Arya, Chapter 2: Energy budget near the surface

2.1 Energy fluxes at an ideal surface

Flux = amount per unit time passing through a unit area

Energy flux units: J s^{-1} m^{-2} or W m^{-2}.

Ideal surface:
Smooth, horizontal, homogeneous, extensive, opaque

Types of energy fluxes:

Net radiation flux \( R_n \)
sensible heat flux \( H \)
latent heat flux \( H_L \)
soil (or water) heat flux \( H_S \)

\( R_n \): mainly solar (downward) in day, IR (upward) at night
\( H \): due to \( \Delta T = T - T_s \)
mode: conductive near surface, turbulent (or convective) beyond a few mm.
\( H \) is upward in day, downward at night.
$H_L$: due to evaporation, evapotranspiration, or condensation (dew) at the surface. Occurs due to gradients of water vapor. Heating or cooling of the surface occurs due to latent heat of evaporation during the phase change.

Boisen ratio $= \frac{H}{H_L} = B$.

$H_L = L_e E$, $L_e = 2.5 \times 10^6$ J kg$^{-1}$. $E$: evaporate rate

$H_G$: By conduction through ground (soil, rock). But in water, conduction occurs only at the interface; then convection transports the energy further.

Layer affected by diurnal variations:
- < 1 m for land
- several 10's m for lakes, oceans,
- (What else is different about energy fluxes into water?)

2.2 Energy Balance Eqs.

2.2.1 Surface Energy Budget (SEB)

\[ \begin{align*}
& \text{Surface} \\
& \downarrow \quad \text{thin interface} \\
& R_N \quad H \quad H_L \\
& \downarrow \quad \text{(no mass, no heat capacity)} \\
& H_G
\end{align*} \]

Conservation of energy then requires net flux into surface $= 0$, or
\[ R_N = H + H_L + H_G, \quad (2.1) \]
where \( R_N > 0 \) toward sfc., others > 0 away from sfc.

**Daytime** (Fig. 2.1(a)):

\[ \begin{align*}
R_N & \quad \downarrow H \quad H_L \\
& \quad \downarrow \quad \downarrow \quad \uparrow \\
& \quad \downarrow H_G \\
& \quad R_N, H, H_L, H_G > 0 \quad \text{typically}
\end{align*} \]

**Nighttime** (Fig. 2.1(b)):

\[ \begin{align*}
R_N & \quad \uparrow H \quad H_L (\text{dew}) \\
& \quad \uparrow \quad \uparrow \\
& \quad \uparrow H_G \\
& \quad R_N, H, H_L, H_G < 0 \quad \text{typically}
\end{align*} \]

Magnitudes smaller than in day except for \( H_G \).

Diurnal cycle of SEB is “forced” by \( R_N, H, H_L, H_G \) are responses to this forcing.

Local SEB depends on many factors:
- Insolation; cloud amount, height; near-sfc. atmospheric \( T, q, u, v \); sfc. characteristics such as wetness.
plant cover, albedo; topography (slope, orientation, etc.)

Determination of $H$, $H_L$ from $R_n$, $H_0$:
1. Measure $R_n$ and $H_0$.
2. Estimate Bowen ratio, $B = H/H_L$.
3. Use Eq (2.1):

$$H = \frac{R_n - H_0}{1 + B^{-1}}$$  \hspace{1cm} (2.2)

$$H_L = \frac{R_n - H_0}{1 + B}$$  \hspace{1cm} (2.3)

(Verify that $H/H_L = B$.)

Example Prob. 1
For Wangara exp. (central Australia):

- $H_0 = 0.30$ (day), 0.52 (nite)
- $R_n$

Assume $B = 5.0$. Est. $H$, $H_L$ for

- $R_n = 250 \text{ W m}^{-2}$ (day), $-55 \text{ W m}^{-2}$ (nite).

Solve

- $H_0 = 0.30 \times 250 = 75 \text{ W m}^{-2}$ (day)
- $= 0.52 \times -55 = -29 \text{ W m}^{-2}$ (nite)

From (2.2), (2.3):

$$H = \frac{250 - 75}{1 + 0.2} = 146 \text{ W m}^{-2} \text{ (day)}$$

$$H = \frac{-55 + 29}{1 + 0.2} = -32 \text{ W m}^{-2} \text{ (nite)}$$
\[ H_L = \frac{250 - 75}{1 + 5} = 29 \text{ W m}^{-2} \text{ (day)} \]
\[ H_L = \frac{-55 + 29}{1 + 5} = -4.4 \text{ W m}^{-2} \text{ (nite)} \]

(Verify that energy balance is satisfied.)

day: \[ 250 = 146 + 29 + 75 \]
nite: \[ -55 = -22 - 5 - 28 \]

\underline{Water surface SEB:} \ H \ll H_L \text{; little diurnal variation due to large heat capacity of water layer affected; } R_N \text{ is difficult to measure since solar radiation penetrates surface.}

2.2.2 **Energy budget of a layer**

\[ R_N = H + H_L + H_6 + \Delta H_s \quad (2.4) \]

Energy budget now includes a storage rate term, \( \Delta H_s \):

\[ \Delta H_s = \int \frac{\partial}{\partial z} (\rho c T) \, dz \quad (2.5) \]

\( \rho \): density, \( c \): specific heat capacity, \( T \): temperature.
Example Problem 2

\[ R_N = 400 \text{ W/m}^2 \]

\[ H^2 \quad H_L^2 \]

\[ (\frac{\partial T}{\partial z})_m = 0.05 \text{ K/day} \]

\[ \Delta H_S = \rho c D \left( \frac{\partial T}{\partial z} \right)_m \]

\[ = \frac{1000 \text{ kg/m}^3 \times 4180 \text{ J/kg K} \times 30 \text{ m} \times 0.05 \text{ K/day}}{\frac{1}{86400 \text{ s/day}}} \]

\[ = 121 \text{ W/m}^2 \]

So \[ H + H_L = R_N - \Delta H_S = 400 - 121 = 279 \text{ W/m}^2 \]

Then, using \[ H_L = H/B, \quad H + H/B = 279 \text{ W/m}^2 \]

\[ H = \frac{279}{1 + \frac{1}{B}} = \frac{279}{1 + 10} = 25.4 \text{ W/m}^2 \]

\[ H_L = H/B = 254 \text{ W/m}^2 \]

\[ E = H_L/Le = \frac{254 \text{ W/m}^2}{2.5 \times 10^6 \text{ J/kg K}^{-1}} = 1.02 \times 10^{-4} \text{ kg m}^{-2} \text{s}^{-1} \]

\[ \frac{E}{\rho w} = \frac{1.02 \times 10^{-4}}{10^3} \times \frac{86400 \text{ s}}{\text{day}} = 8.8 \text{ mm/day} \]

2.2.3 Energy budget of a control volume

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