Shear instability occurs at high Reynolds numbers

\[ Re = \frac{VL}{\nu} \]

"High" means \( Re > 10^3 \). ABL:

\( V = 10 \text{ m/s}, \ L = 10^3 \text{ m}, \ \nu = 10^{-5} \text{ m}^2\text{s}^{-1} \)

so \( Re = \frac{10 \times 10^3}{10^{-5}} = 10^9 \).

Inviscid shear flows are unstable only if they have an inflection point where \( \frac{d^2U}{dz^2} = 0 \), and one definitely unstable if vorticity \( \frac{dU}{dz} \) has an extremum inside shear layers.

\[ U(z) \]

\[ \text{inflection point} \]

Ekman layer profile has an inflection point (Arya Fig. 7.3).

Shear instability example: Von Karman vortex street (van Dyke, p. 56).
Instability of stratified shear layer

cannot occur if stratification is large enough so that

\[ Ri = \frac{N^2}{(dU/dz)^2} > \frac{1}{4} \]

throughout shear layer. For \( Ri < \frac{1}{4} \), instability usually occurs.

\[ N^2 = -\frac{g}{\rho} \frac{dp}{dz} = -\frac{g}{\theta} \frac{d\theta}{dz} \]

consider a generalization of mixing two parcels of volume \( V \) at different heights:

<table>
<thead>
<tr>
<th>Height</th>
<th>Density</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower parcel: ( z = -\delta z )</td>
<td>( \rho - \delta \rho )</td>
<td>( -\delta U )</td>
</tr>
<tr>
<td>Upper parcel: ( z = +\delta z )</td>
<td>( \rho + \delta \rho )</td>
<td>( \delta U )</td>
</tr>
</tbody>
</table>

\[ SU = \frac{dU}{dz} \delta z, \quad S\rho = \frac{d\rho}{dz} \delta z \]

Initial energy = \( V \) = volume

\[ E_i = KE_i + PE_i \]

\[ = V \left\{ \rho (\delta U)^2 + 2 \frac{g}{\rho} \delta \rho \delta z \right\} \]

mix in density and momentum:

<table>
<thead>
<tr>
<th>Height</th>
<th>Density</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>( -\delta z )</td>
<td>( \rho )</td>
</tr>
<tr>
<td>Upper</td>
<td>( +\delta z )</td>
<td>( \rho )</td>
</tr>
</tbody>
</table>
Final energy =

\[ E_f = KE_f + PE_f \]

\[ = 0 + \nu \left[ \rho g (-d\bar{z}) + \rho g (d\bar{z}) \right] \]

so change in energy is

\[ \Delta E = E_f - E_i \]

\[ = \nu \rho (d\bar{z})^2 \left\{ -\left( \frac{d\bar{u}}{d\bar{z}} \right)^2 + 2N^2 \right\} \]

Energy reduction occurs if

\( \left( \frac{d\bar{u}}{d\bar{z}} \right)^2 > 2N^2 \), or

\[ Ri < \frac{1}{2} \]

Energy is transferred to turbulent k.e. less restrictive because momentum is not fully homogenized in shear layer instabilities.

Connection.

We've examined criteria for static instability already.

In a layer of fluid between two plates

\[ T \]

\[ \uparrow \quad h \quad \downarrow \]

\[ T + \Delta T \]
Convection occurs if

\[ Ra = \frac{h^3 \Delta B}{2 \kappa} > 1700 \]

\[ \Delta B = -g \frac{\Delta \rho}{\rho} \]

Result is convection cells:

---

ABL:

\[ \Delta T = 1 \text{ K}, \]
\[ \Delta B = 0.03 \text{ m/s}^2 \]
\[ h = 1000 \text{ m} \]

So

\[ Ra = \frac{0.03 \times 1000}{(1.4 \times 10^{-5})(2 \times 10^{-5})} = 10^{17} \]

Transition to turbulence:

Secondary instabilities

(Van Dyke 102)