Turbulent Kinetic Energy (§5.2) — Lecture 7

- Turbulence is continually losing energy at smallest scales to dissipation by viscosity.
- Thus, there must be continual production of turbulence if it is not to die away.
- Boundary layer turbulence is generated by convection or by wind shear.
- which predominates depends on overall temperature structure of the B.L.
- A convective B.L. has an unstable lapse rate either near the surface or near the B.L. top.
- The unstable lapse rate is produced by surface heating or cloud-top cooling (by radiation).

To study turbulence production we form an eq. for TKE:

(1) Subtract mean momentum eqs. (for $\bar{u}, \bar{v}, \bar{w}$) from eqs. for u, v, w to get eqs. for u', v', w'.

(2) multiply eqs. for u', v', w' by u', v', w', respectively and sum.

(3) Average the result to get an eq. for $\overline{u'^2} + \overline{v'^2} + \overline{w'^2}$, which is $2 \times$ TKE per unit mass.

The resulting eq. is complicated. It can be written symbolically as:

$$\frac{\bar{D}\,(TKE)}{Dt} = \underbrace{MP}_{\substack{\text{mechanical}\\ \text{(shear)}\\ \text{production}}} + \underbrace{BPL}_{\substack{\text{buoyant}\\ \text{production}\\ \text{or loss}}} + \underbrace{TR}_{\substack{\text{redist.}\\ \text{by turb.}\\ \text{transport}\\ \text{\& pressure}\\ \text{forces}}} - \underbrace{\varepsilon}_{\substack{\text{frictional (viscous)}\\ \text{dissipation}\\ >0}} \qquad (5.14)$$

$\underline{BPL}$: conversion between mean flow potential energy and turbulent K.E.

$$\boxed{\text{mean P.E.}} \xleftrightarrow{BPL} \boxed{TKE}$$

For dry air, $BPL = \frac{g}{\theta_0}\overline{w'\theta'}$.

Recall that for a single parcel,

$$\frac{1}{2}(w_2^2 - w_1^2) = \int_{z_1}^{z_2} \frac{g}{\theta_0}\theta'\,dz,$$

where $\theta' = \theta - \bar{\theta}$.

Divide by $\Delta t = \Delta z / w$; where $\Delta z = z_2 - z_1$; and $w \approx \frac{1}{2}(w_1+w_2)$.

$$\frac{1}{2}\frac{(w_2^2 - w_1^2)}{\Delta t} = \frac{w}{\Delta z}\int_{z_1}^{z_1+\Delta z} \frac{g}{\theta_0}\theta'\,dz.$$

By fundamental theorem of calculus, as $\Delta t, \Delta z \to 0$,

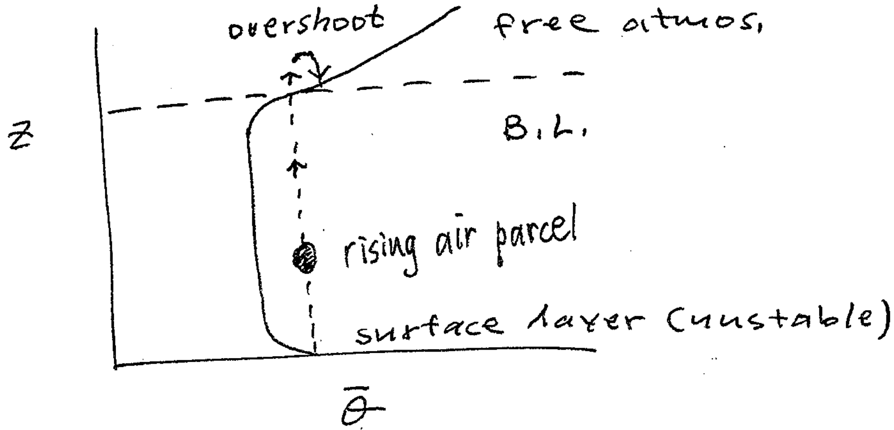
$$\frac{d}{dt}\left(\frac{w^2}{2}\right) = \frac{g}{\theta_0} w\theta'.$$

But $w = \bar{w} + w' \approx w'$ since $\bar{w} \approx 0$, so

$$\boxed{\frac{d}{dt}\left(\frac{w'^2}{2}\right) = \frac{g}{\theta_0} w'\theta'.}$$

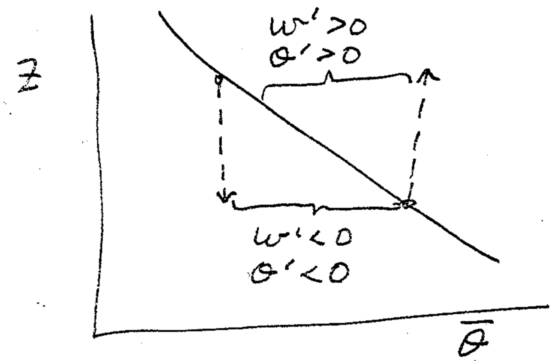
Average to get $\frac{d}{dt}\left(\frac{\overline{w'^2}}{2}\right) = \frac{g}{\theta_0}\overline{w'\theta'}$.

Positive Buoyancy prod. occurs when there is heating at surface so an unstable lapse rate develops:



Stull, Fig. 11.11

Notice that in surface layer, $\overline{w'\theta'} > 0$:



If $\bar{\theta}$ profile is stable, $\overline{w'\theta'} < 0$, which reduces or stops turbulence.

MP:

$$\boxed{\text{mean K.E.}} \xrightarrow{MP} \boxed{\text{turbulent K.E.}}$$

$$MP = -\overline{u'w'}\underbrace{\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z}}_{\text{shear}}$$

(MP > 0 when momentum flux is down gradient of mean momentum.)

Mechanical Production

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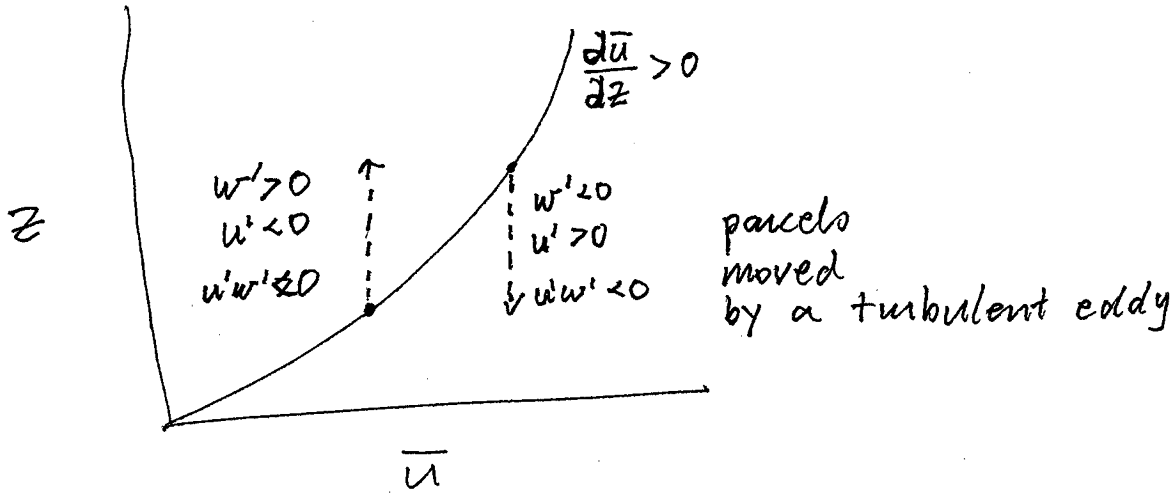
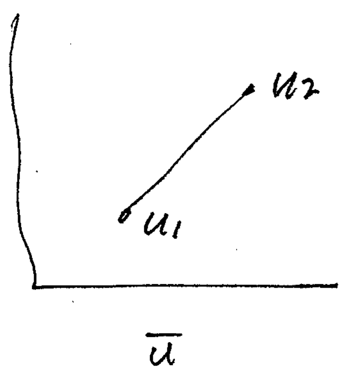


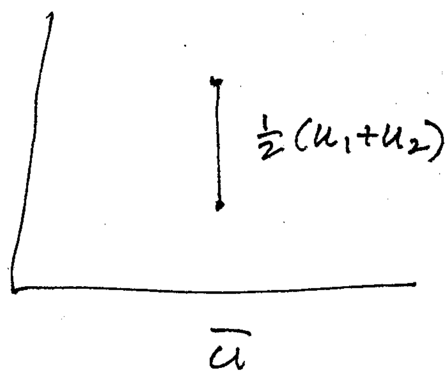
momentum is transferred from region of large $\bar{u}$ to region of low $\bar{u}$, i.e., <u>down gradient</u>; while $\overline{u'w'} < 0$ and $MP > 0$.

Effect of this transfer on mean momentum and mean K.E.

before turbulence



after turbulence



mean K.E. for layer =

$$\frac{1}{2}\left(u_1^2 + u_2^2\right)$$

$$2 \times \frac{1}{2}\left[\frac{1}{2}(u_1 + u_2)\right]^2 = \frac{1}{4}\left[u_1^2 + u_2^2 + 2u_1u_2\right]$$

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What is change in mean K.E. for layer? Before - After =

$$\frac{1}{2}(u_1^2+u_2^2)-\left[\frac{1}{2}(u_1+u_2)\right]^2$$

$$=\frac{u_1^2}{2}+\frac{u_2^2}{2}-\frac{u_1^2}{4}-\frac{u_2^2}{4}-\frac{u_1u_2}{2}$$

$$=\frac{u_1^2}{4}+\frac{u_2^2}{4}-\frac{u_1u_2}{2}$$

$$=\frac{1}{4}(u_1^2+u_2^2-2u_1u_2)$$

$$=\frac{1}{4}(u_1-u_2)^2.$$

Thus, regardless of sign of u_1-u_2, mean K.E. decreases due to mixing by turbulence.

Assign this derivation as a h.w. exercise.

If layer is statically stable, can turbulence exist? only if MP is large enough: (as measured by flux Richardson number)

$$Rf \equiv -\frac{BPL}{MP}.$$

— If BL is unstable, then $BPL > 0$, so $Rf < 0$, & turbulence is produced by convection.

— If BL is stable, obs. suggest that

$$Rf \leq 0.25$$

is req'd to maintain turbulence.

— Since MP depends on shear, it always becomes large close to surface.

— As static stability increases, depth of turbulent layer decreases.

— This explains diurnal cycle of BL depth:

At night, a strong temperature inversion may be produced by radiative cooling of the surface, and the BL depth may be only a few dm (decameters) deep, since turbulence is suppressed at higher levels where MP is small and $BPL \leq 0$.