Lecture 9. **Nonlocal BL parameterizations for clear unstable boundary layers**

*In this lecture…*

- Nonlocal correction to $K$-profile approach for dry convective boundary layers
- Gradient-correction (e.g. Holtslag-Boville or YSU) and EDMF parameterizations

**Motivation: Countergradient heat transport in the DCBL**

Observations and LES of surface-heated dry convective BLs (e.g. dashed line in Fig. 9.1) show that over much of the upper half of the boundary layer ($0.4 < z/z_i < 0.8$), the $\theta$ gradient is very slightly positive even though the heat flux is also upward, opposite to the expectation from downgradient turbulent diffusion. Nonlocal schemes account for this effect by adding a correction term to scalar fluxes in convective boundary layers.

Fig. 9.1: Left: Dashed line shows LES of DCBL $\theta$ profile. Right: Solid line shows corresponding heat flux profile. From Cuijpers and Holtslag (1998).

This approach can be motivated by considering the budget equation for the flux of an advected scalar $a$ in a surface-heated convective BL. Holtslag and Moeng (1991) started by taking $a = \theta$ (potential temperature) and examining the prognostic heat flux equation:

$$
\frac{\partial}{\partial t} w'\theta' = - \frac{\partial}{\partial z} \mathbf{w}' \mathbf{w}' \theta' - \frac{\partial}{\partial z} \frac{\partial \theta}{\partial z} + \frac{g}{\theta_0} \frac{\partial \theta'}{\partial z} - \frac{1}{\rho_0} \frac{\partial \rho'}{\partial z}
$$

(9.1)
Using a 96³ gridpoint LES of a dry convective boundary layer under a 4 K inversion, they determined profiles of the four terms on the RHS (Fig. 9.2, left). Based on these profiles and theoretical arguments, they

1. Neglected storage (LHS)

2. Modeled the pressure-covariance term as:

\[ P = -\frac{1}{\rho_0} \frac{\partial \theta'}{\partial z} = -a \frac{g}{\theta_0} \frac{\theta' \theta'}{\tau}, \quad a = \frac{1}{2}. \]

3. Modeled the turbulent transport term for DCBLs (based on Fig. 9.2 left) as

\[ T = -\frac{\partial}{\partial z} w' w' \theta' = P + bw_{w} w' \theta'_{0}, \quad b = 2 \]

Putting these assumptions together into Eqn. (9.1), we obtain

\[ 0 = -w' w' \frac{\partial \theta'}{\partial z} + 2 \frac{w_{w}}{h} \frac{w' \theta'}{\theta_{0}} + (1 - 2a) \frac{g}{\theta_{0}} \frac{\theta' \theta'}{\tau} - 2 \frac{w' \theta'}{\tau} \]

so...
\[
\bar{w}'\theta'(z) = -\frac{\tau}{2} \bar{w}' \frac{\partial \theta'}{\partial z} + \frac{\tau w_{*}}{h} \bar{w}'_0, \quad (9.2)
\]

That is, the heat flux has a downgradient component with diffusivity
\[
K_a(z) = \frac{\tau}{2} \bar{w}'(z) \quad (9.3)
\]
and a second nonlocal term proportional to the surface heat flux. The nonlocal term is thus seen to derive from the combined turbulent transport, pressure-covariance, and buoyancy contributions to the heat flux tendency. This derivation suggests that the diffusivity should scale with the vertical velocity variance profile, which is well-measured and easily simulated with LES (Fig. 9.2, right):
\[
\bar{w}'w' = 2.8w_z^2(1 - Z)^2, \quad Z = z/h \quad (9.4)
\]

The timescale \( \tau \) can be determined by noting that at \( Z = 0.4 \), there is no vertical \( \theta \) gradient but the heat (buoyancy) flux is 0.5 times the surface value, so
\[
0.5\bar{w}'\theta'_0 = \frac{b\tau}{2} \frac{w_{*}}{h} \bar{w}'\theta'_0 \quad \Rightarrow \quad \tau = \frac{0.5}{w_{*}} h
\]

This gives the diffusivity profile
\[
K_a(z) = 0.7w_z z \left(1 - \frac{z}{h}\right)^2 \quad (9.5)
\]

**Gradient correction schemes**

The above scale analysis applies only to a dry, nearly shear-free convective boundary layer. To handle sheared boundary layers and achieve the appropriate log-layer scaling near the surface, BL parameterizations handle the nonlocal term somewhat differently. In one class of ‘gradient-correction’ methods, the turbulent flux of an advected scalar \( a \) is modelled using a \( K \)-profile with a nonlocal correction \( \gamma_a \) added for advected scalars in convective boundary layers
\[
\bar{w}'a' = -K_a \left(\frac{\partial a}{\partial z} - \gamma_a\right), \quad 0 < z < h \quad (9.6)
\]

The nonlocal term on the right is interpreted as being due to boundary-layer filling convective eddies which distribute the surface flux of \( a \) upward regardless of the local gradient of \( a \). If the surface flux of \( a \) is positive, the nonlocal term produces a BL within which \( a \) decreases less with height than if pure first-order closure were used.

An example of this type of scheme is the Holtslag-Boville (1993) scheme used in the CAM3 and CAM4 climate models. In this scheme, the eddy diffusivity is specified using a \( K \)-profile based on the vertical velocity variance of a CTBL,
\[
K_a(z) = kw_z(1 - z/h)^2, \quad (9.7)
\]
where \( k = 0.4 \) is the von Karman constant, but using a scaling velocity that also applied for stable and neutral boundary layers.
\[
w_z^2 = \text{Pr}(u_z^3 + c_1w_z^3), \quad c_1 = 0.6, \quad \text{Pr} = 1 \text{ (neutral)- 0.6 (pure convective)} \quad (9.8)
\]
The nonlocal term is modelled:

\[ \gamma_a = A \frac{w_u \langle w' \Delta' \rangle_0}{w_t^2 h}, \quad A = 7.2 \] (9.9)

Since the nonlocal flux is proportional to \( w_u/w_t \), it is only active in unstable boundary layers where the convective velocity \( w_u > 0 \). In stable or neutral BLs, the parameterization reduces to a \( K \)-profile eddy diffusivity scheme. The nonlocal flux is largest near the center of the boundary layer, with a maximum value

\[ \langle w' \Delta' \rangle_{\text{nonlocal, max}} = K_{a, \text{max}} \gamma_a = 0.43(w_u/w_t) \langle w' \Delta' \rangle_0 \text{ at } z = h/3 \]

Related nonlocal approaches

The MRF scheme used in the NCEP GFS model (Hong and Pan 2006) and the YSU scheme popular in the WRF (Hong et al. 2006) are structurally similar to the Holtslag-Boville scheme. In addition to the two terms in HB, the YSU scheme also includes a flux explicitly representing the effects of entrainment at the boundary layer top \( h \):

\[ \langle w' \Delta' \rangle_e(z) = \left( \frac{z}{h} \right)^3 \langle w' \Delta' \rangle(h) \]

\[ \langle w' \Delta' \rangle(h) = -w_e \Delta a \]

The entrainment rate is picked roughly following the Moeng and Sullivan closure:

\[ -w_e \Delta b = \langle w' b' \rangle(h) = 0.15 w_t \rho/h \]

This approach regulates the entrainment at the top of the boundary layer better than HB or the MRF scheme, and seems to give better overall results than other WRF PBL parameterizations over land sites (e. g. Hu et al. 2010).

**EDMF scheme**

A related nonlocal approach for convective boundary layers, EDMF (Eddy Diffusion-Mass Flux) parameterization (Siebesma et al. 2007), is used in the ECMWF weather forecasting model. In a dry-convective boundary layer, the vertical velocity has a positively-skewed pdf, implying that updrafts tend to be narrower and more intense than downdrafts, hence presumably more vertically organized. Siebesma et al. separated out vertical fluxes associated with these strongest updrafts, covering a horizontal area fraction \( A \sim 0.05-0.1 \) of the horizontal area. They treated these fluxes using a ‘mass-flux’ term in which the scalar flux is represented using the mean updraft velocity \( w_u(z) \) and mean scalar value \( a_u(z) \) in these updrafts and compensating uniform downward motion across the remaining fraction \( 1 - A \) of the domain:

\[ \langle w' \Delta' \rangle_{\text{MF}} = A w_u(a_u - \bar{a}) + (1 - A) w_d(a_d - \bar{a}) \]

\[ = A w_u(a_u - \bar{a}) + (1 - A) \frac{-A w_u}{(1 - A)} \frac{-A(a_u - \bar{a})}{(1 - A)} = A w_u(a_u - \bar{a}) \text{ if } A << 1. \]
The term \( M = Aw_u \) is called the updraft mass flux (strictly speaking it is the upward volume flux of \( a \) in the organized updrafts per unit horizontal area). This approach was taken from cumulus parameterization, where it is attractive because the cloudy updrafts are typically much more intense than the subsidence around them.

Other eddies are assumed to be less vertically organized and are treated using eddy diffusion. Thus, the overall turbulent transport is assumed to have the form:

\[
\frac{w' a'}{-K(z)\frac{d\bar{a}}{dz} + M(z)\{a_u(z) - \bar{a}(z)\}},
\]

The form of \( K(z) \) is similar to the HB form (9.2). The mass flux and the value of \( a_u \) are calculated from a differential equation describing turbulent mixing into the organized updrafts, again using ideas transferred from cumulus parameterization:

\[
\frac{da_u}{dz} = \varepsilon(z)(\bar{a} - a_u)
\]

\[
(1 - 2\mu)\frac{d}{dz}\left(\frac{w_u^2}{2}\right) = B - b\varepsilon w_u^2, \quad \text{where} \quad \mu = 0.15 \quad \text{accounts for pressure forces,} \quad b = 0.5.
\]

\( B \) is the updraft buoyancy, and based on LES, the lateral entrainment rate into the updraft is

\[
\varepsilon(z) = 0.4\left(\frac{1}{z} + \frac{1}{h - z}\right)
\]

and \( h \) is determined as the height at which \( w_u \) goes to zero. Initial updraft scalar excesses near the surface are proportional to the corresponding surface flux divided by a diagnosed vertical velocity variance at the lowest grid level.

In all the above schemes, the underlying ideas are important to appreciate, but the details and numerical implementation on a discrete grid (often relegated to appendices of papers) are also critical to their success.

References