Flux-Gradient model (Holton 5.3.2)

In neutral or stably stratified BLs, wind speed & direction may vary sig. w/ ht., so ML model is not approp.

Need a model for turb. mom. fluxes.

Trad. appr. - assume eddies act like molec. diffus., so flux = gradient of mean. Then

\[
\begin{align*}
\overline{u'w'} &= - K_m \frac{\overline{w'}}{\overline{z'}} \\
\overline{v'w'} &= - K_m \frac{\overline{v'}}{\overline{z'}} \\
\overline{\theta'w'} &= - K_h \frac{\overline{\theta'}}{\overline{z'}}
\end{align*}
\]

$K_m$: eddy viscosity (\(m^2 s^{-1}\))

$K_h$: eddy diffusivity (\(m^2 s^{-1}\))

Limitations:

- $K_m$ depends on flow, unlike molec. viscosity.

- Constant $K$ is a poor approx. in BL.

- Basis is invalid in many cases because eddies are as large as BL depth, so flux not => mean grad.
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\[ u'w' = - K_m \frac{\partial \bar{u}}{\partial z} \]
\[ v'w' = - K_m \frac{\partial \bar{u}}{\partial z} \]
\[ \theta'w' = - K_h \frac{\partial \bar{\theta}}{\partial z} \]

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(Flux-gradient model)

**Mixing length model** (Holton 5.3.3)

is simplest appr. for est. k.

**Assumption:** A parcel carries mean props. From orig. level for a distance $\tilde{z}'$, then mix - like avg. molec. travels mean free path before colliding & exch. mom.

Disp. creates a turbul. fluct. that depends on $\tilde{z}'$ and grad. of mean prop.

E.g.,

$$\Theta' = - \tilde{z}' \frac{d\overline{\Theta}}{dz}$$

$$u' = - \tilde{z}' \frac{d\overline{u}}{dz}$$

$$v' = - \tilde{z}' \frac{d\overline{v}}{dz}$$

$\tilde{z}' > 0$ for upward, etc.

**Apply to get**

$$-\overline{w'w'} = \overline{w'} \tilde{z}' \frac{d\overline{u}}{dz} = \overline{w' \tilde{z}'} \frac{d\overline{u}}{dz}$$

e tc.
Mixing length model (Holton 5.3.3)
is simplest appr. for estimating $k$.

Assumption: A parcel carries mean props. from orig. level for a
distance $\xi'$, then mix - like avg. molec. travels mean free path
before colliding & exch. mom.
Disp. creates a turbul. flux.
that depends on $\xi'$ and grad.
of mean prop.

E.g.,
\[ \Theta' = -\xi' \frac{d\Theta}{dz} \]
\[ u' = -\xi' \frac{du}{dz} \]
\[ v' = -\xi' \frac{2v'}{dz} \]

$\xi' > 0$ for upward, etc.

Apply to get
\[ -uw' = w' \xi' \frac{du}{dz} = w' \xi' \frac{d\tilde{u}}{dz} \]
etc.
How to get $W$?

Assume buoyancy effects are small, so

$$|w'| \sim |v'|$$  \hspace{1cm} (isotropic eddies)

$$w' \sim w' \left| \frac{\partial \bar{V}}{\partial z} \right|$$

$$\frac{v'}{\bar{V}} + \frac{\bar{V}}{\bar{V}} = \bar{V}$$

Now,

$$-uw' = \frac{\bar{V}'}{\bar{Z}} \left| \frac{\partial \bar{V}}{\partial z} \right| \frac{\bar{W}'}{\bar{Z}}$$

$$= \frac{\bar{V}'}{\bar{Z}} \left| \frac{\partial \bar{V}}{\partial z} \right| \bar{W} \frac{\bar{W}}{\bar{Z}} = \frac{K\bar{V}}{\bar{Z}}$$

so

$$K\bar{V} \equiv \frac{\bar{V}'}{\bar{Z}} \left| \frac{\partial \bar{V}}{\partial z} \right| = \bar{L}^2 \left| \frac{\partial \bar{V}}{\partial z} \right|$$

mixing length

$$l = \left( \frac{\bar{V}'}{\bar{Z}} \right)^{1/2}$$

rms parcel displacement,

a measure of local average eddy size.

$\Rightarrow$ Large eddies, greater shear, $\Rightarrow$
more turbulent mixing.
How to get $w'$?

Assume buoyancy effects are small, so

$$|w'| \sim |v'|$$  
(isotropic eddies)

$$\nu = \text{horiz. wind}$$

$$\frac{\partial \tilde{V}}{\partial Z}$$

$$\frac{\partial \tilde{U}}{\partial Z}$$

$$\frac{\partial \tilde{V}}{\partial Z}$$

Now,

$$-uw' = \bar{w}' \left( \frac{\partial \tilde{V}}{\partial Z} \right) \bar{U} = \frac{\partial \tilde{V}}{\partial Z} \bar{U} \bar{U} = \frac{\partial \tilde{V}}{\partial Z} \bar{U}$$

$$= \frac{\partial \tilde{V}}{\partial Z} \bar{U}$$

$$\Rightarrow \text{Km} \equiv \frac{\partial \tilde{V}}{\partial Z} \bar{U} = l^2 \left| \frac{\partial \tilde{V}}{\partial Z} \right|$$

mixing length:

$$l \equiv \left( \frac{\tilde{v}^2}{2} \right)^{1/2}$$

rms parcel displacement - a measure of average eddy size.

$$\Rightarrow \text{Large eddies, greater shear, } \Rightarrow \text{more turbulent mixing}$$