Atm Sci 6620 Due Wednesday, Oct 2

Homework Set 3 (Monin-Obukhov theory)

1. (Adapted from Arya, p. 180) The following measurements of mean wind and potential temperature were taken around noon during the 1968 Kansas field program:

z (m)	2	4	8	16	32
$\bar{u}$ (m s^{-1})	5.81	6.70	7.49	8.14	8.66
$\bar{\theta}$ (K)	307.20	306.65	306.28	305.88	305.62

We wish to calculate the surface fluxes of heat and moisture using the Monin-Obuhkov relations $\phi_h = \phi_m^2 = \{1 - 16z/L\}^{-1/2}$ for an unstable surface layer. To proceed:

(a) Calculate the gradients of $\bar{u}$ and $\bar{\theta}$ by differencing between successive heights. In the surface layer, the profiles tend to vary roughly logarithmically with height, so it is better to difference using ln(z) as the height coordinate. Note that $du/dz = z^{-1}\, du/d(\ln z)$ and similarly for $\bar{\theta}$. Numbering the levels 1 (2 m) to 5 (32 m), the estimated gradient of u between levels 1 and 2 would be

$$\left(\frac{\Delta u}{\Delta z}\right)_{21} = \frac{1}{z_m}\left(\frac{\Delta u}{\Delta \ln z}\right)_{21} = \frac{1}{z_m}\left(\frac{u_2 - u_1}{\ln z_2 - \ln z_1}\right)$$

at a height z_m such that $\ln z_m = (\ln z_1 + \ln z_2)/2 = (\ln 2 + \ln 4)/2$, i. e. at $z_m = 2.82$ m. This works out to a gradient $(\Delta u/\Delta z)_{12} = 0.45$ s^{-1}. Neglecting virtual effects on buoyancy, use your calculated gradients to find Ri vs. z.

(b) From this data, estimate the Obhukov length L (note that Ri = z/L in an unstable surface layer).

(c) Using the data from the lowest two heights and your L from (b) , calculate the friction velocity, the sensible heat flux, and the surface roughness length z_0. Does the implied roughness length seem appropriate for a field of wheat stubble? Take the air density = 1.2 kg m^{-3} and $C_p = 10^3$ J kg^{-1} K^{-1}. Can we also estimate the thermal roughness length z_T from the given data?

2. An oceanographic research ship is stationed 100 km off the California coast. It is measuring a wind speed of 10 m s^{-1} and an air temperature of 286.9 K at a height of 10 m above sea level (i.e. 287 K if adiabatically displaced to the sea surface). The ocean surface temperature is also 287 K. Neglect virtual effects, so we have a neutral surface layer.

(a) Using the bulk aerodynamic approach, with $C_{DN} = (0.75 + 0.067u_{10})\text{x}10^{-3}$, find the surface stress and the friction velocity (use the same reference air density as before)?

(b) What is the roughness length z_0 (use Charnock's formula)?

(c) If you based C_{DN} on this z_0, rather than the bulk formula of (a), how different would the result be (this checks the consistency of the two approaches).

(d) The saturation mixing ratio at the sea-surface is 9.3 g kg^{-1} , while the mixing ratio at 10 m elevation is 7.0 g kg^{-1}. Using a bulk formula with $C_{qN} = 1.3\text{x}10^{-3}$, calculate the surface latent heat flux.

(e) Over water the thermal and moisture roughness lengths do not increase with wind speed. One formulation, used by ECMWF is that $z_q = 0.62\nu/u_*$, where $\nu = 1.4\text{x}10^{-5}$ m^2s^{-1}. What value would this z_q and your z_0 from (b) imply for C_{qN} ? Compare with (d).

(f) An airplane flies overhead at 30 m elevation. Assuming this is still within the surface layer, what mean wind speed, temperature and mixing ratio would the aircraft measure?

3. Now the ship moves over a coastal upwelling zone, where the 10 m wind and air temperature remain as above but the SST is 284 K.

(a) Starting with the bulk formula of (a) and $C_{HN} = 1.3\text{x}10^{-3}$, make a first guess at the sensible heat flux and Obuhkov length L. This won't be exact, but will be good enough for our purposes.

(b) Using this to specify z/L in the formulas on the middle of page 6.4 of the notes, recalculate u_* and the sensible heat flux (use $z_T = 0.4\nu/u_*$). By what percentage is the surface stress changed by moving over the colder SST?