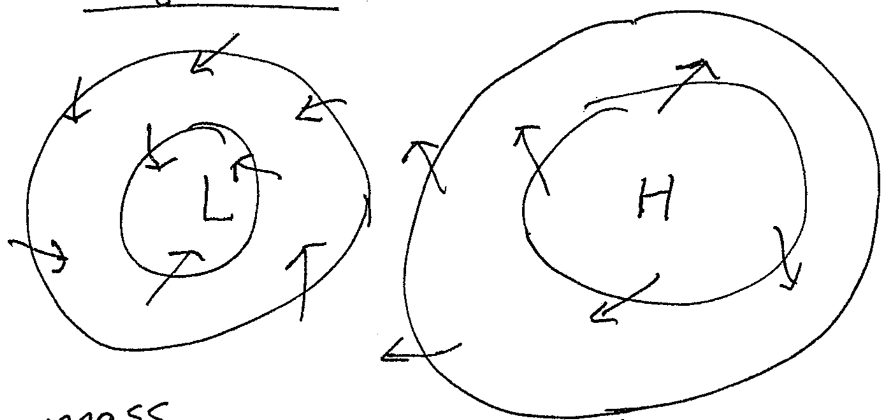
Secondary circulations & Spin-Down
(Holton 5.4).

Fig. 5.6 (cross-isobar flow)



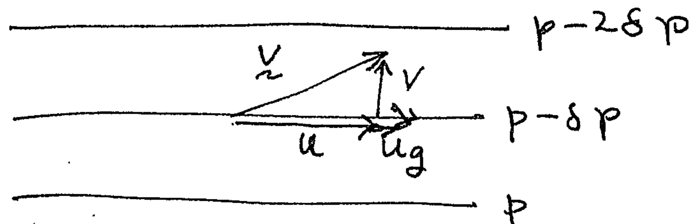
surface wind and pressure

mass convergence → upward motion

divergence → downward motion

Esti. magnitude of induced vertical motion:

If $v_g = 0$,
cross-isobar flow $= \rho_0 v$.



Vertically integrated mass transport:

$$M = \int_0^h \rho_0 v \, dz = \rho_0 v h \quad \text{(mixed layer)}$$

$$M = \int_0^{De} \rho_0 v \, dz = \int_0^{De} \rho_0 u_g \exp\left(-\frac{\pi z}{De}\right) \sin\left(\frac{\pi z}{De}\right) dz$$

$De = \pi/\gamma$, E.L. depth (Ekman layer)

Calculate w at B.L. top:

$$w(h) = -\int_0^h \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz \,, \quad w(0) = 0.$$

Assume $v_g = 0$; subst. u, v from either M.L. sol. or Ekman spiral sol'n.

For m.L. (Problem 5.10)

$$\boxed{\begin{aligned} \bar{u} &= u_g - k_s|\bar{V}|\bar{v} \\ \bar{v} &= k_s|\bar{V}|\bar{u} \end{aligned}}$$

can drop $(\bar{\ })$ for consistency with previous notation

Assume $|V| =$ const. indep. of x, y, $u_g = u_g(y)$.

$$\boxed{\begin{aligned} \frac{\partial u}{\partial x} &= -\underbrace{k_s|V|}_{\equiv k}\frac{\partial v}{\partial x} = 0 \\ \frac{\partial v}{\partial y} &= k_s|V|\frac{\partial u}{\partial y} = \frac{k}{1+k^2}\frac{\partial u_g}{\partial y} \end{aligned}}$$

solve for u first...

$u = u_g - kv = u_g - ku$, so $u = \dfrac{u_g}{1+k^2}$.

$v = ku = \dfrac{k}{1+k^2} u_g$

$\partial v/\partial x = 0$, $\partial u/\partial y = \dfrac{\partial u_g/\partial y}{1+k^2}$

$$\boxed{w(h) = -\int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\, dz = -\frac{hk}{1+k^2}\frac{\partial u_g}{\partial y}}$$

We have for m.L.: $\boxed{M = \int \rho_0 v h = \rho_0 h \frac{k}{1+k^2} u_g,}$

so combine above 2 eqs. to get

$\boxed{\rho_0 w(h) = -\dfrac{\partial M}{\partial y} \Rightarrow \text{mass flux out of BL} = \text{convergence of cross-isobar mass flux,}}$

Note that $\boxed{-\partial u_g/\partial y = \zeta_g}$, geostrophic vorticity (a measure of curvature of pressure field).

So

$$\boxed{\begin{aligned} w(h) &= \frac{hk}{1+k^2}\zeta_g \quad \text{for } \underline{m.L.} \\ w(D_e) &= \left|\frac{K_m}{2f}\right|^{1/2}\zeta_g \quad \text{for Ekman Layer} \end{aligned}}$$

vorticity (vertical) at BL top

$\sim \xi_g$.

BL communicates w/ FA via forced secondary circul. that usu. domin. over turb. mixing.

typical:

$\zeta_g \sim 10^{-5}\ s^{-1}$

$f \sim 10^{-4}\ s^{-1}$

$D_e \sim 1$ km,

$w \sim$ few mm/s

Tea cup circul.

Spin-down time for atmos. vortex

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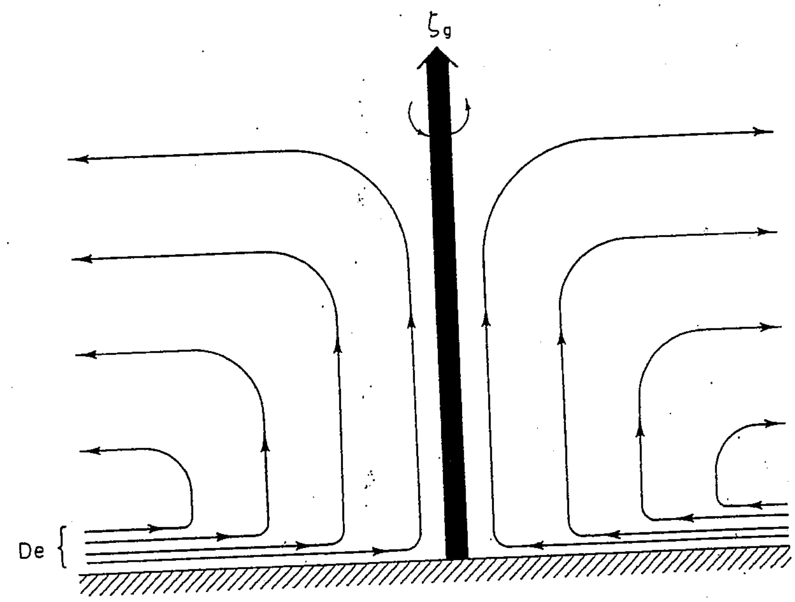

Fig. 5.7 Streamlines of the secondary circulation forced by frictional convergence in the planetary boundary layer for a cyclonic vortex in a barotropic atmosphere. The circulation extends throughout the full depth of the vortex.

Barotropic vorticity eq.

$$\boxed{\frac{D\zeta_g}{Dt} = -f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = f\,\frac{\partial w}{\partial z}}$$

$$\zeta_g \equiv -\frac{\partial u_g}{\partial y} + \frac{\partial v_g}{\partial x}$$

Subst. for w (De):
and integrate from top of EL to tropopause:

$$\boxed{\frac{D\zeta_g}{Dt} = -\left|\frac{f K_m}{2H^2}\right|^{1/2} \zeta_g}$$

Integ.

$$\boxed{\zeta_g(t) = \zeta_g(0)\exp\left(-t/\tau_e\right)}$$

$$\boxed{\tau_e = H\left(\frac{2}{f K_m}\right)^{1/2}}$$

$H = 10$ km

$f =$

$K_m = 10\ m^2 s^{-1}$

$$\boxed{\tau_e \sim 4 \text{ days}}$$