Note that in our notes, both log and ln indicate the natural log function.

1. (c) If $\zeta = z/L$, $du/dz = \phi_m(\zeta)u_*/kz$. At any midpoint level z_m, this equation can be solved for u_* in terms of known quantities. Use the lowest midpoint where the shear is largest.

The roughness height z_0 can be deduced from the M-O formula for the wind profile:

$$u(z) = \frac{u_*}{k}\left[\log\left(\frac{z}{z_0}\right) - \Psi_m\left(\frac{z}{L}\right)\right].$$

2. (a) In the formula for C_{DN}, the coefficient for u_{10} has units of $(\mathrm{m/s})^{-1}$.

(b) Charnock's formula is given on page 8 of the slides on M-O theory: `http://www.inscc.utah.edu/~krueger/6220/Met5220_0915.pdf`

(c) The neutral drag coefficient (C_{DN}) and the roughness length (z_0) are related by the formula given on page 3 of the summary slides on the surface layer: `http://www.inscc.utah.edu/~krueger/6220/Met5220_0915%20SUMMARY.pdf`

(d) See page 5 of the summary slides on the surface layer. Latent heat flux is the moisture (water vapor) flux X latent heat of evaporation (L), where $L = 2.5 \times 10^6$ J/kg.

(e) The formula for C_{qN} is just like the one for C_{DN} used in problem (c) except that z_0 is replaced by z_q.

(f) If the surface is neutrally stratified, the temperature lapse rate is dry adiabatic ($dT/dz = -9.8$ K/km).

To estimate the water vapor mixing ratio (q) at 30 m, assume that q has a log profile up to 30 m, and use the known values of q at the ocean surface and at 10 m. The log profile for q is

$$q(z) - q(0) = \frac{q_*}{k}\log(z/z_q),$$

where q_* is a surface-layer scale for q analogous to u_*.

3. (a) See page 5 of the summary slides on the surface layer. You may assume that the potential temperature (θ) at 10 m is the temperature of 10 m air displaced adiabatically ($dT/dz = -9.8$ K/km) to the surface. Sensible heat flux is the potential temperature flux given on page 6 X specific heat at constant pressure (c_p) where $c_p = 1004$ J/(kg K).

(b) The formula for Ψ_h was not given in the slides, but it is in the Bretherton Lecture Notes, page 6.4. For $0 < \zeta$ (stable),

$$\Psi_h = -\beta\zeta,$$

where $\beta = 5$ (given on page 6.2). There is a typo in the Bretherton notes: The second equation for Ψ_m on page 6.4 is actually for Ψ_h.