Liquid Virtual Dry Static Energy

The virtual dry static energy is

$$s_v = c_p T_v + gz, \tag{1}$$

where the virtual temperature

$$T_v = T(1 + \delta q_v - q_l) \tag{2}$$

takes into account the effects on air density of both water vapor (with mixing ratio q_v) and liquid water (with mixing ratio q_l). Here $\delta = 0.608$.

As moist (and dry) adiabatically conserved quantities, we use the total water mixing ratio $q_t = q_v + q_l$ and the moist static energy

$$h = c_p T + gz + L q_v. \tag{3}$$

We can use (1), (2), and (3) to write

$$s_v = h - \epsilon L q_t - [1 - (1 + \delta)\epsilon] L q_v, \tag{4}$$

where

$$\epsilon \equiv \frac{c_p T}{L}.$$

We can form a conserved quantity called the liquid virtual dry static energy by subtracting $[1 - (1 + \delta)\epsilon] L q_l$ from both sides of (4):

$$s_{vl} \equiv s_v - [1 - (1 + \delta)\epsilon] L q_l = h - (1 - \delta\epsilon) L q_t = h - \mu L q_t, \tag{5}$$

where $\mu \equiv 1 - \delta\epsilon$. In unsaturated air, $s_{vl} = s_v$.