Flux-Gradient Model (Holton 5.3.2)

In neutral or stably stratified BLs, wind speed & direction may vary sig. w/ ht., so ML model is not approp.

Need a model for turb. mom. fluxes,

Trad. appr. – assume eddies act like molec. diffus. so flux ∼ gradient of mean. Then

$$\overline{u'w'} = -K_m \frac{\partial \bar{u}}{\partial z}$$

$$\overline{v'w'} = -K_m \frac{\partial \bar{v}}{\partial z}$$

$$\overline{\theta'w'} = -K_h \frac{\partial \bar{\theta}}{\partial z}$$

$$\left.\begin{array}{l} K_m: \text{ eddy viscosity } (m^2\, s^{-1}) \\ K_h: \text{ eddy diffusivity } (m^2\, s^{-1}) \end{array}\right\} \text{K-theory.}$$

Limitations:

- K_m depends on flow, unlike molec. viscosity.
- Constant K is a poor approx. in BL.
- Basis is invalid in many cases because eddies are as large as BL depth, so flux not $\propto$ mean grad.

(Flux-grad. model)

<u>mixing length model</u> (Holton 5.3.3)

is simplest appr. for esti K,

<u>Assumption</u>: A parcel carries mean
props. from orig. level for a
distance ξ', then mix - like avg.
molec. travels mean free path
before colliding & exch. mom.

Displ. creates a turbul. fluct.
that depends on ξ' and grad.
of mean prop.

E.g.,

$$\theta' = -\xi' \frac{\partial \bar{\theta}}{\partial z}$$

$$u' = -\xi' \frac{\partial \bar{u}}{\partial z}$$

$$v' = -\xi' \frac{\partial \bar{v}}{\partial z}$$

$\xi' > 0$ for upward, etc.

Apply to get

$$-\overline{u'w'} = \overline{w'\xi' \frac{\partial \bar{u}}{\partial z}} = \overline{w'\xi'} \frac{\partial \bar{u}}{\partial z}$$

etc.

How to get w'?

Assume buoy. effects are small, so

$|w'| \sim |\underset{\sim}{V}'|$ (isotropic eddies)

$\underset{\sim}{V}$ = horiz. wind

$$w' \approx \xi' \left| \frac{\partial \bar{V}}{\partial z} \right|$$

$$\underset{\sim}{V}' + \underset{\sim}{\bar{V}} = \underset{\sim}{V}$$

Now,

$$-\overline{u'w'} = \overline{\xi' \left| \frac{\partial \underset{\sim}{\bar{V}}}{\partial z} \right| \xi' \frac{\partial \bar{u}}{\partial z}}$$

$$= \overline{\xi'^2} \left| \frac{\partial \underset{\sim}{\bar{V}}}{\partial z} \right| \frac{\partial \bar{u}}{\partial z} = K_m \frac{\partial \bar{u}}{\partial z}$$

so $$K_m \equiv \overline{\xi'^2} \left| \frac{\partial \bar{V}}{\partial z} \right| = \overline{l^2} \left| \frac{\partial \bar{V}}{\partial z} \right|$$

mixing length:

$$l \equiv \left(\overline{\xi'^2}\right)^{1/2}$$

rms parcel displacement — a measure of $\sim$ avg. eddy size.

$\Rightarrow$ Large eddies, greater shear, $\Rightarrow$ more turb mixing