Shear instability occurs at high Reynolds numbers

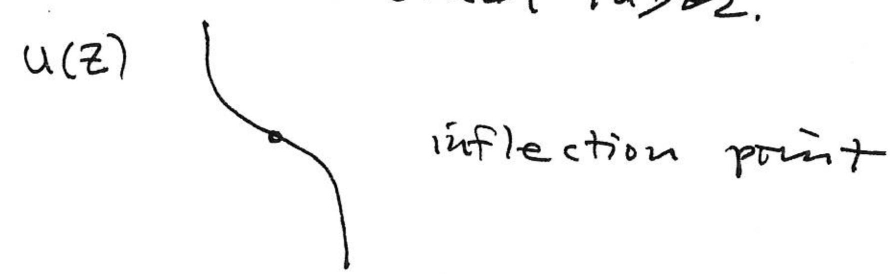
$$Re = \frac{VL}{\nu}.$$

"High" means $Re > 10^3$. ABL:

$V = 10$ m/s, $L = 10^3$ m, $\nu = 10^{-5}\ m^2 s^{-1}$

so $$Re = \frac{10^1 \times 10^3}{10^5} = 10^9.$$

Inviscid shear flows are unstable only if they have an <u>inflection point</u> where $\frac{d^2u}{dz^2} = 0$, and are definitely unstable if vorticity $\frac{du}{dz}$ has an extremum inside shear layer.



Ekman layer profile has an inflection point (Arya Fig. 7.3).

Shear instability example: Von Karman vortex street (van Dyke, p. 56).

Instability of stratified shear layer

cannot occur if stratification is large enough so that

$$Ri = \frac{N^2}{(dU/dz)^2} > \frac{1}{4}$$

throughout shear layer. For $Ri < \frac{1}{4}$, instability usually occurs.

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} = +\frac{g}{\theta}\frac{d\theta}{dz}.$$

consider a generalization of mixing two parcels of volume V at different heights:

	Height	density	velocity
Lower parcel:	$z = -\delta z$	$\rho - \delta\rho$	$-\delta U$
Upper parcel:	$+\delta z$	$\rho + \delta\rho$	δU

$$\delta U = \frac{dU}{dz}\delta z, \qquad \delta\rho = \frac{d\rho}{dz}\delta z.$$

Initial energy = (V = volume)

$$E_i = KE_i + PE_i$$

$$= V\{\rho(\delta U)^2 + 2g\,\delta\rho\,\delta z\}$$

mix in density and momentum:

	Ht	density	velocity
Lower	$-\delta z$	ρ	0
upper	$+\delta z$	ρ	0

Final energy =

$$E_f = KE_f + PE_f$$

$$= 0 + V\{\rho g(-\delta z) + \rho g(\delta z)\}$$

$$= 0$$

so change in energy is

$$\Delta E = E_f - E_i$$

$$= V\rho(\delta z)^2\left\{-\left(\frac{dU}{dz}\right)^2 + 2N^2\right\}$$

Energy reduction occurs if

$$\left(\frac{dU}{dz}\right)^2 > 2N^2,$$ or

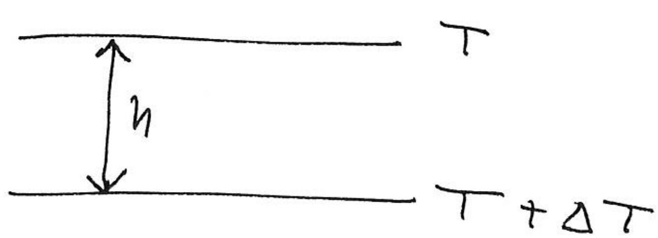
$$Ri < \frac{1}{2}.$$

Energy is transferred to turbulent K.E. Less restrictive because momentum is not fully homogenized in shear layer instabilities.

Convection.

We've examined criteria for static instability already.

In a layer of fluid between two plates

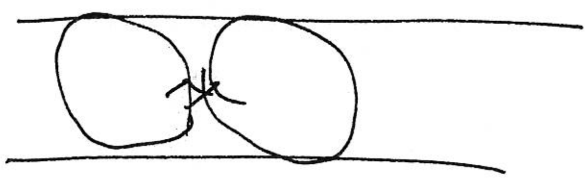


convection occurs if

$$Ra = \frac{h^3 \Delta B}{\nu \kappa} > 1700$$

$$\Delta B = -g \frac{\Delta \rho}{\rho},$$

Result is convection cells:



ABL :

$\Delta T = 1$ K,

$\Delta B = 0.03$ m s^{-2}

$h = 1000$ m

so

$$Ra = \frac{0.03 \quad 1000}{(1.4 \times 10^{-5})(2 \times 10^{-5})} = 10^{17}.$$

<u>Transition to turbulence</u>:

Secondary instabilities.

(van Dyke 102)