A Simple Model of Evaporatively Driven Downdrafts

Atmospheric Sciences 6150

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For precipitating convection, we have the following set of equations for potential temperature, θ, mixing ratios of water vapor, w, cloud water, l, and rainwater, r;

$$\frac{d\theta}{dt} = \frac{L}{c_p\bar{\pi}}(C - E_r) + D_\theta \tag{1}$$

$$\frac{dw}{dt} = -(C - E_r) + D_w \tag{2}$$

$$\frac{dl}{dt} = C - A_r + D_l \tag{3}$$

$$\frac{dr}{dt} = \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}\left(\bar{\rho}V_t r\right) - E_r + A_r + D_r \tag{4}$$

The dynamics are governed by the equations for the $x-$, $y-$, and $z-$ velocity components, U, V, and W; and non-dimensional pressure perturbation, π_1:

$$\frac{dU}{dt} = -c_p\bar{\theta}_v\frac{\partial\pi_1}{\partial x} + D_U \tag{5}$$

$$\frac{dV}{dt} = -c_p\bar{\theta}_v\frac{\partial\pi_1}{\partial y} + D_V \tag{6}$$

$$\frac{dW}{dt} = -c_p\bar{\theta}_v\frac{\partial\pi_1}{\partial z} + g\left(\frac{\theta - \bar{\theta}}{\bar{\theta}} + 0.61(w - \bar{w}) - l - r\right) + D_W \tag{7}$$

$$\frac{\partial \pi_1}{\partial t} = -\frac{c_s^2}{c_p \bar{\theta}_v^{\,2}} \left[\frac{\partial}{\partial x}(\bar{\theta}_v U) \frac{\partial}{\partial y}(\bar{\theta}_v V) + \frac{\partial}{\partial z}(\bar{\theta}_v W) \right]. \tag{8}$$

where θ_v is the virtual potential temperature, C is the net condensation rate, E_r is the rain evaporation rate, A_r is the cloud-to-rain water conversion rate, V_t is the mass-weighted average terminal velocity of rain drops (defined to be positive), and D_i represents the effects of turbulent mixing. Overbars indicate hydrostatic, reference state values.

The equations that govern the rates of change with height of a parcel's thermodynamic properties are:

$$\frac{d\theta}{dz} = \gamma \tilde{C} - \lambda(\theta - \theta_e) \tag{9}$$

$$\frac{dw}{dz} = -\tilde{C} - \lambda(w - w_e) \tag{10}$$

$$\frac{dl}{dz} = \tilde{C} - \tilde{A}_r - \lambda(l - l_e) \tag{11}$$

$$\frac{dr}{dz} = \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} V_t r) - \tilde{E}_r + \tilde{A}_r - \lambda(r - r_e) \tag{12}$$

In (9)-(12), the process rates are per unit *increase in height,* and $\gamma \equiv L/(c_p \bar{\pi})$.

This set governs the properties of an air parcel as it undergoes ascent or descent.

The net condensation rate, $\tilde{C}$, in (9)-(11), is implicitly determined by the saturation adjustment algorithm. The autoconversion rate, $\tilde{A}_r$, in (11) represents the net conversion of cloud water to rain, that is, both the loss of condensate by precipitation falling out of a parcel and the gain of condensate as precipitation falls into the parcel from above:

$$\tilde{A}_r \equiv \left(\frac{dl}{dz} \right)_{\text{conversion to rain}} = -\tilde{C}_2 \, l, \tag{13}$$

for $dz/dt > 0$ only, with $\tilde{C}_2 = 0.15 \text{ km}^{-1}$. The rate was empirically determined from a cloud-resolving model simulation and is expected to vary when, for example, vertical shear is included.

Entrainment is represented by the last term in each of (9)-(12). The fractional rate of entrainment per unit height is denoted by λ, while θ_e, w_e, l_e, and r_e represent the entrained values of potential temperature and the mixing ratios of water vapor, cloud water, and rain water.

We derived the following equation for the parcel vertical velocity, W, by parameterizing the vertical perturbation pressure gradient acceleration:

$$\frac{1}{2}\frac{dW^2}{dz} = aB - b\,\lambda\,W^2, \tag{14}$$

where

$$B \equiv g\left(\frac{\theta-\bar{\theta}}{\bar{\theta}} + 0.61(w-\bar{w}) - l - r\right).$$

The values used for a range from 1/3 to 1, while for b, 2 has been widely used.

Convective downdrafts are primarily produced and maintained by the drag and evaporation of precipitation. Because precipition particles are large and have low concentrations compared to cloud droplets, they usually are not able to evaporate rapidly enough to maintain saturation, As a consequence, convective downdrafts are typically subsaturated. Precipitation particles also fall rapidly with respect to the air, so that a parcel model is generally not appropriate for convective downdrafts.

However, we can construct a one-dimensional column model that is more realistic than a parcel model while still being much simpler than a multi-dimensional model. In this model, we predict the values of the model variables as a function of both height and time. The column equations are similar to (1)-(4) and (7), except that each total derivative is replaced with a local time derivative and vertical advection term:

$$\frac{\partial\theta}{\partial t} = -W\frac{\partial\theta}{\partial z} + \frac{L}{c_p\bar{\pi}}(C-E_r) + D_\theta \tag{15}$$

$$\frac{\partial w}{\partial t} = -W\frac{\partial w}{\partial z} - (C-E_r) + D_w \tag{16}$$

$$\frac{\partial l}{\partial t} = -W\frac{\partial l}{\partial z}C - A_r + D_l \tag{17}$$

$$\frac{\partial r}{\partial t} = -W\frac{\partial r}{\partial z} + \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(\bar{\rho}V_t r) - E_r + A_r + D_r \tag{18}$$

$$\frac{\partial W}{\partial t} = -W\frac{\partial W}{\partial z} - c_p\bar{\theta_v}\frac{\partial\pi_1}{\partial z} + g\left(\frac{\theta-\bar{\theta}}{\bar{\theta}} + 0.61(w-\bar{w}) - l - r\right) + D_W \tag{19}$$

and each entrainment term D_ϕ has the form

$$D_\phi = -\lambda|W|(\phi - \phi_e).$$

For the case of a subsaturated downdraft, we can further simplify the set (15)-(19) by setting $l = C = A_r = 0$ and by neglecting the vertical perturbation pressure gradient acceleration:

$$\frac{\partial \theta}{\partial t} = -W\frac{\partial \theta}{\partial z} - \frac{L}{c_p\bar{\pi}}E_r - \lambda|W|(\theta - \theta_e) \tag{20}$$

$$\frac{\partial w}{\partial t} = -W\frac{\partial w}{\partial z} + E_r - \lambda|W|(w - w_e) \tag{21}$$

$$\frac{\partial r}{\partial t} = -W\frac{\partial r}{\partial z} + \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(\bar{\rho}V_t r) - E_r - \lambda|W|(r - r_e) \tag{22}$$

$$\frac{\partial W}{\partial t} = -W\frac{\partial W}{\partial z} + g\left(\frac{\theta - \bar{\theta}}{\bar{\theta}} + 0.61(w - \bar{w}) - r\right) - \lambda|W|W \tag{23}$$

All that remains is to specify the functional forms of E_r and V_t. Emanuel §10.4 describes parameterizations for these based on Marshall-Palmer (or exponential) raindrop size distributions.