Liquid Virtual Dry Static Energy

The virtual dry static energy is

\[ s_v = c_p T_v + gz, \]  

(1)

where the virtual temperature

\[ T_v = T(1 + \delta q_v - q_l) \]  

(2)

takes into account the effects on air density of both water vapor (with mixing ratio \( q_v \)) and liquid water (with mixing ratio \( q_l \)). Here \( \delta = 0.608 \).

As moist (and dry) adiabatically conserved quantities, we use the total water mixing ratio \( q_t = q_v + q_l \) and the moist static energy

\[ h = c_p T + gz + L q_v. \]  

(3)

We can use (1), (2), and (3) to write

\[ s_v = h - \epsilon L q_t - [1 - (1 + \delta)\epsilon] L q_v, \]  

(4)

where

\[ \epsilon \equiv \frac{c_p T}{L}. \]

We can form a conserved quantity called the liquid virtual dry static energy by subtracting \([1 - (1 + \delta)\epsilon] L q_l\) from both sides of (4):

\[ s_{vl} \equiv s_v - [1 - (1 + \delta)\epsilon] L q_l = h - (1 - \delta \epsilon) L q_t = h - \mu L q_t, \]  

(5)

where \( \mu \equiv 1 - \delta \epsilon \). In unsaturated air, \( s_{vl} = s_v \).