neutral conditions ($L = \infty$) . The problem is further complicated over the ocean, where z_0 and z_T depend on u_*. A popular simplification is to use a computationally efficient approximation to the iterative solution, e.g. Louis 1979, *Bound. Layer Meteor.*, **17**, 187-202.

Scaling for the entire boundary layer- the turbulent Ekman layer (Garratt, 3.2)

In general, the BL depth h and turbulence profile depend on many factors, including history, stability, baroclinicity, clouds, presence of a capping inversion, etc. Hence universal formulas for the velocity and thermodynamic profiles above the surface layer (i. e. where transports are primarily by the large, BL-filling eddies) are rarely applicable.

However, a couple of special cases are illuminating to consider. The first is a well-mixed BL (homework), in which the fluxes adjust to ensure that the tendency of θ, q, and velocity remain the same at all levels. Well mixed BLs are usually either strongly convective, or strongly driven stable BLs capped by a strong inversion. As will be furher discussed in later lectures, mixed layer models incorporating an entrainment closure for determining the rate at which BL turbulence incorporates above-BL air into the mixed layer are widely used.

The other interesting (though rarely observable) case is a steady-state, neutral, barotropic BL. This is the turbulent analogue to a laminar Ekman layer. Here, the fundamental scaling parameters are $G = |\mathbf{u}_g|$, f, and z_0. Out of these one can form one independent nondimensional parameter, the surface Rossby number $\mathrm{Ro}_s = G/fz_0$ (which is typically 10^4 -10^8). The friction velocity (which measures surface stress) must have the form

$$u_*/G = F(\mathrm{Ro}_s) \tag{6.23}$$

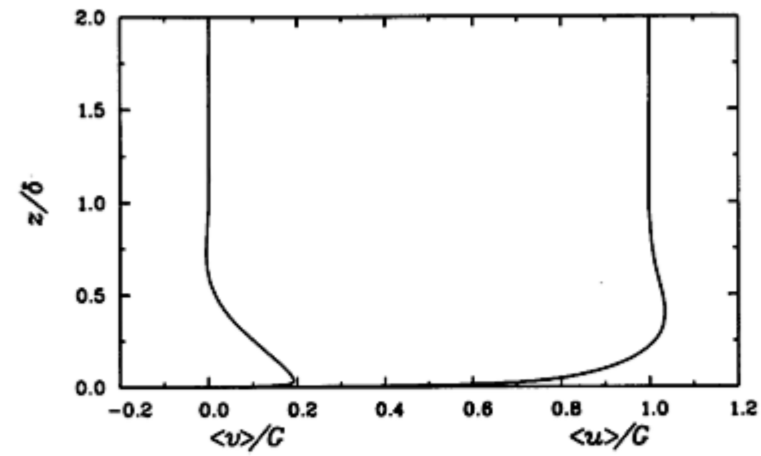
Hence, one can also regard u_*/G (which has a typical value of 0.01-0.1) as a proxy nondimensional control parameter in place of Ro_s. The steady-state BL momentum equations are

$$f(u - u_g) = -\frac{d}{dz}\overline{v'w'}, \tag{6.24}$$

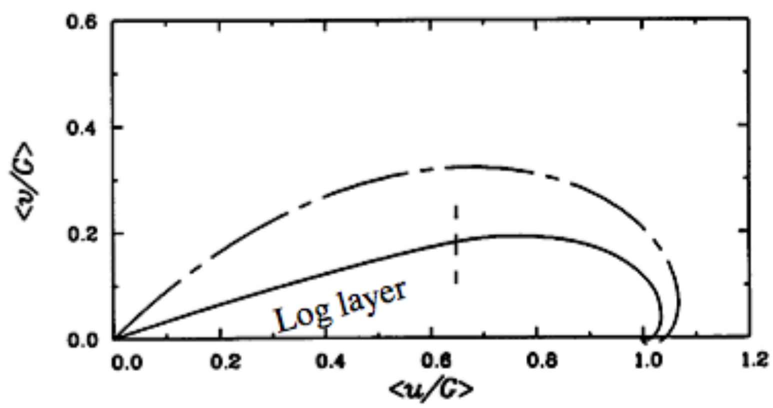
$$f(v - v_g) = \frac{d}{dz}\overline{u'w'}. \tag{6.25}$$

On the next page are velocity and momentum flux profiles from a direct numerical simulation (384×384×85 gridpoints) in which $u_*/G = 0.053$ (Coleman 1999, *J. Atmos. Sci*, **56**, 891-900). The geostrophic wind is oriented in the x direction, and is independent of height (the barotropic assumption). Height is nondimensionalized by $\delta = u_*/f$. In the thin surface layer, extending up to $z = 0.02\delta$, the wind increases logarithmically with height without appreciable turning (this is most clearly seen on the wind hodograph), and is turned at 20° from geostrophic (this angle is an increasing function of u_*/G) The neutral BL depth, defined as the top of the region of significantly ageostrophic mean wind, is

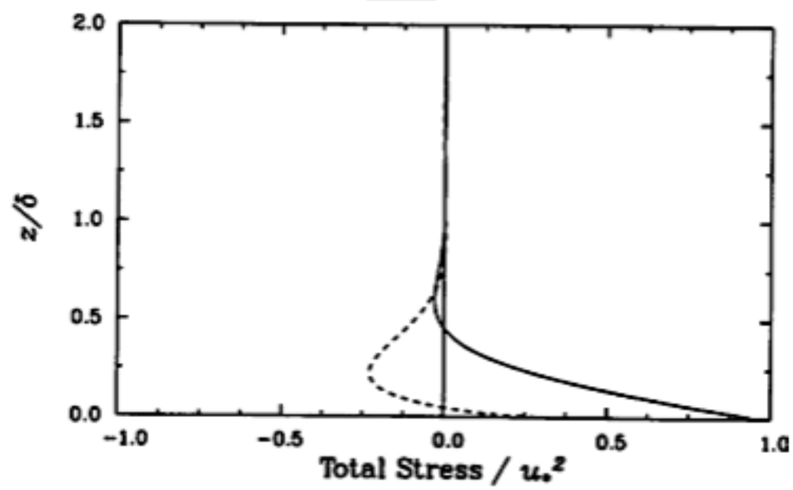
$$h_N = 0.8u_*/f. \tag{6.26}$$



Wind profiles in a neutral barotropic BL with
$u_*/G = 0.053$ (Coleman 1999).



Wind hodograph (dashed = Ekman layer). Log (surface) layer is part of profile to right of dashes.



Stress profiles in geostrophic coordinate system.
Solid = in direction of $\mathbf{u}_g$, dashed = transverse dir.

Fig. 6.5: Wind and stress profiles in a numerically simulated turbulent barotropic Ekman layer

For $u_* = 0.3$ m s^{-1} and $f = 10^{-4}$ s^{-1}, $H_N = 2.4$ km. Real ABLs are rarely this deep because of stratification aloft, but fair approximations to the idealized turbulent Ekman layer can occur in strong winds over the midlatitude oceans. The wind profile qualitatively resembles an Ekman layer with a thickness $0.12u_*/f$, except much more of the wind shear is compressed into the surface layer.

The profiles of ageostrophic wind and momentum flux depend only very weakly on Ro_s above the surface layer. Below we show a scaling using u_* and f that collapses these into universal profiles. These wind and stress profiles can be matched onto a z_0-dependent surface log-layer; the matching height (i. e. the top of the surface layer) and the implied surface wind turning angle depend upon z_0; in this way the profiles can apply to arbitrary Ro_s.

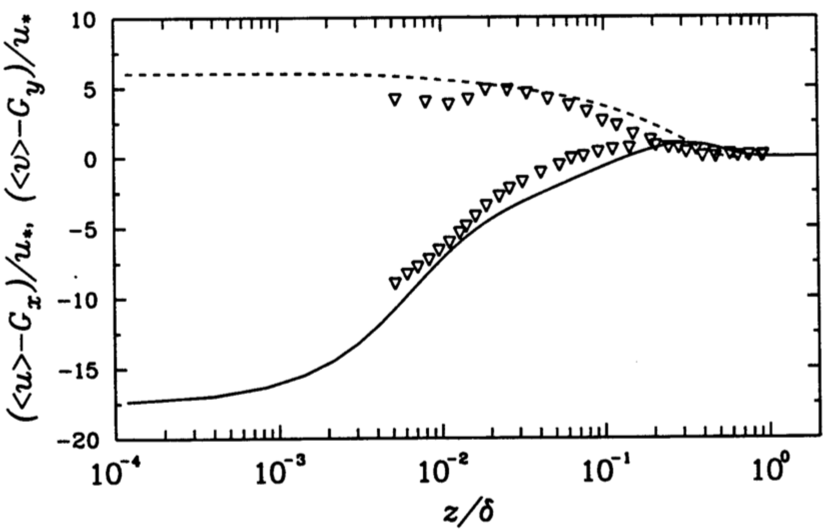


Fig. 6.6: Scaled ageostrophic wind (solid: LES; triangles: lab expt.) for a turbulent Ekman layer. A log-profile in the surface layer ($z/\delta < 0.02$) matches onto universal profiles above.

As we go up through the boundary layer, the magnitude of the momentum flux will decrease from u_*^2 in the surface layer to near zero at the BL top, so throughout the BL, the momentum flux will be $O(u_*^2)$, and the turbulent velocity perturbations u', w' should scale with u_* to be consistent with this momentum flux). We assume that the BL depth scales with $\delta = u_*/f$. These scalings suggest a nondimensionalization of the steady state BL momentum equations (6.24) and (6.25):

$$\frac{u - u_g}{u_*} = -\frac{d\left(\overline{v'w'}/u_*^2\right)}{d\left(z/\delta\right)} \tag{6.27}$$

$$\frac{v - v_g}{u_*} = \frac{d\left(\overline{u'w'}/u_*^2\right)}{d\left(z/\delta\right)} \tag{6.28}$$

If we adopt a coordinate system in which the x axis is in the direction of the surface-layer wind, the boundary conditions on the momentum flux are

$$\overline{u'w'}/u_*^2 \to 1 \text{ and } \overline{v'w'}/u_*^2 \to 0 \text{ as } z/\delta \to 0 \text{ (i. e. at surface layer top)} \tag{6.29}$$

$$\overline{u'w'}/u_*^2 \to 0 \text{ and } \overline{v'w'}/u_*^2 \to 0 \text{ as } z/\delta \to \infty \tag{6.30}$$

If we assume that the momentum flux depends only on wind shear and height, this is consistent with universal **velocity defect laws**:

$$\boxed{\frac{u-u_g}{u_*} = F_x(z/\delta), \quad \frac{v-v_g}{u_*} = F_y(z/\delta)} \ . \tag{6.31}$$

and similarly for momentum flux scaled with u_*^2. These universal profiles can then be deduced from either lab experiments or numerical simualtions of turbulent Ekman layers. . The figure below shows that Coleman's simulations and laboratory experiments with different parameters are consistent with the same F_x and F_y, supporting their universality. One can see that at h_N = 0.8δ, the velocity defects are very close to zero (geostrophic flow), while at $z \approx 0.02\delta$, the v defect has flattened out with $F_y(0) \approx 5$. This corresponds to the top of the surface layer.

In the surface layer , these universal functions cease to apply and the logarithmic wind profile $u(z) = (u_*/k) \ln(z/z_0)$, $v(z) = 0$ must match onto the defect laws. In particular, this means that $F_y(0) = -v_g/u_*$, i. e. that $v_g \approx -5u_*$. From the overall geostrophic wind magnitude G, we can deduce the surface wind turning angle α, i. e.

$$\alpha \approx \sin^{-1}(5u_*/G) \ . \tag{6.32}$$

For the case shown, this gives α ≈ 15°, in excellent agreement with the hodograph in Fig. 6.5. Smoother surfaces with lower u_*/G (e. g. ocean) will give smaller turning angles and rougher surfaces will give larger turning angles, as we'd expect. We can also deduce u_g (≈ 0.96G for the case shown). At the top of the surface layer, $u_s = u_g + u_*F_x(0) \approx u_g - 5u_* \approx (0.96\text{-}0.27)G = 0.7G$, again in good agreement with the plotted hodograph once it is rotated into coordinates along/transverse to the surface wind. From this we could deduce a precise matching height z_s at which $u_s = (u_*/k) \ln(z_s/z_0)$ between the log-layer and the velocity defect profiles. While this all may seem rather indirect, it provides a way to construct the boundary layer wind and stress profile in any turbulent barotropic Ekman layer. In fact, this would be a wonderful approach to parameterize BLs if they were actually unstratified and barotropic, but this is almost never the case in reality.