Meteorology 6160
Entrainment Rate Parameterization in Shallow Convecting Layers

Consider the TKE budget in the entrainment zone at the top of a clear convective boundary layer capped by a stable interface. In the entrainment zone, transport of TKE into the zone (and possible shear generation of TKE) must balance destruction by entrainment , dissipation, and storage (see TKE budget plot). Dimensional arguments following Tennekes (1973) suggest that for a fully turbulent boundary layer with turbulent velocity scale U and depth z_i, transport, dissipation and entrainment will all be O(U^3/z_i). For a shear-driven boundary layer, the shear production will also be of this order, while the storage term is much smaller if the entrainment zone is strongly stratified. Hence the entrainment buoyancy flux $(\overline{w'\theta_v'})_e$ should scale as

$$-(\overline{w'b'})_e = AU^3/z_i, \quad (1)$$

where A is an empirical constant. For a discontinuous inversion with a buoyancy jump $\Delta\theta_v$,

$$-(\overline{w'\theta_v'})_e = w_e \Delta\theta_v. \quad (2)$$

By substituting (2) into (1), we obtain

$$w_e = \frac{AU^3}{z_i \Delta\theta_v}, \quad (3)$$

which can be expressed in terms of a bulk interfacial Richardson number $\mathrm{Ri} = z_i \Delta\theta_v / U^2$ as

$$\frac{w_e}{U} = \frac{A}{z_i \Delta\theta_v / U^2} = \frac{A}{\mathrm{Ri}}. \quad (4)$$

For buoyancy-driven boundary layers, U can be taken as the convective velocity scale w_* (Deardorff, 1980). This is obtained from the vertically integrated TKE equation by assuming that buoyancy generation and dissipation balance:

$$\int_o^{z_i} \frac{g}{\theta_0} \overline{w'\theta_v'}\, dz = \int_o^{z_i} \epsilon\, dz$$

and that

$$\epsilon = \frac{w_*^3}{2.5 z_i},$$

which lead to

$$w_*^3 = 2.5 \int_o^{z_i} \frac{g}{\theta_0} \overline{w'\theta_v'}\, dz. \tag{5}$$

In a clear convective boundary layer, $\overline{w'\theta_v'}$ is a linear function of height above the surface, z:

$$\overline{w'\theta'} = (\overline{w'\theta_v'})_s(1 - z/z_i) + (\overline{w'\theta'})_e(z/z_i), \tag{6}$$

where $(\overline{w'\theta_v'})_s$ is the surface buoyancy flux. This can be used in (5), along with (1) and (2), to obtain an equation for w_e in terms of $(\overline{w'\theta_v'})_s$ and $\Delta\theta_v$:

$$w_e = \text{[exercise for the student]} \tag{7}$$

References

Deardorff (1980)
Tennekes (1973) *A First Course in Turbulence.*