Boundary Layer Meteorology
ATMOS 5220/6220

09.15.2011

Neutral conditions

$$\overline{(u'w')}_s = u_*^2$$

$$u_*^2 = \overline{(u'w')}_s = K_m \frac{\partial \overline{U}}{\partial z}$$

u_*-Friction velocity
k-von Karman const.
z-height
z_0-roughness length

$$K_m = l^2 \left| \frac{\partial \overline{U}}{\partial z} \right| \qquad l = k \cdot z$$

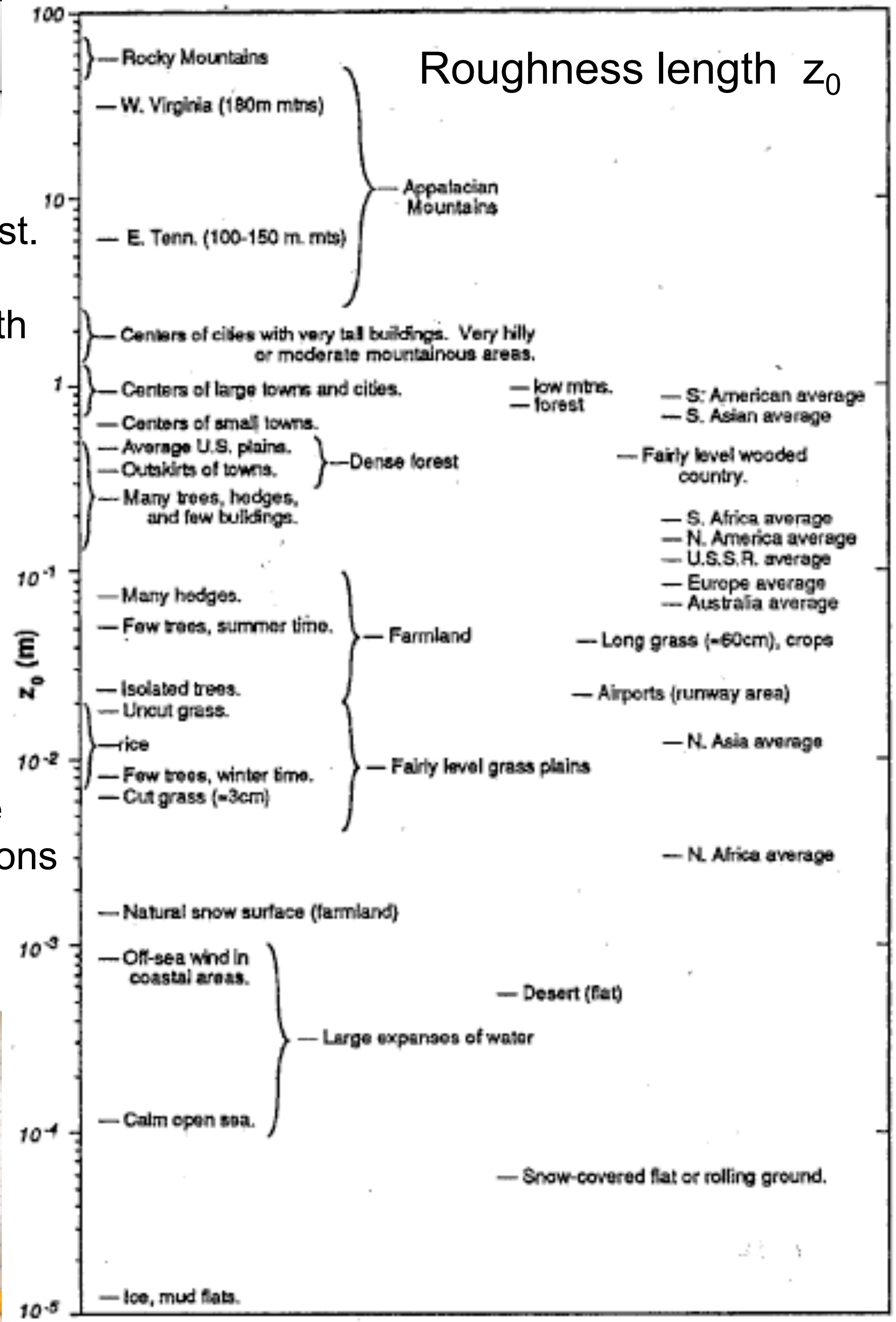
$$u_*^2 = k^2 z^2 \left| \frac{\partial \overline{U}}{\partial z} \right|^2 \rightarrow \left| \frac{\partial \overline{U}}{\partial z} \right| = \frac{u_*}{k \cdot z}$$

$$U(z) = \frac{u_*}{k} \log\left(\frac{z}{z_0}\right)$$

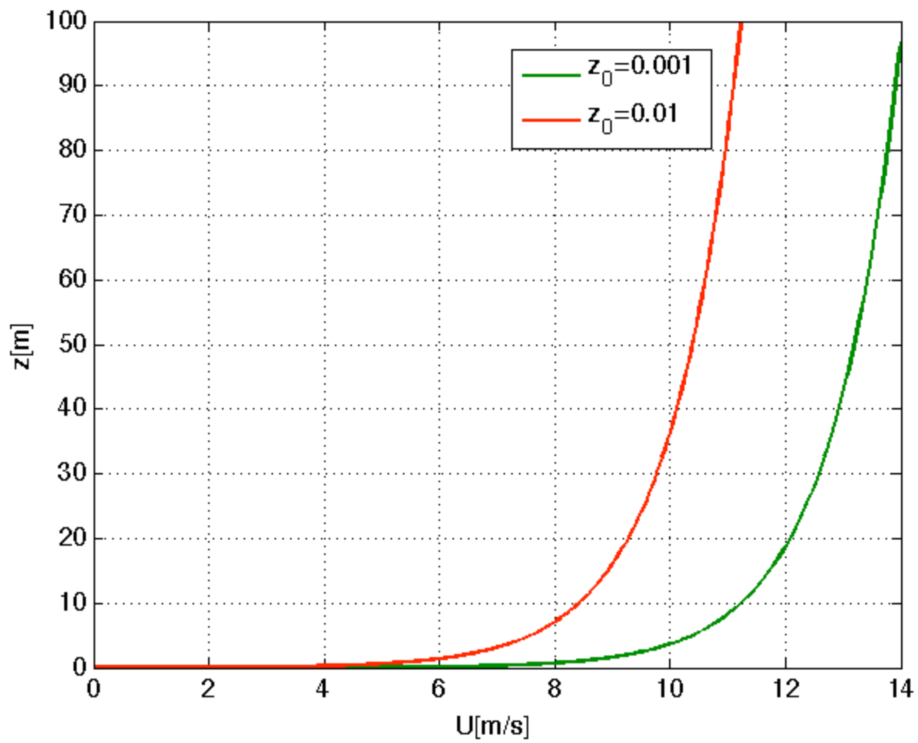
Logarithmic wind profile valid for **neutral** conditions

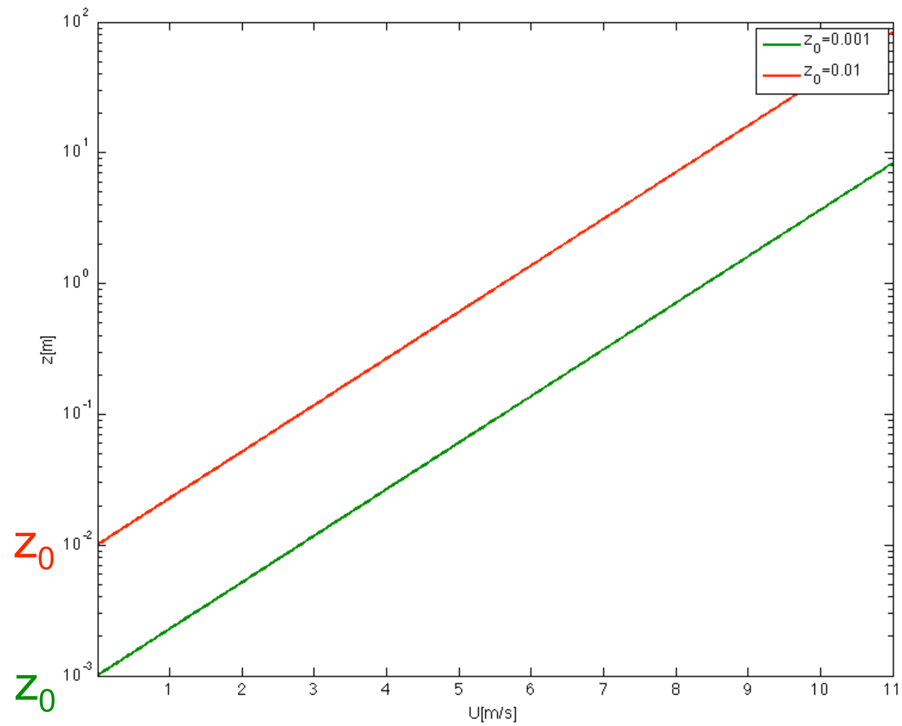
$$\rho \cdot \overline{(u'w')}_s = \tau$$

surface stress



Logarithmic wind profiles, $u_*=0.5$ m/s:





Aerodynamic bulk formula ($\tau = \rho$) $C_D \cdot U^2$

$$\rho \cdot \left(\overline{u'w'}\right)_s = \tau$$

surface stress

$$\left(\overline{u'w'}\right)_s = u_*^2$$

$$U(z) = \frac{u_*}{k} \log\left(\frac{z}{z_0}\right)$$

$$\left(\overline{u'w'}\right)_s = \frac{k^2 \cdot \left[U(z)\right]^2}{\left[\log\left(\frac{z}{z_0}\right)\right]^2}$$

$$u_* = \frac{k \cdot U(z)}{\log\left(\frac{z}{z_0}\right)}$$

$$\tau = \rho \cdot \left(\overline{u'w'}\right)_s = \rho \frac{k^2}{\left[\log\left(\frac{z}{z_0}\right)\right]^2} \cdot \left[U(z)\right]^2$$

drag coefficient C_D

$$\tau = \rho \cdot C_D \cdot U^2$$

Drag Coefficient (C_D)

$$\tau = \rho \cdot C_D \cdot U^2$$ surface stress

$$(\overline{u'w'})_s = \frac{\tau}{\rho}$$

In practice the drag coefficient is given usually with respect to the wind speed at z=10m and for neutral conditions (C_{DN10})

Typical values of the drag coefficient over the land are significantly larger than over the water

$C_{D\ land} \approx 7\times10^{-3}$

$C_{D\ water} \approx 1\times10^{-3}$

Transfer Coefficients

$$\tau = \rho \cdot (\overline{u'w'})_s = \rho \cdot C_D \cdot U^2$$ bulk formula for momentum

$$\rho \cdot (\overline{w'a'})_s = \rho \cdot C_a \cdot U(z_r) \cdot \left[a_0 - a(z_r)\right]$$ bulk formula for scalar 'a'

transfer coefficient for moisture

$$\rho \cdot (\overline{w'q'})_s = \rho \cdot C_E \cdot U(z_r) \cdot \left[q_0 - q(z_r)\right]$$ bulk formula for moisture

transfer coefficient for heat

$$\rho \cdot (\overline{w'\theta'})_s = \rho \cdot C_H \cdot U(z_r) \cdot \left[\theta_0 - \theta(z_r)\right]$$ bulk formula for heat

Transfer coefficient over water surfaces

Garratt

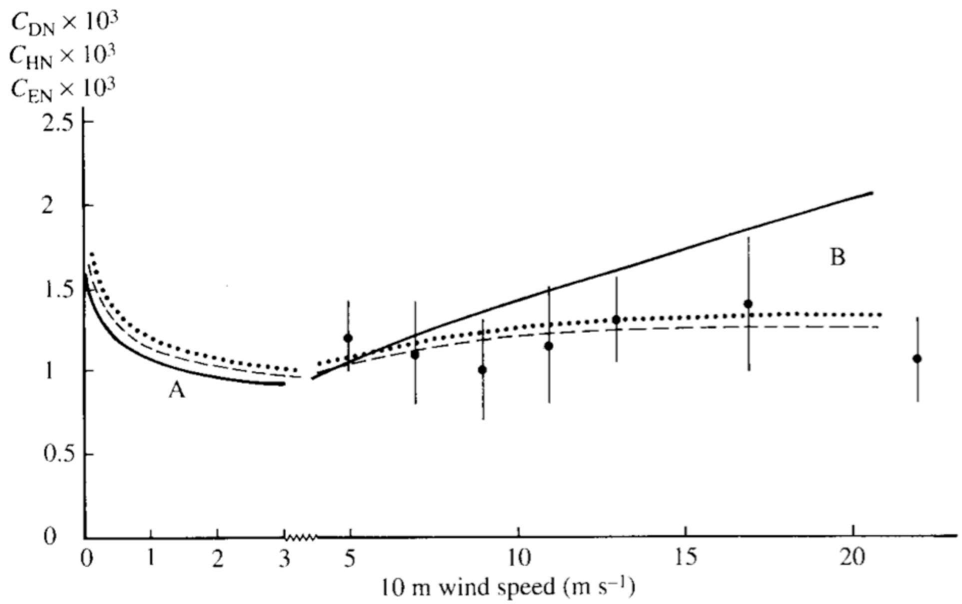


Fig. 4.9 Drag coefficient C_{DN}, heat transfer coefficient C_{HN} and water vapour transfer coefficient C_{EN} as functions of the 10 m wind speed. Curves A are for smooth flow: solid curve C_{DN} (Eq. 4.22); pecked curve, C_{HN} (Eqs. 4.10 and 4.26a); dotted curve, C_{EN} (Eqs. 4.11 and 4.26b). Curves B are for rough flow: solid curve, C_{DN} (Eq. 4.23); pecked curve, C_{HN} (Eqs. 4.10 and 4.27); dotted curve, C_{EN} (Eqs. 4.11 and 4.28). Observational data are from Large and Pond (1982).

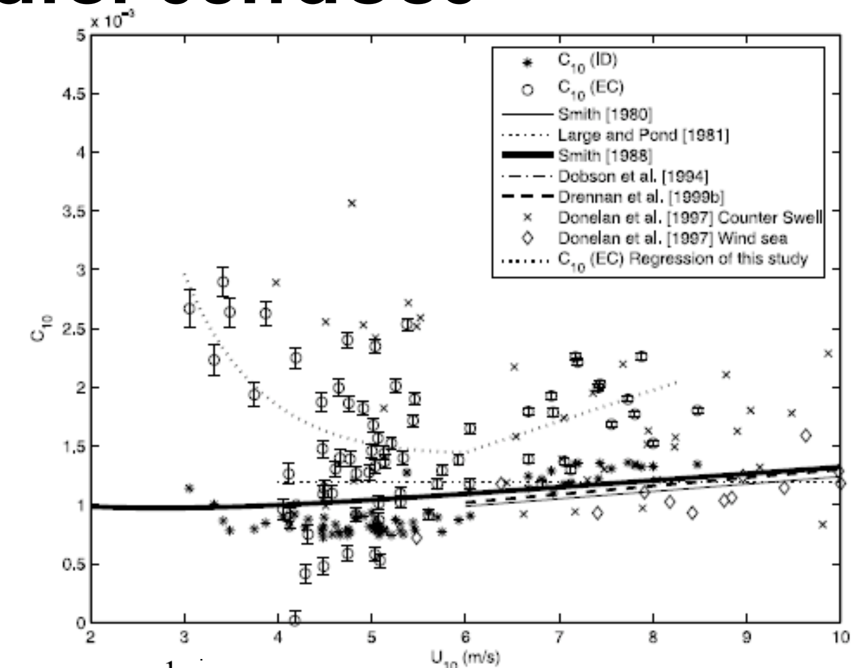
Transfer coefficient over water surfaces

Charnock formula

$z_0 = 0.016\ u_*/g$

TOGA-COARE formula

$z_0 = 0.11 v/u_* + 0.016\ u_*/g$



Large and Pond (1982) formula

$$C_{D,LP} = 1.2 \cdot 10^{-3}, \quad for\ 4 \leq V < 11\ m\,s^{-1}$$

$$C_{D,LP} = \left(0.49 + 0.065\overline{V}\right) \cdot 10^{-3}, \quad for\ 11 \leq \overline{V} \leq 25\ m\,s^{-1}$$

Transfer coefficient over water surfaces

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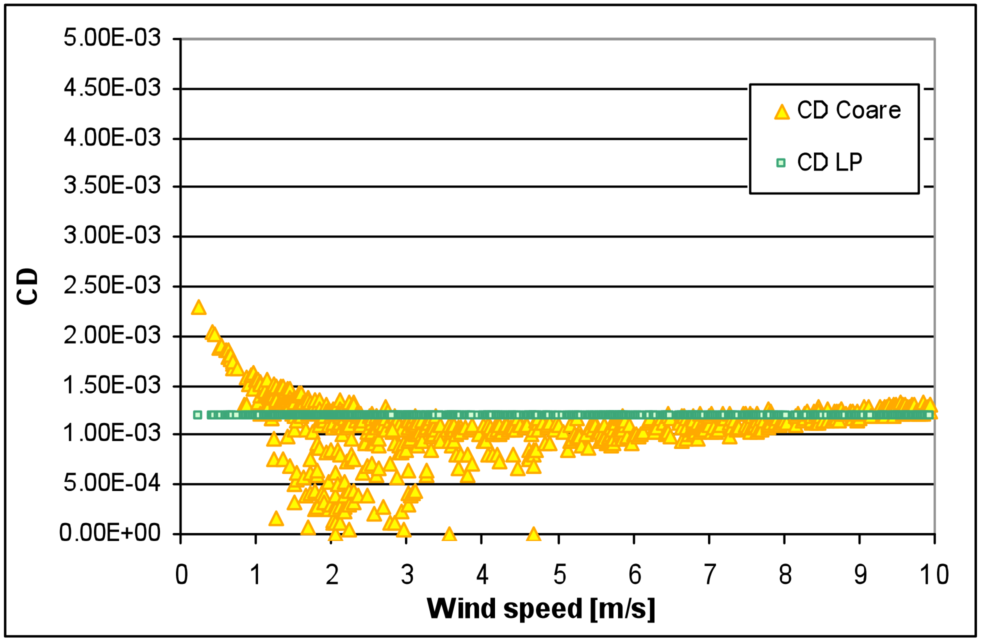

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Task 1

Knowing that the wind speed at 10m above the ocean surface is 12m/s compute using Large and Pond formula:

-the aerodynamic roughness length (z_0)

-friction velocity u_*

-shear stress at the ground τ

-wind speed at 6m

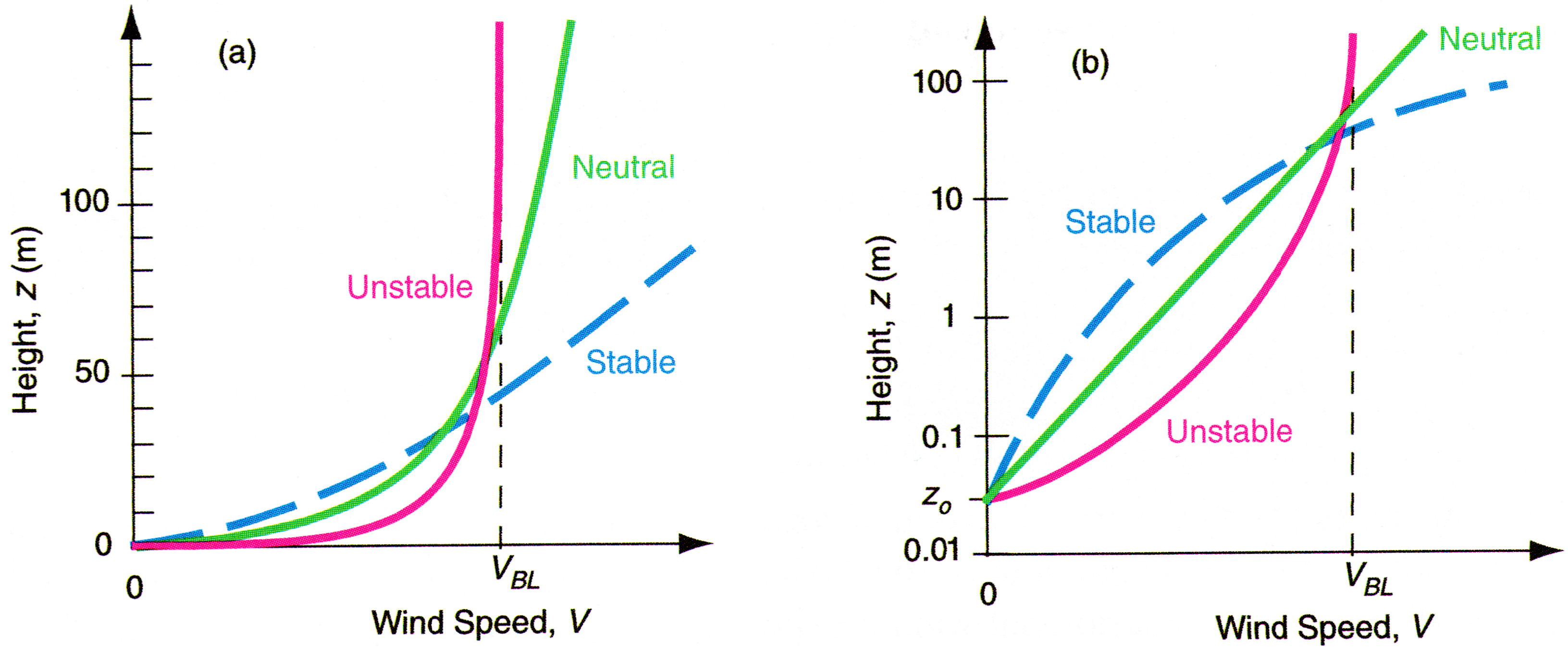
$$(\overline{u'w'})_s = u_*^2$$

$$\tau = \rho \cdot (\overline{u'w'})_s = \rho \cdot C_D \cdot U^2$$

$$C_{D,LP} = 1.2 \cdot 10^{-3}, \qquad for\ 4 \le \overline{V} < 11\ m\,s^{-1}$$

$$C_{D,LP} = \left(0.49 + 0.065\overline{V}\right) \cdot 10^{-3}, \quad for\ 11 \le \overline{V} \le 25\ m\,s^{-1}$$

Is logarithmic profile universal?



Velocity scales:

- Friction velocity:

$$u_* = \left[\overline{u'w'}^2 + \overline{v'w'}^2 \right]^{\frac{1}{4}} \qquad u_*^2 = \left(\overline{u'w'} \right)$$

For one-dimensional case

- Convective velocity scale (Deardorff velocity):

$$w_* = \left[\frac{g \cdot z_i}{T_v} \overline{w'\theta_S'} \right]^{\frac{1}{3}}$$

z_i – height of capping inversion (PBL height)
T_v – virtual temperature
ϑ – potential temperture

Length scales:

- Monin-Obukhov length — L
- Stability parameter: — $\zeta = z/L$
- Height of capping inversion (PBL height): — z_i
- Aerodynamic roughness length — z_0
- Height above the surface — z

Monin-Obukhov Length: Height proportional to the height above the surface at which buoyant production of turbulence first equals mechanical (shear) production of turbulence.

$$L = \frac{-u_*^3}{k \cdot B_0} = \frac{-u_*^3}{w_*^3}\frac{z_i}{k}$$

$$L = \frac{-u_*^3}{k\frac{g}{T_v}\left(\overline{w'\theta'}\right)_s}$$

$$B_0 = \overline{w'b'}_0 = \frac{-u_*^3}{k \cdot L}$$

z_i – height of capping inversion (PBL height)
T_v – virtual temperature
ϑ – potential temperature
k – von Karman constant (0.41)
B_0- surface buoyancy flux

For unstable atmosphere $L < 0$, so $\zeta < 0$
For neutral atmosphere $L \rightarrow \infty$, so $\zeta = 0$
For stable atmosphere $L > 0$, so $\zeta > 0$

Length scales:

- ❑ Height of capping inversion (PBL height): z_i
- ❑ Aerodynamic roughness length z_0
- ❑ Height above the surface z
- ❑ Monin Obukhov length L

$$L = \frac{-u_*^3}{k\frac{g}{T_v}\left(\overline{w'\theta'}\right)_s} = \frac{-u_*^3}{w_*^3}\frac{z_i}{k}$$

$$\frac{L\cdot k}{z_i} = \frac{-u_*^3}{w_*^3}$$

$$when \quad L\cdot k = z_i$$

$$u_* = \left|w_*\right|$$

So below $z=L\cdot k$ mechanical production (u*) dominates

z_i – height of capping inversion (PBL height)
T_v – virtual temperature
ϑ – potential temperature
k – von Karman constant (0.41)

Universal similarity functions and eddy viscosities

Universal similarity functions relate the fluxes of momentum and sensible heat to their mean gradients

universal similarity function for momentum

$$\phi_m(\varsigma) = \frac{k \cdot z}{u_*}\left(\frac{\partial \bar{u}}{\partial z}\right)$$

eddy viscosity for momentum

$$K_m = \frac{-\overline{u'w'}}{\frac{\partial u}{\partial z}} = \frac{u_*^2}{\frac{u_*\phi_m(\varsigma)}{k \cdot z}} = \frac{u_* \cdot k \cdot z}{\phi_m(\varsigma)}$$

universal similarity function for heat

$$\phi_h(\varsigma) = \frac{k \cdot z}{\theta_*}\left(\frac{\partial \bar{\theta}}{\partial z}\right)$$

eddy viscosity for heat

$$K_h = \frac{-\overline{u'\theta'}}{\frac{\partial \theta}{\partial z}} = \frac{u_*\theta_*}{\frac{\theta_*\phi_h(\varsigma)}{k \cdot z}} = \frac{u_* \cdot k \cdot z}{\phi_h(\varsigma)}$$

$\phi_m(\varsigma) < 1$ for unstable conditions

$\phi_m(\varsigma) = 1$ for neutral conditions

$\phi_m(\varsigma) > 1$ for stable conditions

Other similarity functions

$$-(\overline{u'w'}) = u_*^2 = K_m \frac{\partial \overline{U}}{\partial z}$$

$$\frac{\partial \overline{u}}{\partial z} = \frac{-\overline{u'w'}}{K_m}$$

Turbulent Prandtl number:

$$\Pr_t = \frac{K_m}{K_h} = \frac{\phi_h(\varsigma)}{\phi_m(\varsigma)}$$

$$B_0 = \overline{w'b'} = \frac{-u_*^3}{L \cdot k}$$

$$\frac{\partial \overline{b}}{\partial z} = \frac{-\overline{w'b'}}{K_h} = \frac{u_*^3}{L \cdot k \cdot K_h}$$

Gradient Richardson number:

$$Ri = \frac{\left(\frac{d\overline{b}}{dz}\right)}{\left(\frac{d\overline{u}}{dz}\right)^2} = \frac{-\frac{\overline{w'b'}}{K_h}}{\frac{\left(\overline{u'w'}\right)^2}{K_m^2}} = \frac{\frac{u_*^3}{L \cdot k \cdot K_h}}{\frac{u_*^4}{K_m^2}} = \frac{\frac{u_*^3 \phi_h}{L \cdot k^2 u_* z}}{\frac{u_*^4 \phi_m^2}{k^2 u_*^2 z^2}} = \frac{z}{L} \frac{\phi_h}{\phi_m^2}$$

|Ri|<=1 shear production of sub-scale KE dominates

|Ri| > 1 buoyant production dominates

Similarity functions:

- For momentum ϕ_m:

$$\phi_m = \begin{cases} [1-15\varsigma]^{-\frac{1}{4}} & for\ -2 < \varsigma < 0 \quad (unstable) \\ 1 & for \quad \varsigma = 0 \quad (neutral) \\ 1+4.7\varsigma & for \quad 0 \le \varsigma < 1 \quad (stable) \end{cases}$$

$$\phi_m = \begin{cases} [1-16\varsigma]^{-\frac{1}{4}} & for\ -2 < \varsigma < 0 \quad (unstable) \\ 1 & for \quad \varsigma = 0 \quad (neutral) \\ 1+5\varsigma & for \quad 0 \le \varsigma < 1 \quad (stable) \end{cases}$$

Note that for neutral conditions:

$$\phi_m = \left(\frac{k \cdot z}{u_*}\right)\frac{d\overline{U}}{dz} = 1$$

$$\frac{d\overline{U}}{dz} = \left(\frac{u_*}{k \cdot z}\right)$$

So we come back to the logarithmic profile:

$$\overline{U} = \frac{u_*}{k}\log\left(\frac{z}{z_0}\right)$$

Similarity functions:

- For heat ϕ_h:

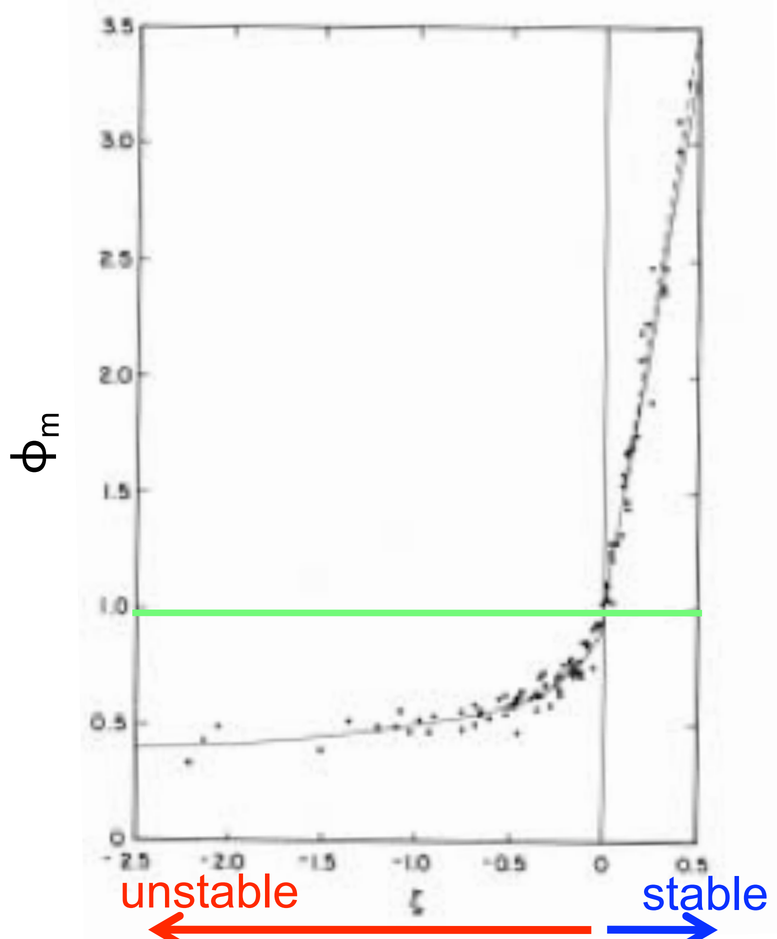
$$\phi_h = \begin{cases} \Pr_{tN}\left(1-9\varsigma\right)^{-\frac{1}{2}} & for\ -2<\varsigma<0 \quad (unstable) \\ 1 & for \quad \varsigma=0 \quad (neutral) \\ \Pr_{tN}+4.7\varsigma & for \quad 0\le\varsigma<1 \quad (stable) \end{cases}$$

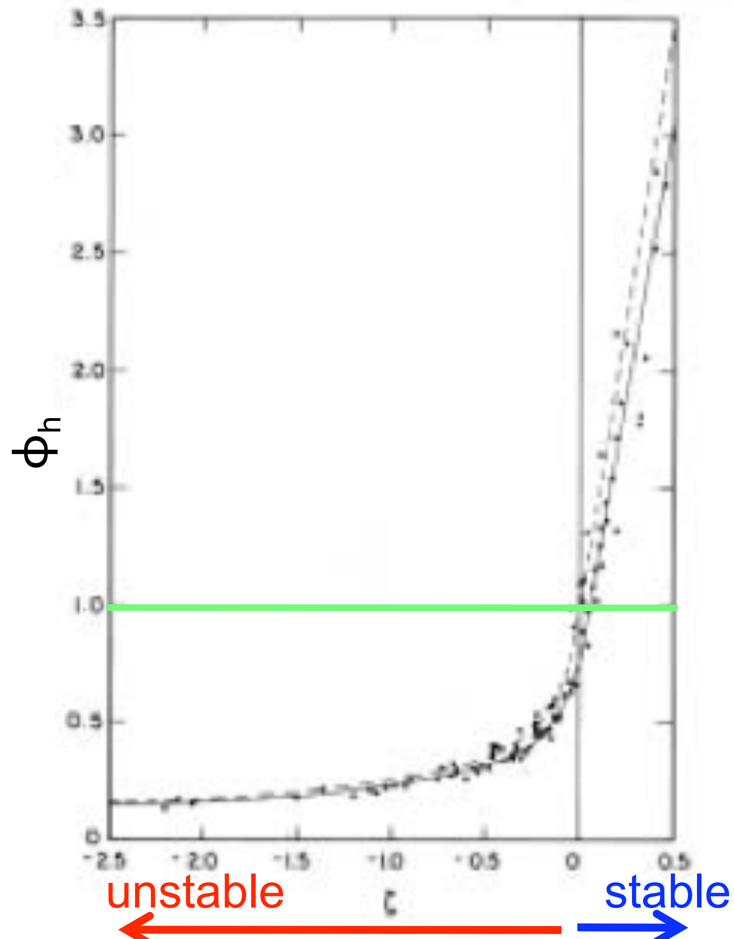
$Pr_{tN}=0.74$

$$\phi_h = \begin{cases} \left(1-16\varsigma\right)^{-\frac{1}{2}} & for\ -2<\varsigma<0 \quad (unstable) \\ 1 & for \quad \varsigma=0 \quad (neutral) \\ 1+5\varsigma & for \quad 0\le\varsigma<1 \quad (stable) \end{cases}$$

$Pr_{tN}=1$

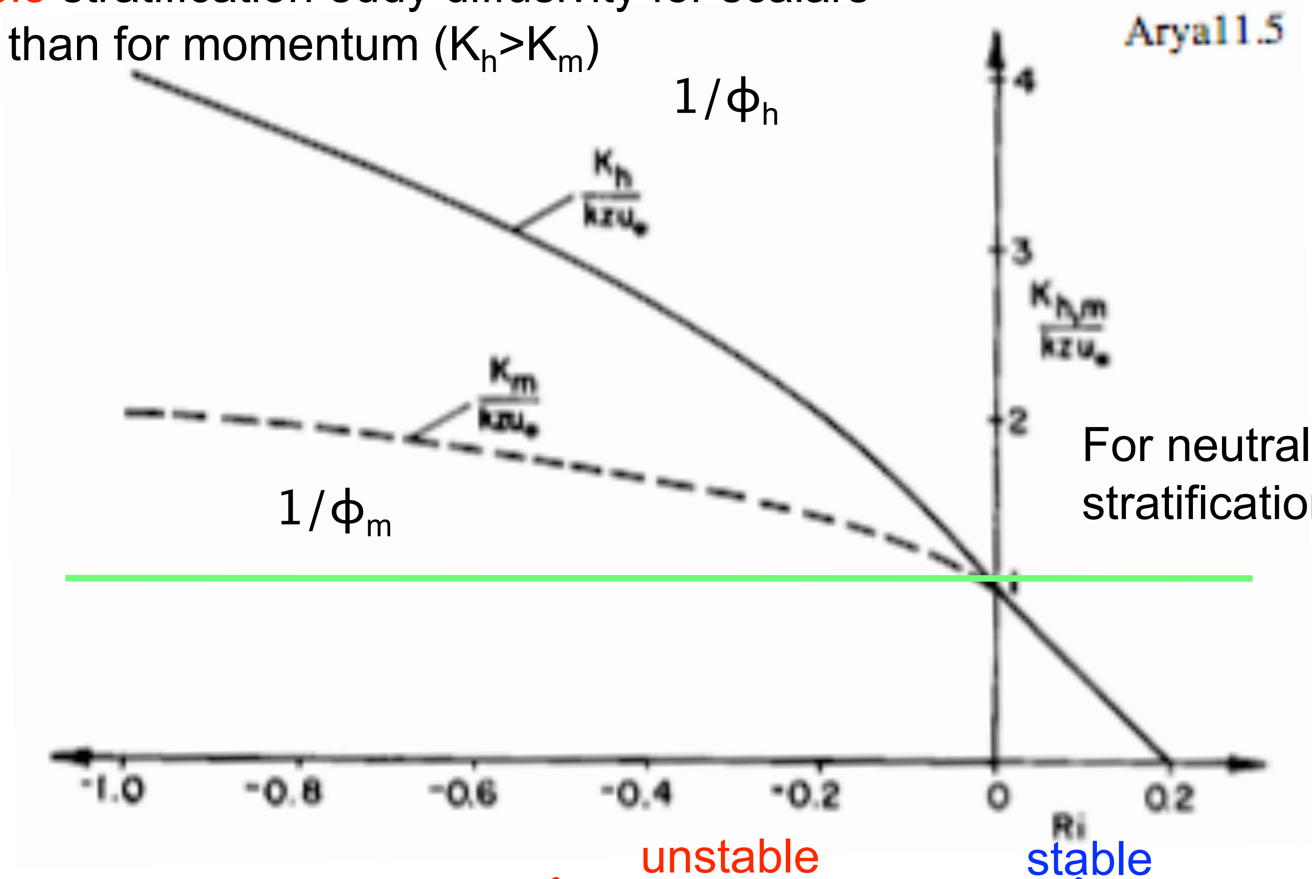
Empirically determined similarity functions vs. stability parameter ζ





Eddy diffusivities as a function of Richardson number

For unstable stratification eddy diffusivity for scalars is greater than for momentum ($K_h > K_m$)



For neutral and stable stratification $K_m=K_h$

unstable ← → stable

In neutral or stable stratification $\phi_m=\phi_h$ $(1/K_m=1/K_h)$

Pressure perturbations do not affect the eddy transport of momentum relative to heat and other scalars $Pr_t=1$.

$$\phi_h = \begin{cases} 1 & \mathit{for} \quad \varsigma = 0 \quad (\mathit{neutral}) \\ 1+5\varsigma & \mathit{for} \quad 0 \le \varsigma < 1 \quad (\mathit{stable}) \end{cases}$$

$$\phi_m = \begin{cases} 1 & \mathit{for} \quad \varsigma = 0 \quad (\mathit{neutral}) \\ 1+5\varsigma & \mathit{for} \quad 0 \le \varsigma < 1 \quad (\mathit{stable}) \end{cases}$$

In unstable stratification $\phi_h < \phi_m$ ($K_h > K_m$)

Eddy diffusivity for scalars is more than for momentum (universal similarity function for momentum (Φ_m) is greater than for scalars (Φ_h)).

$$\phi_m = \left\{ [1-16\varsigma]^{-\frac{1}{4}} \qquad for \quad 0 \le \varsigma < 1 \quad (stable) \right\}$$

$$\phi_h = \left\{ [1-16\varsigma]^{-\frac{1}{2}} \qquad for \quad 0 \le \varsigma < 1 \quad (stable) \right\}$$

Conversion between stability function (ζ) and Richardson number (Ri)

$$\varsigma = \begin{cases} Ri & for \quad -2 \le Ri < 0 \quad (unstable) \\ \dfrac{Ri}{1-5Ri} & for \quad 0 \le Ri < 0.2 \quad (stable) \end{cases}$$

Limiting cases:

1.Neutral limit: Φ_m, Φ_h → 1 as ζ→0 logarithmic profile

2.Stable limit: z-less scaling stable buoyancy forces tend to suppress eddies with a scale > L:

$$K_m = \frac{ku_* z}{\phi_m} \propto (velocity) \times (length) \propto u_* L \Rightarrow \phi_m \propto \frac{z}{L} = \varsigma \qquad \phi_h \propto \frac{z}{L} = \varsigma$$

3.Unstable limit: eddy viscosity scales with the buoyancy flux

$$K_m = \frac{ku_* z}{\phi_m} \propto u_f z = (B_0 z)^{\frac{1}{3}} z \Rightarrow \phi_m \propto \frac{u_*}{u_f} \propto \left(-\frac{z}{L}\right)^{-\frac{1}{3}} = (-\varsigma)^{-\frac{1}{3}}$$

Wind and thermodynamic profiles

- For all cases we can use one formula with stability correction function defined below:

$$\overline{U}(z) = \left(\frac{u_*}{k}\right) \cdot \left[\log\left(\frac{z}{z_0}\right) + \psi_M\left(\frac{z}{L}\right)\right]$$

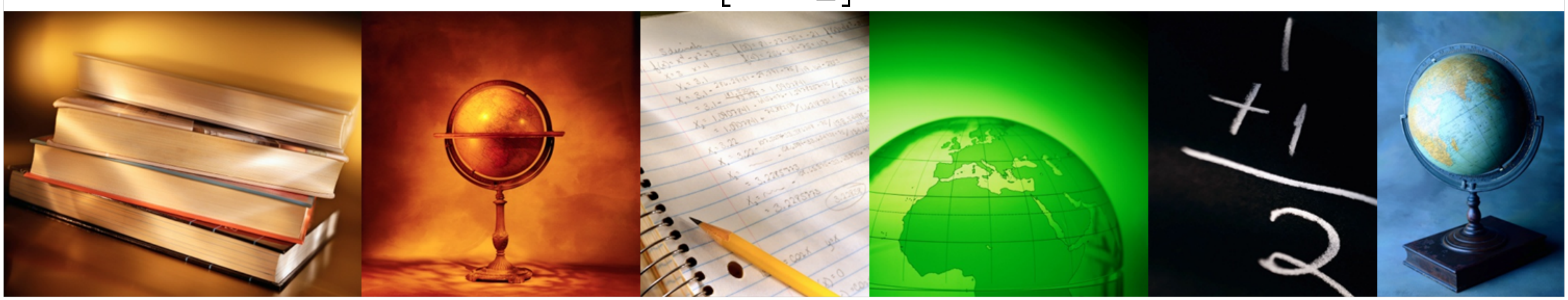
Ψ_M – stability correction function

L – Monin-Obukhov length

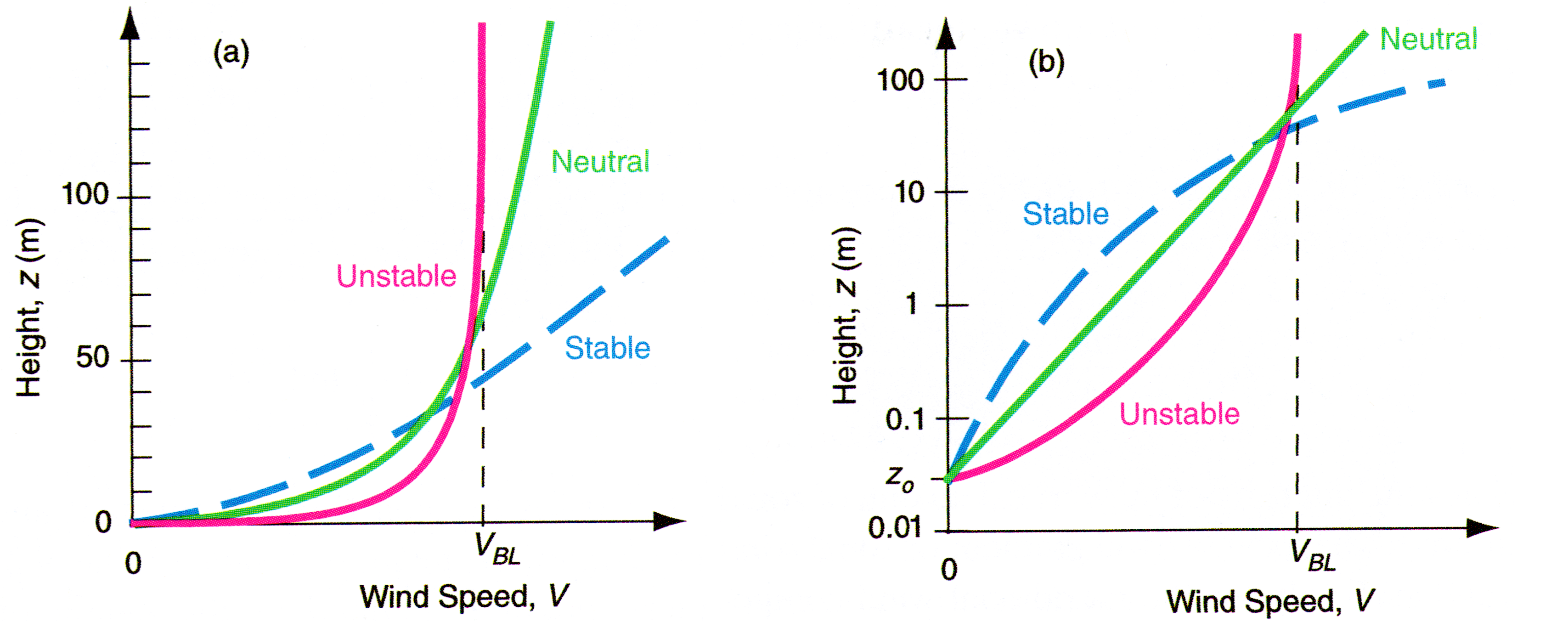
$$\psi_M = \int_0^{\varsigma} \left[1 - \phi_m(\varsigma')\right] d\varsigma' / \varsigma'$$

$$\psi_M = \begin{cases} \left(\dfrac{4.7 \cdot z}{L}\right) & for \quad \dfrac{z}{L} > 0 \quad (stable) \\ 0 & for \quad \dfrac{z}{L} = 0 \quad (neutral) \\ -2\ln\left[\dfrac{1+x}{2}\right] - \ln\left[\dfrac{1+x}{2}\right] - \ln\left[\dfrac{1+x^2}{2}\right] + 2\tan^{-1}(x) - \dfrac{\pi}{2} & for \quad \dfrac{z}{L} < 0 \quad (unstable), \end{cases}$$

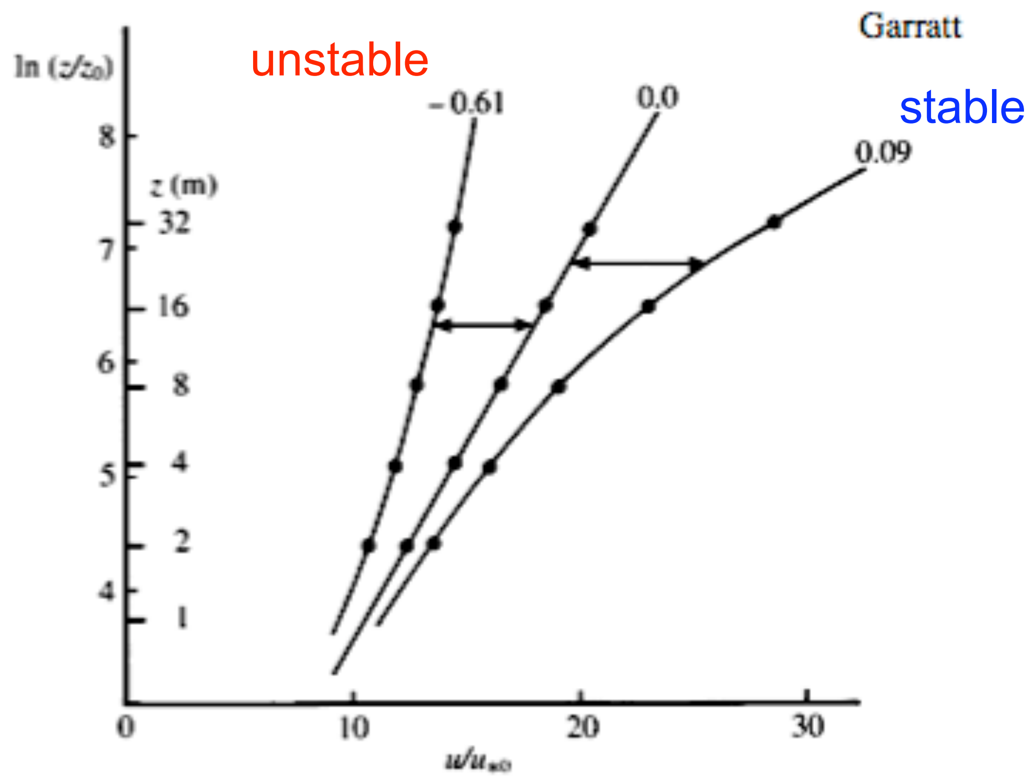
$$where: \ x = \left[1 - 15\frac{z}{L}\right]^{\frac{1}{4}}$$



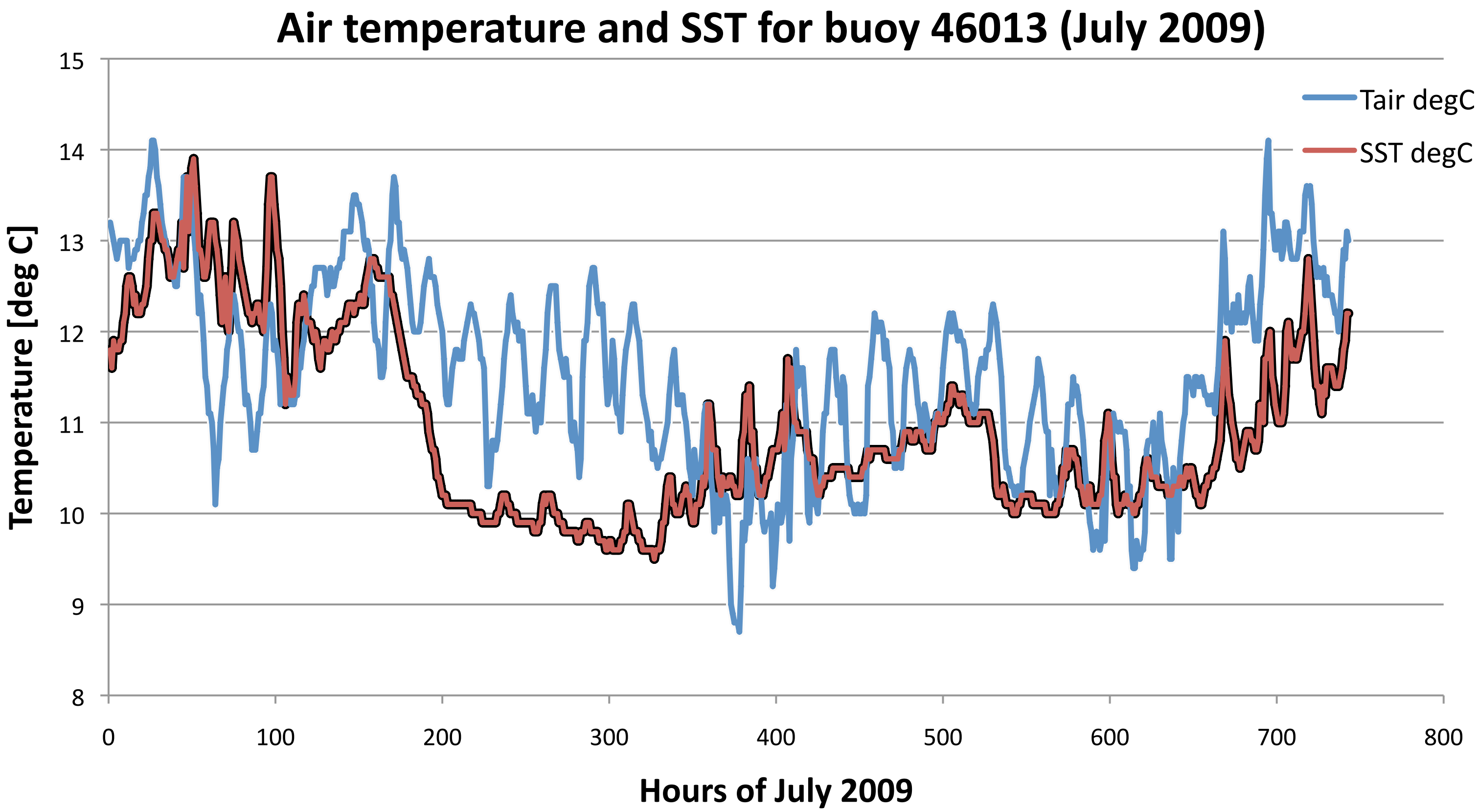
Stability-corrected profiles



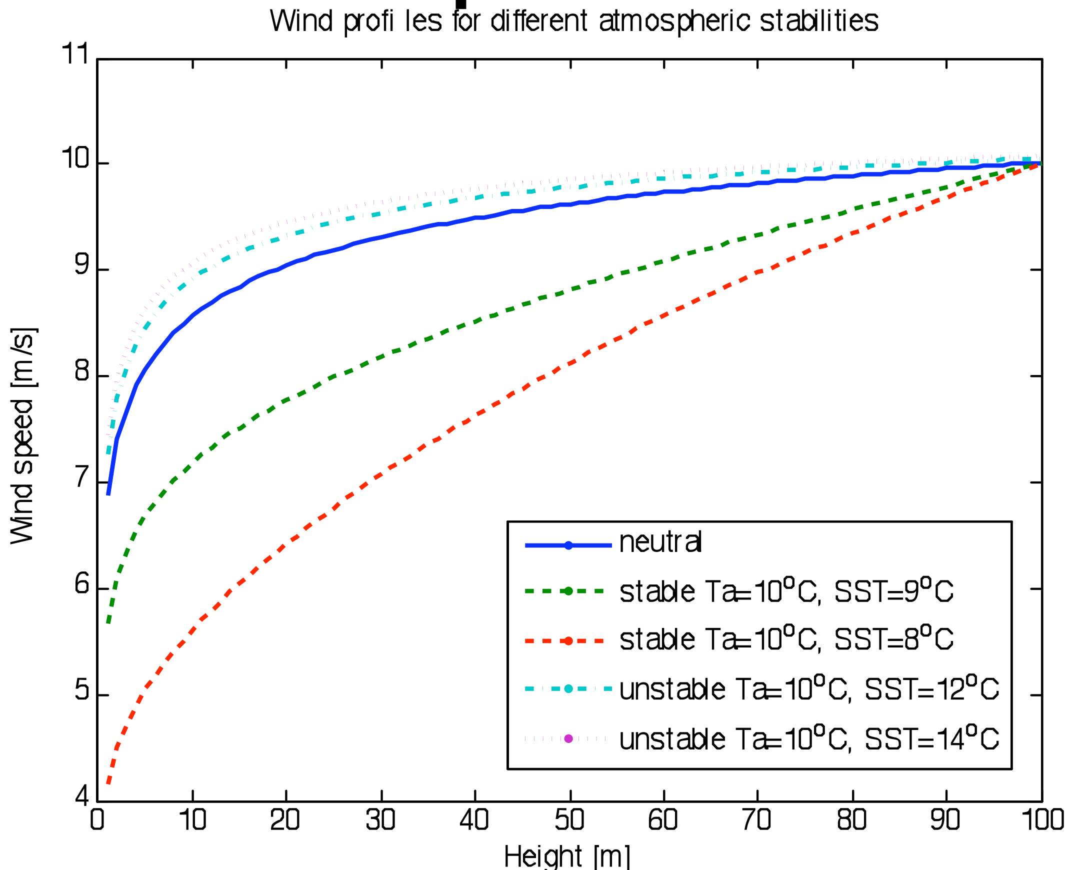
Stability-corrected profiles



Do we really have to take care about stability?



Stability-corrected profiles



Stability-corrected/Neutral drag coefficient as a function of stability

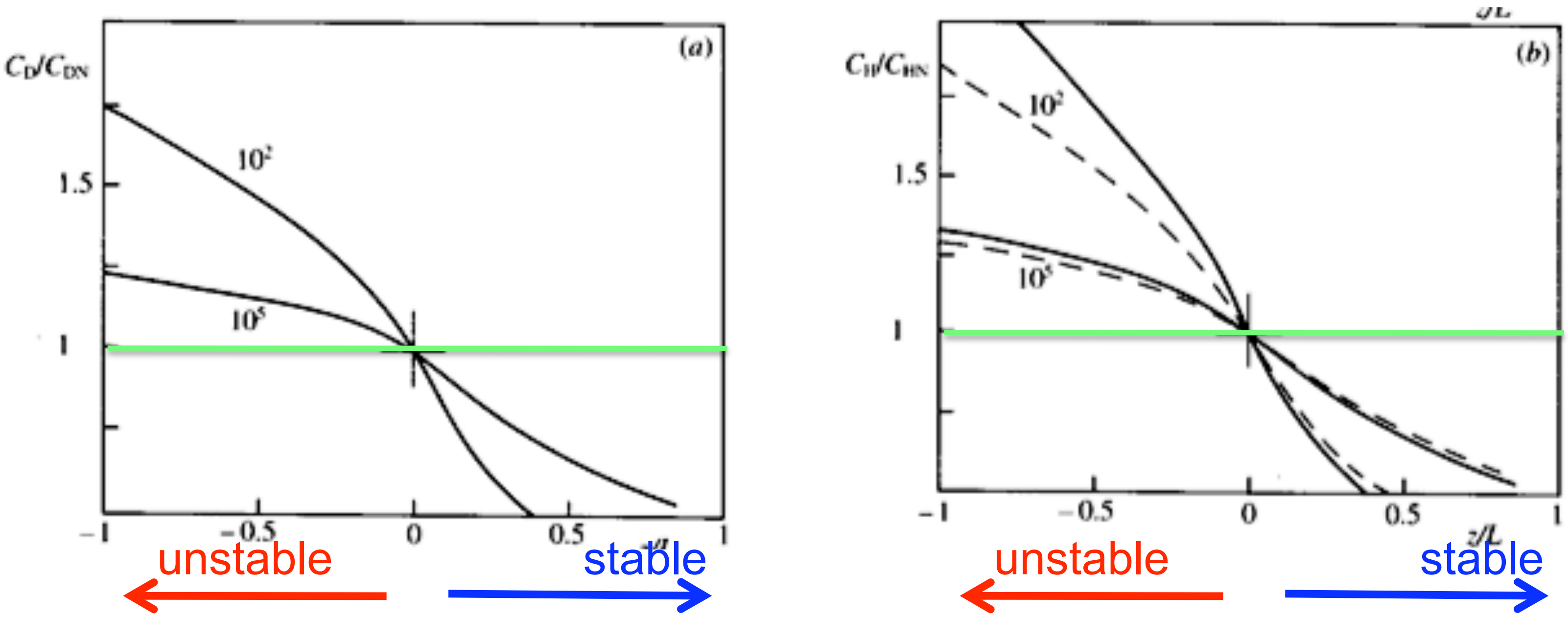
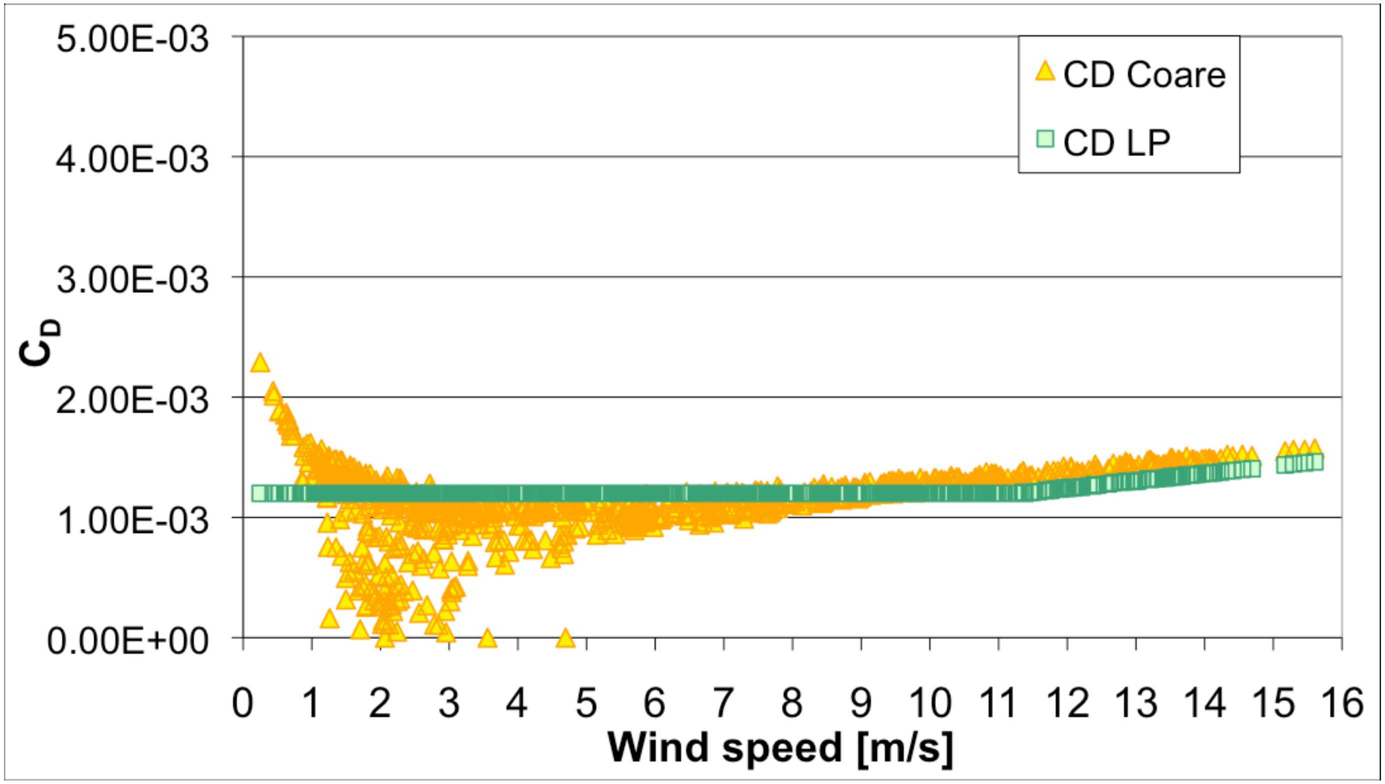


Fig. 3.7 Values of (a) C_D/C_{DN} and (b) C_H/C_{HN} as functions of z/L for two values of z/z_0 as indicated. In (b), the solid curves have $z_0 = z_T$, and the pecked curves have $z_0/z_T = 7.4$ (see Chapter 4).

Drag coefficient vs. wind speed for buoy NDBC 4613 June 2001



Wind profile in non-neutral conditions:

- For stable conditions:

$$\frac{d\overline{U}}{dz} = \left(\frac{u_*}{k \cdot z}\right) \cdot \left[1 + \left(\frac{4.7 \cdot z}{L}\right)\right]$$

$$\phi_M = \left[1 + \left(\frac{4.7 \cdot z}{L}\right)\right] = \left(\frac{k \cdot z}{u_*}\right)\frac{d\overline{U}}{dz}$$

$$\overline{U}(z) = \int \left(\frac{u_*}{k \cdot z}\right) \cdot \left[1 + \left(\frac{4.7 \cdot z}{L}\right)\right] dz$$

$$\phi_M = \begin{cases} 1 + \left(\frac{4.7 \cdot z}{L}\right) & for \quad \frac{z}{L} > 0 \quad (stable) \\ 1 & for \quad \frac{z}{L} = 0 \quad (neutral) \\ \left[1 - \left(\frac{15 \cdot z}{L}\right)\right]^{\frac{1}{4}} & for \quad \frac{z}{L} < 0 \quad (unstable) \end{cases}$$

$$\boxed{\overline{U}(z) = \left(\frac{u_*}{k}\right) \cdot \left[\log\left(\frac{z}{z_0}\right) + 4.7 \cdot \left(\frac{z}{L}\right)\right]}$$

Wind profile in non-neutral conditions:

- For unstable conditions:

$$\phi_M = \left[1-\left(\frac{15\cdot z}{L}\right)\right]^{-\frac{1}{4}} = \left(\frac{k\cdot z}{u_*}\right)\frac{d\overline{U}}{dz}$$

$$\frac{d\overline{U}}{dz} = \left(\frac{u_*}{k\cdot z}\right)\cdot\left[1-\left(\frac{15\cdot z}{L}\right)\right]^{\frac{1}{4}}$$

$$\overline{U}(z) = \int\left(\frac{u_*}{k\cdot z}\right)\cdot\left[1-\left(\frac{15\cdot z}{L}\right)\right]^{\frac{1}{4}}dz$$

$$\phi_M = \begin{cases} 1+\left(\frac{4.7\cdot z}{L}\right) & for \quad \frac{z}{L}>0 \quad (stable) \\ 1 & for \quad \frac{z}{L}=0 \quad (neutral) \\ \left[1-\left(\frac{15\cdot z}{L}\right)\right]^{\frac{1}{4}} & for \quad \frac{z}{L}<0 \quad (unstable) \end{cases}$$

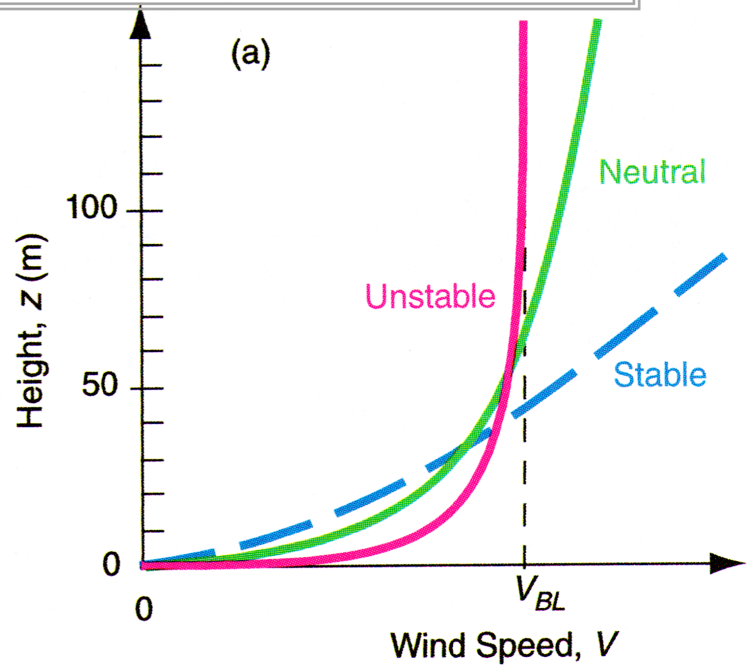
$$\overline{U}(z) = \left(\frac{u_*}{k}\right)\cdot\left\{4\left(1-15\frac{z}{L}\right)^{\frac{1}{4}} + \left[\log\left(1-15\frac{z}{L}\right)^{\frac{1}{4}} - 1\right] - \left[\log\left(1-15\frac{z}{L}\right)^{\frac{1}{4}} + 1\right] - 2\arctan\left[\left(1-15\frac{z}{L}\right)^{\frac{1}{4}}\right]\right\}$$

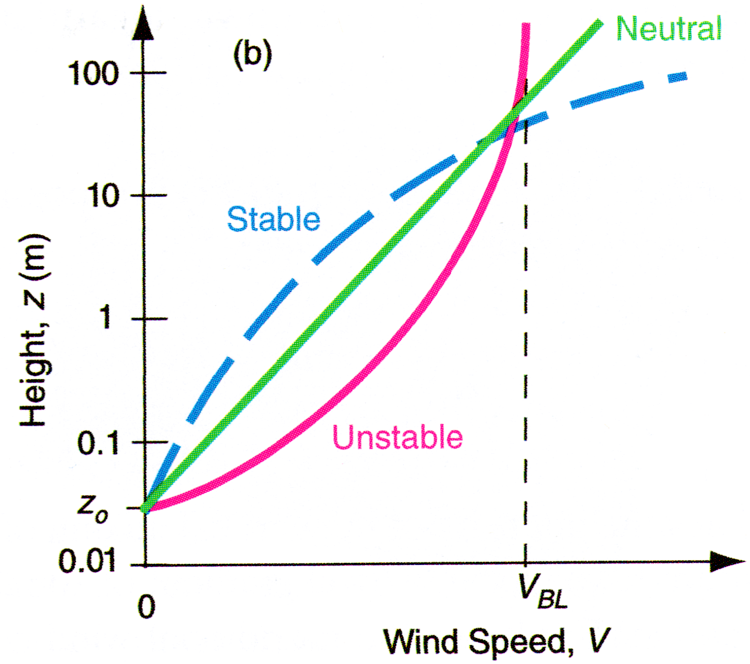
Final result: stability-corrected profiles:

$$\overline{U}(z) = \left(\frac{u_*}{k}\right) \cdot \left[\log\left(\frac{z}{z_0}\right) + \psi_M\left(\frac{z}{L}\right)\right]$$

(but we need Monin-Obukhov length L)

$$L = \frac{-u_*^3}{w_*^3}\frac{z_i}{k}$$





The End