

Find  $u_*$  and  $z_0$  from the following wind profile measurements made during statically neutral conditions at sunset:

$z$ (m)	$\bar{u}$ (m/s)
1	4.6
3	6.0
10	7.6
30	9.0

*Answer:*

$u_* = 0.52$  m/s,  $z_0 = 0.028$  m. To calculate  $u_*$ , apply the log wind profile

$$u = u_*/k \log(z/z_0),$$

at any two heights  $z_1$  and  $z_2$  to obtain

$$u(z_2) - u(z_1) = u_*/k \log(z_2/z_1),$$

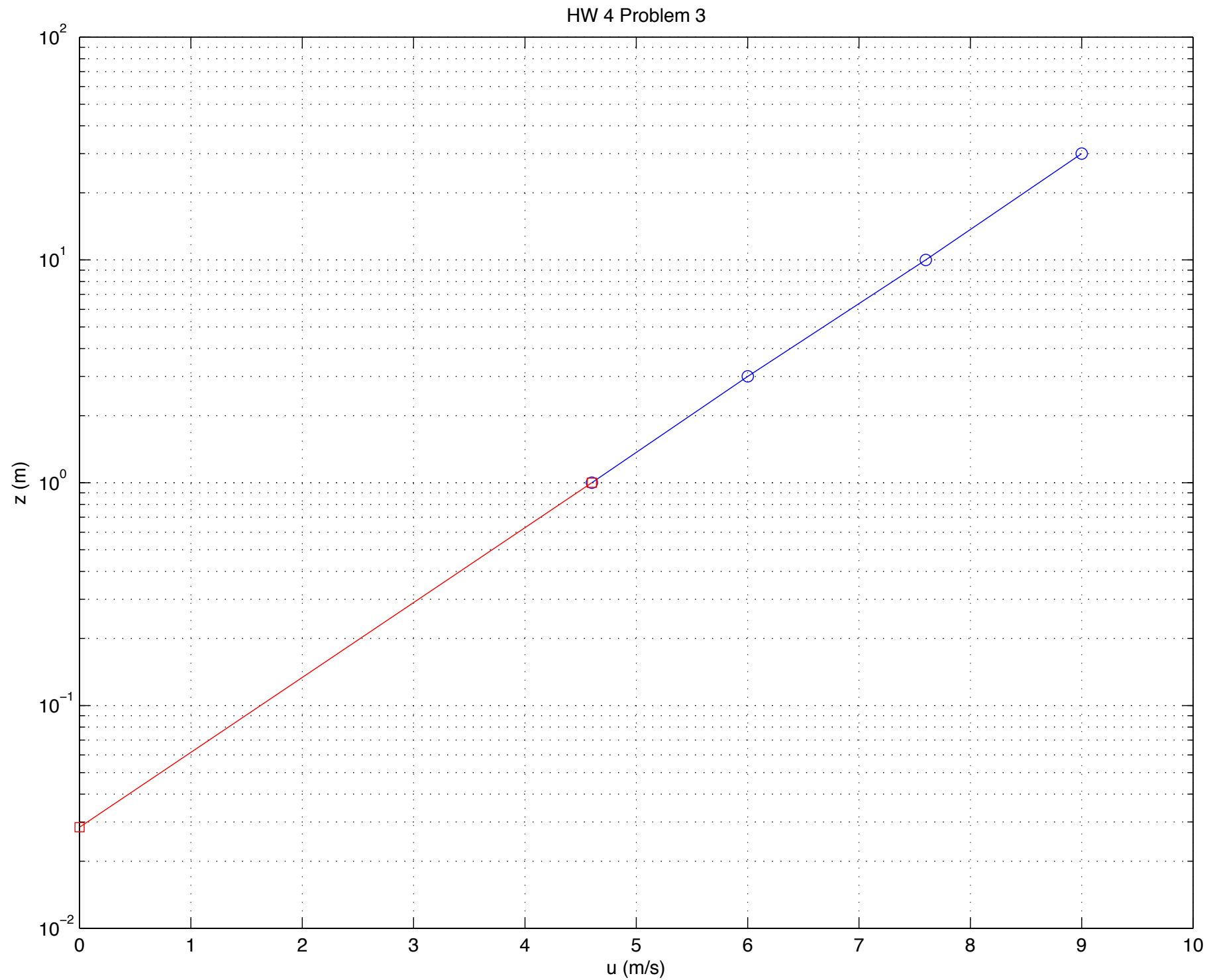
then solve for  $u_*$ :

$$u_* = k \frac{u(z_2) - u(z_1)}{\log(z_2/z_1)}.$$

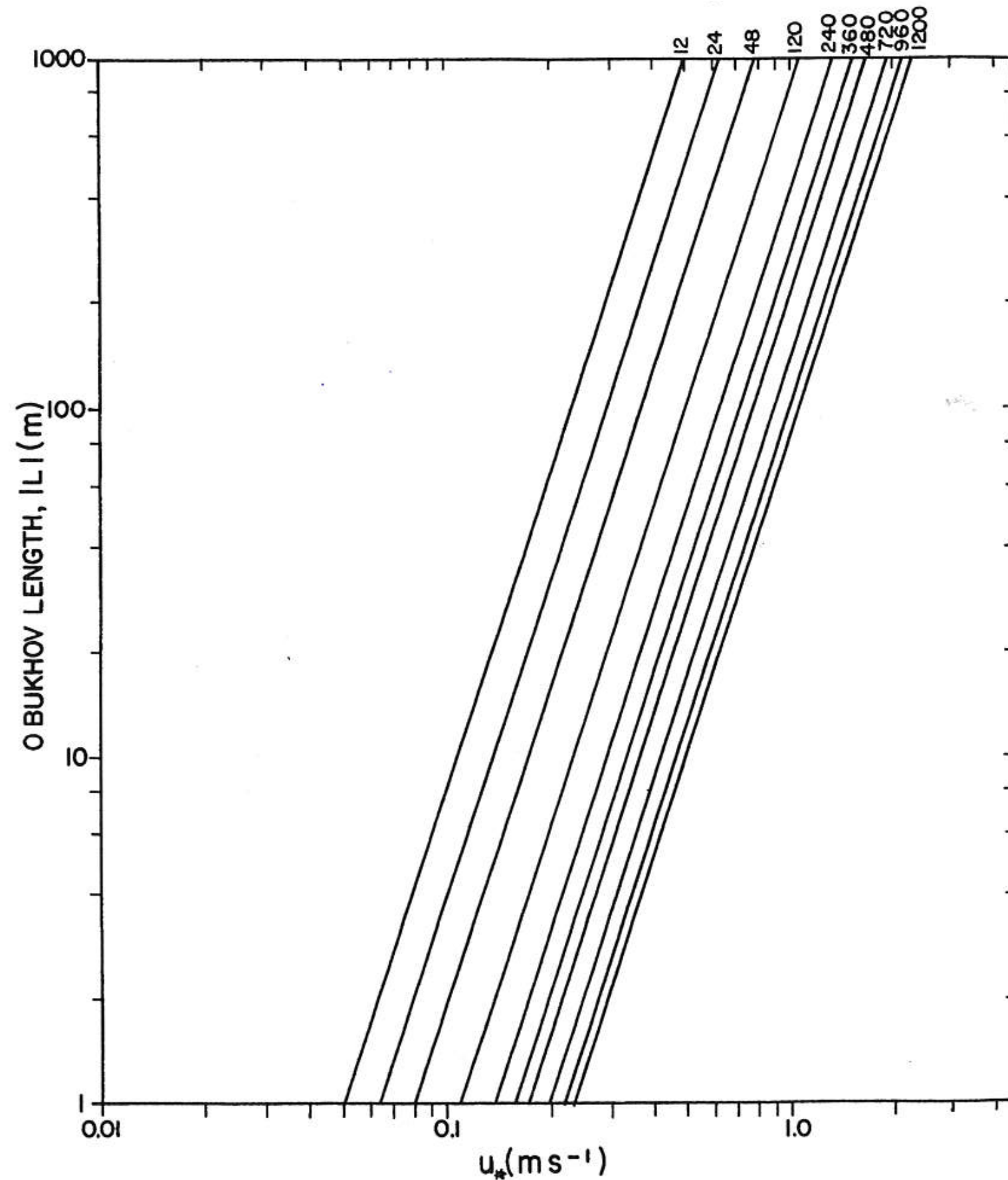
To calculate  $z_0$ , solve the log wind profile at any height  $z$  for  $z_0$ :

$$z_0 = z \exp(-ku(z)/u_*).$$

The graphical solution method is to plot the wind profile  $u$  versus  $\log z$ , then extrapolate the profile to  $u = 0$ . The height at which  $u = 0$  is  $z_0$ .



# Monin-Obukhov length as a function of friction velocity and surface heat flux



# *Wind and thermodynamic profiles*

$$\psi_m(\zeta) = \int_0^{\zeta} [1 - \phi_m(\zeta')] d\zeta' / \zeta'$$

$$= \begin{cases} \ln\left(\left(\frac{1+x^2}{2}\right)\left(\frac{1+x}{2}\right)^2\right) - 2\tan^{-1}x + \frac{\pi}{2}, & \text{for } -2 < \zeta < 0 \text{ (unstable)} \\ -\beta\zeta, & \text{for } 0 \leq \zeta \text{ (stable)} \end{cases}$$

$$\psi_h(\zeta) = \int_0^{\zeta} [1 - \phi_h(\zeta')] d\zeta' / \zeta'$$

$$= \begin{cases} 2\ln\left(\frac{1+x^2}{2}\right), & \text{for } -2 < \zeta < 0 \text{ (unstable)} \\ -\beta\zeta, & \text{for } 0 \leq \zeta \text{ (stable)} \end{cases}$$

# ***Similarity Functions***

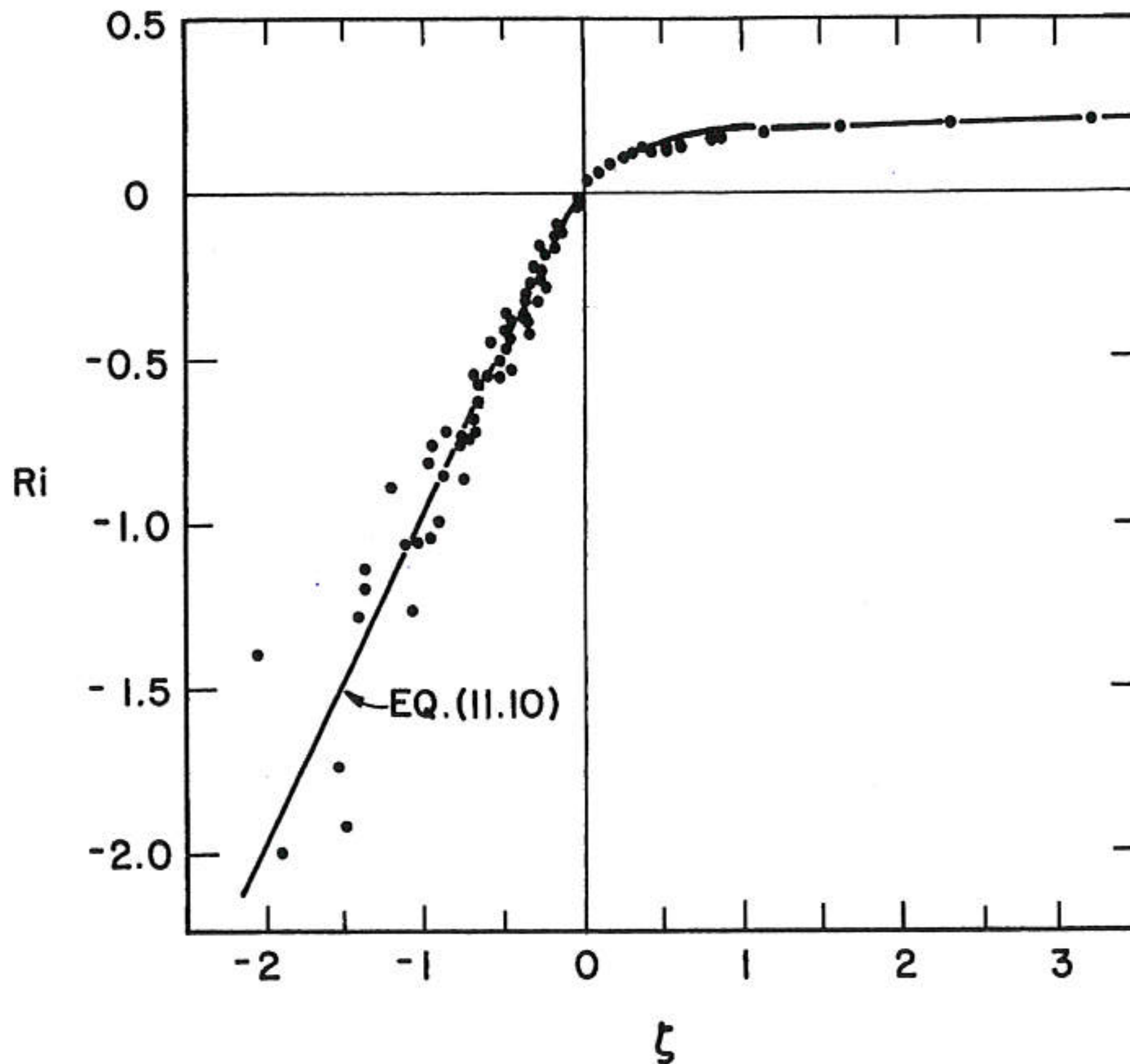
$$(kz/u_*)(\partial \bar{u}/\partial z) = \phi_m(\zeta)$$

$$(kz/\bar{\theta}_*)(\partial \bar{\theta}/\partial z) = \phi_h(\zeta)$$

$$K_m = -\overline{u'w'} / (\partial \bar{u}/\partial z) = u_*^2 / (\phi_m(\zeta) u_*/kz) = ku_*z / \phi_m(\zeta)$$

$$K_h = -\overline{w'\theta'} / (\partial \bar{\theta}/\partial z) = u_*\theta_* / (\phi_h(\zeta) \theta_*/kz) = ku_*z / \phi_h(\zeta)$$

$$\begin{aligned} \text{Ri} &= (-d\bar{b}/dz) / (d\bar{u}/dz)^2 \\ &= (\overline{w'b'}_0 / K_h) / (\overline{u'w'}_0 / K_m)^2 \\ &= (B_0\phi_h(\zeta) / ku_*z) / (u_*^2\phi_m(\zeta)/ku_*z)^2 \\ &= \zeta\phi_h/\phi_m^2 \end{aligned}$$

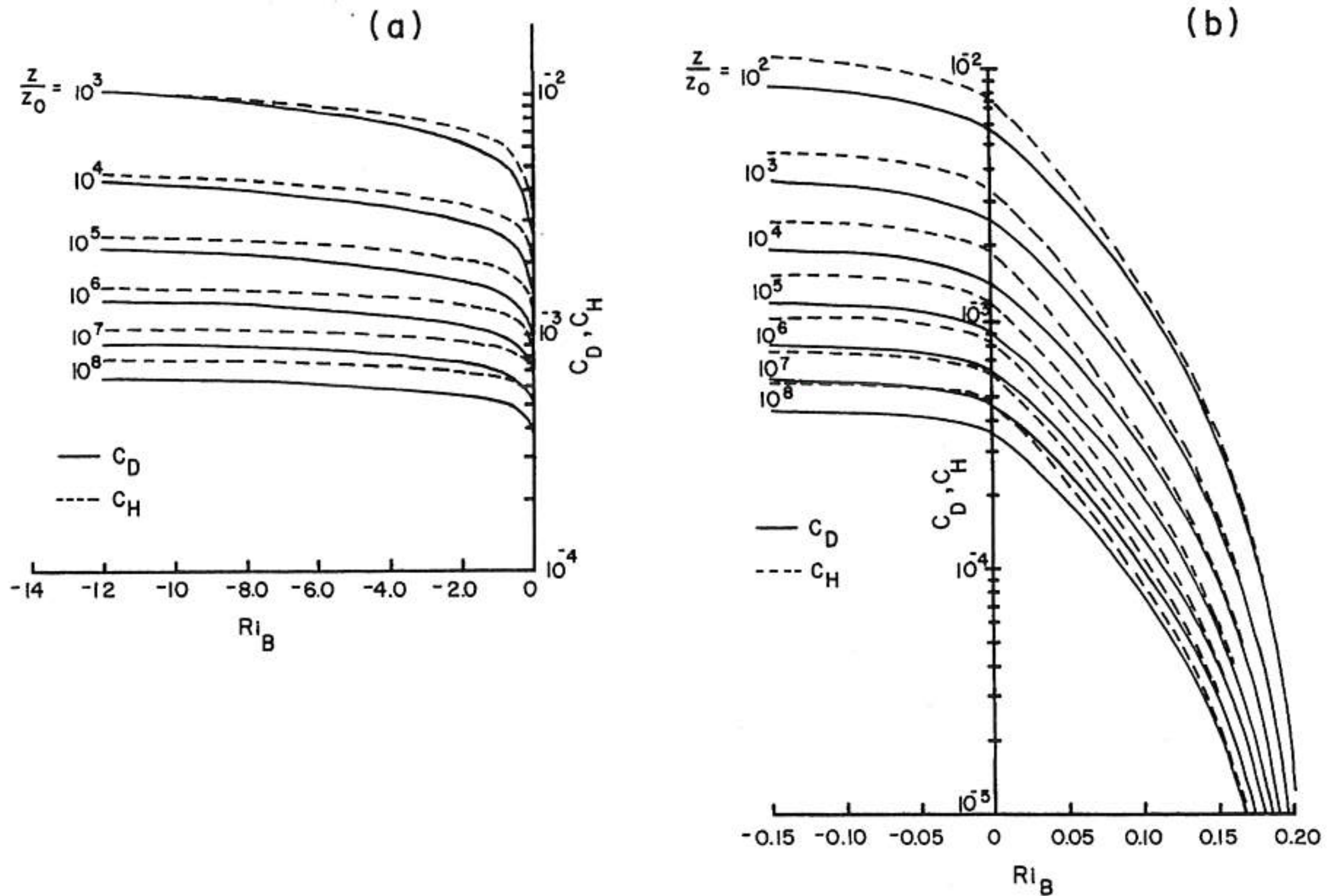


**Figure 11.4** The Richardson number as a function of the M–O stability parameters. Equation (11.10) (—) is compared with Kansas data. [After Businger *et al.* (1971).]

## *Bulk aerodynamic coefficients for non-neutral conditions*

$$C_D = \frac{k^2}{[\ln(z/z_0) - \psi_m(z/L)]^2}$$

$$C_H = \frac{k^2}{[\ln(z/z_{T0}) - \psi_m(z/L)][\ln(z/z_{T0}) - \psi_h(z/L)]}$$



**Figure 11.6** Variation of surface drag and heat transfer coefficients with surface roughness and the bulk Richardson number for (a) unstable conditions, and (b) near-neutral and stable conditions. [After Arya (1977).]