Cumulus Parameterization: Cloud Ensemble Budget Equations

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Entrainment and Detrainment

The cloud can be horizontally expanding or shrinking. Therefore the mass gained or lost by the cloud per unit height and time is

$$\frac{\partial M_i}{\partial z} + \frac{\partial}{\partial t}(\rho \sigma_i).$$

where σ_i is the fractional area covered by the ith cloud and $M_i = \rho \sigma_i w_i$ is the mass flux of the ith cloud.

Entrainment occurs when mass is added to the cloud. Then the entrainment rate (the rate of mass addition per unit height and time) is

$$E_i = \frac{\partial M_i}{\partial z} + \frac{\partial}{\partial t}(\rho \sigma_i) > 0.$$

Detrainment occurs when mass is lost from the cloud. Then the detrainment rate (the rate of mass loss per unit height and time) is

$$D_i = -\left(\frac{\partial M_i}{\partial z} + \frac{\partial}{\partial t}(\rho \sigma_i)\right) > 0.$$

Budget Equations for a Single Cloud

Entrainment layer The budget equations for the entrainment layer of the ith cloud are

$$\textbf{mass: } \frac{\partial}{\partial t}(\rho\sigma_i) = E_i - \frac{\partial M_i}{\partial z}$$

$$\textbf{dry static energy: } \frac{\partial}{\partial t}(\rho\sigma_i s_i) = E_i\tilde{s} - \frac{\partial}{\partial z}(M_i s_i) + LC_i + Q_{Ri}$$

$$\textbf{water vapor: } \frac{\partial}{\partial t}(\rho\sigma_i q_i) = E_i\tilde{q} - \frac{\partial}{\partial z}(M_i q_i) + C_i$$

where C_i is the net condensation rate in the ith cloud and Q_{Ri} is the radiative heating rate in in the ith cloud.

Detrainment layer The budget equations for the detrainment layer of the ith cloud are

$$\textbf{mass: } \frac{\partial}{\partial t}(\rho\sigma_i) = -D_i - \frac{\partial M_i}{\partial z}$$

$$\textbf{dry static energy: } \frac{\partial}{\partial t}(\rho\sigma_i s_i) = -D_i s_{Di} - \frac{\partial}{\partial z}(M_i s_i) + LC_i + Q_{Ri}$$

$$\textbf{water vapor: } \frac{\partial}{\partial t}(\rho\sigma_i q_i) = -D_i q_{Di} - \frac{\partial}{\partial z}(M_i q_i) + C_i$$

where s_{Di} and q_{Di} are the dry static energy and water vapor mixing ratio of the air detrained from the ith cloud, respectively. In general, $s_{Di} \neq s_i$ and $q_{Di} \neq q_i$.

Budget Equations for a Cloud Ensemble

For simplicity, assume that the cloud ensemble consists of a single cloud type. Then $\sigma = \sum_i \sigma_i$ is the fractional area of the cloud ensemble, $M_c = \sum_i M_i$, $s_c = s_i$, and $q_c = q_i$. Also assume that the cloud ensemble is in a statistically steady state so

$$\frac{\partial}{\partial t}(\rho\sigma) = \frac{\partial}{\partial t}(\rho\sigma s_c) = \frac{\partial}{\partial t}(\rho\sigma q_c) = 0.$$

Then,

$$E = \sum_i E_i = \sum \frac{\partial M_i}{\partial z} = \frac{\partial M_c}{\partial z} > 0,$$

and

$$D = \sum_i D_i = -\sum \frac{\partial M_i}{\partial z} = -\frac{\partial M_c}{\partial z} > 0.$$

The resulting budget equations for the entrainment layer of the cloud ensemble are

$$\textbf{mass: } 0 = E - \frac{\partial M_c}{\partial z}$$

$$\textbf{dry static energy: } 0 = E\tilde{s} - \frac{\partial}{\partial z}(M_c s_c) + LC + Q_{R_c}$$

$$\textbf{water vapor: } 0 = E\tilde{q} - \frac{\partial}{\partial z}(M_c q_c) + C$$

where $C = \sum_i C_i$.

The corresponding budget equations for the detrainment layer of the cloud ensemble are

$$\textbf{mass: } 0 = -D - \frac{\partial M_c}{\partial z}$$

$$\textbf{dry static energy: } 0 = -Ds_c - \frac{\partial}{\partial z}(M_c s_c) + LC + Q_{R_c}$$

$$\textbf{water vapor: } 0 = -Dq_c\frac{\partial}{\partial z}(M_c q_c) + C$$

where we have assumed for simplicity that $s_{Di} = s_i$ and $q_{Di} = q_i$.