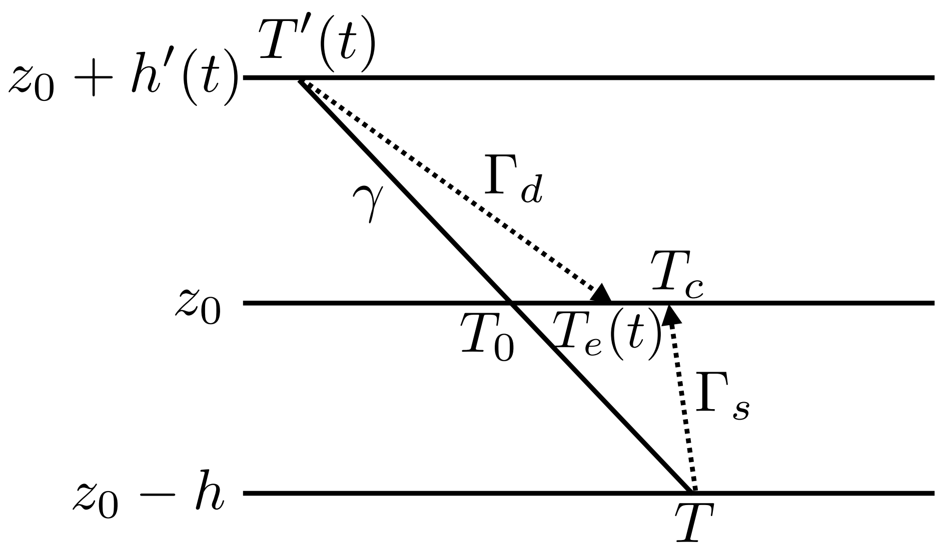
The Slice Method

Atmospheric Sciences 6150

Consider an ensemble (or population) of cumulus clouds consisting of cloudy (saturated) updrafts with temperature T_c at height z_0, and speed w_c and fractional area σ at heights $z \geq z_0$, but below cloud top height, z_t. The clouds are embedded in a clear (unsaturated) environment with temperature T_e at height z_0, and initial lapse rate $\gamma = -dT/dz$ at heights between z_0 and z_t. Assume that the air between the clouds at height z_0 remains unaffected by mixing with air from the cloudy updrafts during the time period of interest. This is a good assumption when the cumulus cloud top level z_t is sufficiently far above z_0, and the time period of interest is sufficiently small.



1. Cumulus cloud updrafts are non-hydrostatic, and are primarily driven by buoyancy, B, so that we can write
$$\frac{dw_c}{dt} \approx B.$$
If we ignore contributions to B from water vapor and condensate loading, B can be expressed mathematically in terms of T_c and T_e. Do so.

   Answer:
$$g\frac{T_c - T_e}{T_e}.$$

2. The horizontally averaged vertical velocity (averaged over cloudy and clear regions) is $\bar{w}$ at heights at heights between z_0 and z_t. Express the vertical velocity of the air between the clouds, w_e, in terms of $\bar{w}$, w_c, and σ.

   Answer:

   Conservation of mass requires
$$\bar{w} = \sigma w_c + (1 - \sigma) w_e,$$
   where w_e is the vertical velocity in the clear air between the cloudy updrafts. Solving for w_e gives
$$w_e = \frac{\bar{w} - \sigma w_c}{1 - \sigma}. \tag{1}$$

3. Assume that the cloudy updrafts originate in a boundary layer whose properties do not change with time, such as might be encountered over an ocean region. Under these conditions, show that
$$\frac{d(T_c - T_e)}{dt} = -\left(\frac{g}{c_p} - \gamma\right)\frac{\sigma w_c - \bar{w}}{1 - \sigma},$$
   where g is the acceleration of gravity and c_p is the specific heat capacity at constant pressure.

   Answer:

   In this case, the temperature of a cloudy updraft at a given height does not change with time, so that T_c is constant, and $d(T_c - T_e)/dt = -dT_e/dt$. The diagram shows that the air between the clouds at height z_0 and time t has descended from a height $z_0 + h'(t)$ and warmed due to adiabatic compression:
$$T_e(t) = T_0 + (\Gamma_d - \gamma)h'(t),$$

where $T_0 = T_e(0)$ and $\Gamma_d = g/c_p = 9.8$ K/km is the dry adiabatic lapse rate.

Take the time derivative to get

$$\frac{dT_e}{dt} = (\Gamma_d - \gamma)\frac{dh'(t)}{dt}.$$

Recognizing that $dh'(t)/dt = -w_e$, this becomes

$$\frac{dT_e}{dt} = -(\Gamma_d - \gamma)w_e. \tag{2}$$

This could also be obtained by applying the adiabatic thermodynamic equation,

$$\frac{\partial T}{\partial t} = -w(\Gamma_d + \frac{\partial T}{\partial z}),$$

to the air between the clouds.

Now use (1) to substitute for w_e in (2) to get

$$\frac{dT_e}{dt} = (\Gamma_d - \gamma)\frac{\sigma w_c - \bar{w}}{1 - \sigma}. \tag{3}$$

Finally, use $d(T_c - T_e)/dt = -dT_e/dt$ and $\Gamma_d = g/c_p$ in (3) to get

$$\frac{d(T_c - T_e)}{dt} = -\left(\frac{g}{c_p} - \gamma\right)\frac{\sigma w_c - \bar{w}}{1 - \sigma}.$$

4. Assume that a sufficient condition for the existence of cumulus clouds at height z_0 is $T_c > T_e$. What is the lifetime of the cumulus ensemble (time until $T_e = T_c$) when $T_c > T_e$ initially and $\bar{w} = 0$?

   Answer:

   Set $\bar{w} = 0$ in (3) to obtain

$$\frac{dT_e}{dt} = (\Gamma_d - \gamma)\frac{\sigma w_c}{1 - \sigma}.$$

   Integrate $T_e(t)$ from T_0 to T_c to get the time, t. The r.h.s. is a constant, so we get

$$t = (T_0 - T_c)\left(\frac{dT_e}{dt}\right)^{-1}.$$

5. Observations show that a typical value of σ is about 0.1. Based on your knowledge of the typical characteristics of cumulus clouds and of the lower atmosphere when cumulus clouds are present, estimate the characteristic lifetime of a cumulus ensemble (not of an individual cumulus cloud) when $\bar{w} = 0$.

   Answer:

   Typical values are $T_c - T_e = 1$ K, $w_c = 2$ m/s, and $\gamma = 6.5$ K/km. Plugging these values into the answer for Problem 4, we get 1364 s = 23 minutes.

6. Assuming that, initially, $T_c > T_e$, under what conditions could a cumulus ensemble persist indefinitely. Give an example, and suggest values of the relevant parameters.

   Answer:

   To obtain $dT_e/dt = 0$, we must have either $\gamma = \Gamma_d$ or $w_e = 0$. Rarely is $\gamma = \Gamma_d$ through a deep layer of the troposphere, so this condition is not the relevant one. The condition for $w_e = 0$ is

$$\bar{w} = \sigma w_c.$$

   For $\sigma = 0.1$ and $w_c = 2$ m/s, $\bar{w} = 0.2$ m/s. For $\sigma = 0.01$ and $w_c = 20$ m/s, $\bar{w} = 0.2$ m/s.

7. Express dT_e/dt, as given by (3),

$$\frac{dT_e}{dt} = (\Gamma_d - \gamma)\frac{\sigma w_c - \bar{w}}{1 - \sigma},$$

   in terms of the dry static energy of the environment, $s_e = c_p T_e + gz$, when $\sigma \ll 1$.

   Answer:

$$\frac{1}{c_p}\frac{ds_e}{dt} = \frac{dT_e}{dt},$$

$$\frac{1}{c_p}\frac{\partial s_e}{\partial z} = \frac{\partial T_e}{\partial z} + \frac{g}{c_p} = \Gamma_d - \gamma,$$

   and

$$\frac{\sigma w_c - \bar{w}}{1 - \sigma} \approx \sigma w_c - \bar{w},$$

   so (3) becomes

$$\frac{ds_e}{dt} = -\bar{w}\frac{\partial s_e}{\partial z} + \sigma w_c \frac{\partial s_e}{\partial z}.$$