# Atmospheric Sciences 6220
# Exercise 4: Mixed Layer Wind
# Due Oct 23, 2013

1. Use the mixed layer expressions for $\bar{u}$ and $\bar{v}$ to calculate $\bar{u}$, $\bar{v}$, $|\bar{\mathbf{V}}|$, and the cross-isobar angle of $\bar{\mathbf{V}}$ for $\bar{u}_g = 10$ m/s and $\kappa_s = 0$, 0.02, 0.05, 0.1, 0.2, and 0.4 s/m. *Hint:* Derive a quadratic equation for $|\bar{\mathbf{V}}|^2 \equiv \bar{u}^2 + \bar{v}^2$ in terms of $\kappa_s$ and $\bar{u}_g$.

   (a) Use the Matlab plot command QUIVER(X,Y,U,V,S) to plot $\bar{\mathbf{V}}$ for all cases on a single plot with X=Y=S=0. QUIVER(0,0,U,V,0) plots velocity vectors as arrows with components (u,v).
   (b) What happens to the wind speed and cross-isobar angle as the drag coefficient $\kappa_s$ increases?

2. Schematically draw the boundary layer airflow in the midlatitudes of the northern hemisphere as the air flows from
   (a) a region of lower roughness to a region of higher roughness (such as from ocean to land) during which the value of $\kappa_s$ increases, and
   (b) a region of higher roughness to a region of lower roughness (such as from land to ocean) during which the value of $\kappa_s$ decreases.

3. Suppose that the horizontal distribution of geopotential at the top of the mixed layer can be expressed in the form

$$\Phi(x, y) = \Phi_0 - f_0 U_0 y + A \sin kx \sin ly,$$

   where $\Phi_0 = 9800$ m$^2$ s$^{-2}$, $f_0 = 10^{-4}$ s$^{-1}$, $U_0 = 5$ m s$^{-1}$, $A = 1500$ m$^2$ s$^{-2}$, $k = \pi L^{-1}$, $l = \pi L^{-1}$, and $L = 6000$ km.

   (a) The MATLAB script **mixed_layer_wind2.m** contains the code to compute the geostrophic wind from the geopotential using

$$f\mathbf{V}_g = \mathbf{k} \times \nabla_p \Phi,$$

   and to plot the geopotential contours and the geostrophic wind vectors as separate plots. *Run the script without modification to make these plots.* You should compare your plots to those posted on the class web page. *Modify the code so that the geopotential contours and the wind vectors are overlaid on the same plot.* You may compare your plots to those posted on the class web page. You should notice that the wind vectors are parallel to the geopotential contours.

(b) The demonstration script **mixed_layer_wind1.m** uses an iterative technique to determine the wind in the mixed layer. (The direct solution that you used for problem 1 applies only if $v_g = 0$, which is not the case for this problem.) Running the script without modification will make a plot of the mixed layer wind components for a range of geostrophic winds. Insert the relevant parts of this script into your script from part (a) to determine the horizontal distribution of wind vectors in the mixed layer for the case where $\kappa_s = 0.05$ and the geostrophic wind field is the one calculated in part (a). *Plot the mixed layer wind vectors with the geopotential contours overlaid, similar to the plot made for part (a).* You may compare your plots to those posted on the class web page. You should notice that the mixed layer wind vectors cross the geopotential contours toward lower heights.

(c) Use the following formula (derived in the lecture notes) to compute the the vertical velocity at the top of the mixed layer for the horizontal distribution of geopotential used in parts (a) and (b) when the depth of the mixed layer, $h$, is 1 km:

$$w(h) = \frac{h\kappa_s|\mathbf{V}|}{1 + (\kappa_s|\mathbf{V}|)^2}\zeta_g,$$

where $|\mathbf{V}|$ is the mixed layer wind speed, and $\zeta_g$ is the geostrophic vorticity:

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y},$$

which can be obtained *analytically* from $\mathbf{V}_g$. The script **mixed_layer_wind1.m** contains $\mathbf{V}_g$ derived analytically from $\Phi(x, y)$. *Make two contour plots, one of $\zeta_g$ and one of $w(h)$, on the same page.* You may compare your plots to those posted on the class web page. Describe the locations of the extremes of $\zeta_g$ and $w(h)$ relative to the mixed layer wind and geopotential fields.