The Convective Mass Flux Model

σ : updraft fraction

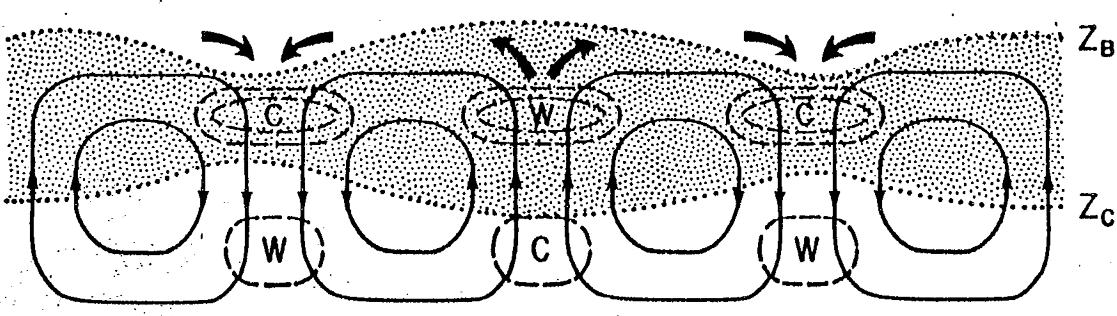
$$\bar{\psi} = \sigma\psi_u + (1-\sigma)\psi_d$$

The convective flux F_ψ is

$$F_\psi = \rho\overline{w'\psi'} = M_c(\psi_u - \psi_d)$$

where

$$\begin{aligned} M_c &\equiv \rho\sigma(1-\sigma)(w_u - w_d) \\ &= \rho\sigma(w_u - \bar{w}) \\ &\approx \rho\sigma w_u \end{aligned}$$



Schubert et al. 1979

Meteorology 6150: Exercise 9
Shallow-layer Convection: Mass Flux Model for Convective Fluxes
Due December 5, 2000

This exercise is based on the three non-linear simulations that you performed for Exercise 7. Do all calculations at the w-levels. For potential temperature, you will need to average vertically to obtain values at the w-levels.

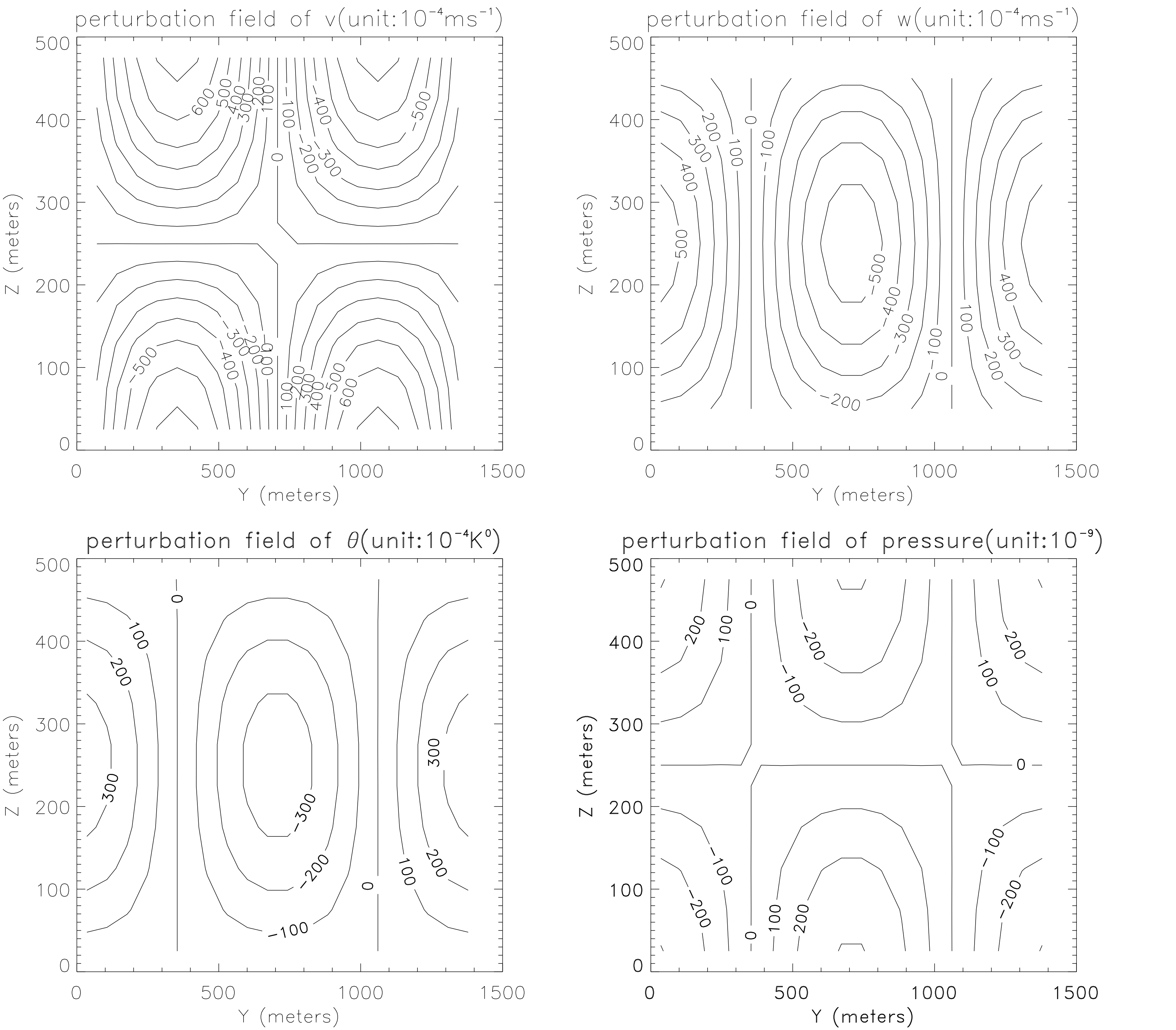
1. Calculate and plot the profile of the steady-state average updraft speed w_u for each case.

2. Calculate and plot the profile of the steady-state average updraft fractional area σ for each case.

3. Calculate and plot the profile of the steady-state average updraft potential temperature θ_u, downdraft potential temperature θ_d, and their difference $\theta_u - \theta_d$, for each case.

4. Plot the updraft mass flux profile $M_u = \sigma w_u$ for each case.

5. Estimate the convective heat flux profile $\overline{w'\theta'}$ using $M_u(\theta_u - \theta_d)$ for each case. Compare to the actual convective heat flux profiles that you obtained in Exercise 7, part 3.



perturbation field of v(unit:10⁻⁴ms⁻¹)
perturbation field of w(unit:10⁻⁴ms⁻¹)
perturbation field of θ(unit:10⁻⁴K⁰)
perturbation field of pressure(unit:10⁻⁹)
Z (meters)
Y (meters)

At $z = \frac{H}{2}$, $H=1$ — <u>solution for linear convection</u>

$$W_1(z) = A_1 \sin(1 \cdot \pi \cdot 0.5)$$
$$= A_1 \sin(\pi/2)$$
$$\underline{W_1 = A_1}$$

So $w(y, 0.5) = W_1 \cos(k_c y) = A_1 \cos(k_c y)$

$k_c = \frac{\pi}{\sqrt{2}}$, $\boxed{L = 2\sqrt{2}}$

$\cancel{y\, k_c\, 2\pi} = \cancel{\frac{2\pi}{L}}$

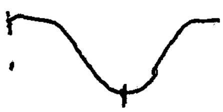
$k_c y = 2\pi$ when $y = \frac{2\pi}{k_c} = \frac{2\pi}{\pi/\sqrt{2}}$, $y = 2\sqrt{2}$

At $z = 0.5$:

$$g\frac{\theta'}{\theta_0} \equiv B = \frac{\nu}{k_c^2}\left(\cancel{\frac{d^2}{dz^2}} - k_c^2\right)^2 W_1 \cos(k_c y)$$
$$= \frac{\nu}{\cancel{k_c^2}}\left(k_c^2\right)^{\cancel{2}} W_1 \cos(k_c y)$$
$$= \nu k_c^2 A_1 \cos(k_c y) \equiv B_1 \cos(k_c y)$$

~~use above to get~~ true flux (at $z = \frac{H}{2}$)

$$\overline{w'\theta'} = \overline{w(\theta - \bar{\theta})} = \overline{w\theta} - \bar{w}\bar{\theta} = \overline{w\theta} \quad (\text{since } \bar{w} = 0)$$
$$= \frac{1}{L}\int_0^L w\theta \, dy = \frac{1}{L}\int_0^L A_1 B_1 \cos^2(k_c y)\, dy$$
$$= \frac{1}{2L}\int_0^L A_1 B_1 \left(\cos^2(k_c y) + \sin^2(k_c y)\right) dy$$
$$= \frac{1}{2L}\int_0^L A_1 B_1 \, dy = \frac{A_1 B_1}{2}.$$



mass flux model: $\overline{w'\theta'} = \sigma\left(\overline{w'\theta'}\right)_u + (1-\sigma)\left(\overline{w'\theta'}\right)_d$

$\left(\overline{w'\theta'}\right)_u = w_u(\theta_u - \bar{\theta})$, $\overline{w'\theta'}_d = w_d(\theta_d - \bar{\theta})$, so

$$\overline{w'\theta'} = \sigma w_u \theta_u - \sigma w_u \bar{\theta} + (1-\sigma) w_d \theta_d - (1-\sigma) w_d \bar{\theta}$$
$$= \sigma w_u \theta_u + (1-\sigma) w_d \theta_d - \bar{\theta}\underbrace{\left[\sigma w_u + (1-\sigma) w_d\right]}_{= \bar{w} = 0}.$$

$$w_u = \frac{1}{L/2}\int_0^{L/2} A_1 \sin(k_c y)\, dy = \frac{2}{L} A_1 \left(-\frac{1}{k_c}\cos(k_c y)\right)\Big|_0^{L/2}$$
$$= \frac{2}{L} A_1 \left(-\frac{\pi}{\sqrt{2}}\right)^{-1}\left[-1-1\right] = \cancel{A_1 \frac{2}{2\sqrt{2}}\frac{\sqrt{2}}{\pi} 2} = \cancel{A_1\pi}$$

$k_c L/2 = \boxed{\frac{\pi}{\sqrt{2}}\frac{2\sqrt{2}}{2} = \pi}$

$\underline{w_u = A_1 \frac{2}{\pi}}$: ~~incorrect~~ $\Big] = A_1 \frac{2}{\pi}$

$$w_d = -w_u$$

$$\theta_u = B_1 \pi$$

$$\theta_d = -\theta_u$$

$$\overline{w'\theta'} = \sigma w_u \theta_u + (1-\sigma) w_d \theta_d$$

$$= \sigma w_u \theta_u + (1-\sigma) w_u \theta_u$$

$$= w_u \theta_u$$

$$= A_1 \frac{2}{\pi} \, B_1 \frac{2}{\pi}$$

$$\boxed{\overline{w'\theta'} = A_1 B_1 \frac{4}{\pi^2}}$$

$$\frac{1}{\pi}\int_0^{\pi} \sin\theta \, dy = \frac{1}{\pi}\int_0^{\pi} d(-\cos\theta) = \frac{1}{\pi}(-\cos\theta)\Big|_0^{\pi}$$

$$= \frac{1}{\pi}(1+1) = \frac{2}{\pi}$$

Ratio of mass flux model to true flux: (at $z = H/2$)

$$\frac{A_1 B_1 \, 4/\pi^2}{A_1 B_1 / 2} = \frac{8}{\pi^2} = \underline{0.81}$$