (vertical structure)

Have previously derived (in 2 ways)

$$(9.23)' \quad \frac{d\bar{u}}{dz} = \frac{u_*}{kz}, \quad \text{which integrates to}$$

$$(9.22) \quad \bar{u} = \frac{u_*}{k} \log(z/z_0) \quad \text{log wind profile.}$$

N.B. $V = \bar{u}$ in WH.

Ex. 9.3(a) What is eddy viscosity K?

$$\overline{u'w'} = -K_m\, \partial\bar{u}/\partial z, \text{ or}$$

$$u_*^2 = K_m \frac{u_*}{kz}, \text{ so}$$

$$K_m = u_* k z.$$

Can generalize (9.23)' to

$$\phi_m = \frac{kz}{u_*}\frac{d\bar{u}}{dz}. \qquad (9.23)$$

For neutral flow, $\phi_m = 1$.

Under stable conditions, that are still turbulent, $(z/L > 0)$

$$\phi_m = 1 + \frac{\cancel{8.1}}{5}\frac{z}{L} \qquad (9.25)$$

(WH 9.3.3)

For unstable stratification:

$$(9.26) \qquad \phi_m = \left[1 - 15\,\frac{z}{L}\right]^{-\frac{1}{4}} \qquad \text{(error in text, p.394)}$$

<u>Ex. 9.4</u> Integrate (9.25) to derive $\bar{u}(z)$ for stable conditions. Assume $\bar{u}(z_0) = 0$; u_* , L : constants.

$$(9.25) \qquad \frac{kz}{u_*}\frac{d\bar{u}}{dz} = 1 + \beta\,\zeta\,\frac{z}{L} \qquad \zeta \equiv \frac{z}{L},\ \beta = 8$$

$$d\bar{u} = \frac{u_*}{k}\left[\frac{dz}{z} + \beta\,\frac{dz}{L}\right]$$

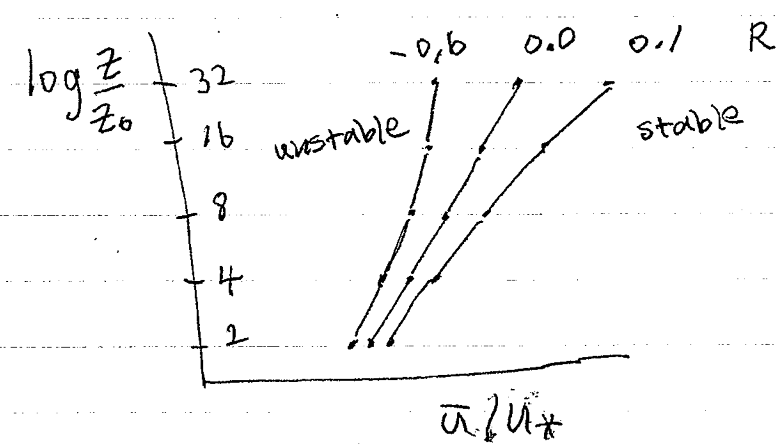
Integrate from z_0 to z:

$$\bar{u}(z) - \bar{u}(z_0) = \frac{u_*}{k}\left[\log\left(\frac{z}{z_0}\right) + \beta\,\frac{z - z_0}{L}\right]$$

$$\frac{\bar{u}(z)}{u_*} = \frac{1}{k}\left[\log(z/z_0) + \beta\,\frac{z}{L}\right] \qquad z_0 \ll z$$

"log - linear" profile.

See (WH Fig 9.17) CSB p. 615:



See WH problem 9.21: plot $\bar{u}(z)$ for $L = \infty, 100, -10$ m. $z_0 = 0.1$ m