The goal of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data. Estimation of power spectra is useful in a variety of applications, including the detection of signals buried in wide-band noise.

The power spectrum of a stationary random process \( x_n \) is mathematically related to the correlation sequence by the discrete-time Fourier transform. In terms of normalized frequency, this is given by

\[
S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} R_{xx}(m)e^{-j\omega m}
\]

This can be written as a function of physical frequency \( f \) (e.g., in hertz) by using the relation \( \omega = 2\pi f/f_s \), where \( f_s \) is the sampling frequency.

\[
S_{xx}(f) = \sum_{m=-\infty}^{\infty} R_{xx}(m)e^{-2\pi jfm/f_s}
\]

The correlation sequence can be derived from the power spectrum by use of the inverse discrete-time Fourier transform:

\[
R_{xx}(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(\omega)e^{j\omega m} d\omega = \frac{f_s}{2\pi} \int_{-f_s/2}^{f_s/2} S_{xx}(f)e^{2\pi jfm/f_s} df
\]

The average power of the sequence \( x_n \) over the entire Nyquist interval is represented by

\[
R_{xx}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xx}(\omega) d\omega = \frac{f_s}{2\pi} \int_{-f_s/2}^{f_s/2} S_{xx}(f) df
\]

The quantities

\[
P_{xx}(\omega) = \frac{S_{xx}(\omega)}{2\pi} \quad \text{and} \quad P_{xx}(f) = \frac{S_{xx}(f)}{f_s}
\]

from the above expression are defined as the power spectral density (PSD) of the stationary random signal \( x_n \).

The average power of a signal over a particular frequency band \([\omega_1, \omega_2]\), \(0 \leq \omega_1 < \omega_2 \leq \pi\), can be found by integrating the PSD over that band:

\[
P_{[\omega_1, \omega_2]} = \int_{\omega_1}^{\omega_2} P_{xx}(\omega) d\omega = \int_{\omega_1}^{-\omega_1} P_{xx}(\omega) d\omega + \int_{-\omega_2}^{\omega_2} P_{xx}(\omega) d\omega
\]

You can see from the above expression that \( P_{xx}(\omega) \) represents the power content of a signal in an infinitesimal frequency band, which is why we call it the power spectral density.
The units of the PSD are power (e.g., watts) per unit of frequency. In the case of $P_{xx}(\omega)$, this is watts/rad/sample or simply watts/rad. In the case of $P_{xx}(f)$, the units are watts/hertz. Integration of the PSD with respect to frequency yields units of watts, as expected for the average power $\bar{P}_{[\omega_1, \omega_2]}$.

For real signals, the PSD is symmetric about DC, and thus $P_{xx}(\omega)$ for $0 \leq \omega < \pi$ is sufficient to completely characterize the PSD. However, in order to obtain the average power over the entire Nyquist interval it is necessary to introduce the concept of the one-sided PSD.

The one-sided PSD is given by

$$P_{\text{onesided}}(\omega) = \begin{cases} 
0, & -\pi \leq \omega < 0 \\
2P_{xx}(\omega), & 0 \leq \omega < \pi
\end{cases}$$

The average power of a signal over the frequency band $[\omega_1, \omega_2]$, $0 \leq \omega_1 < \omega_2 \leq \pi$, can be computed using the one-sided PSD as

$$\bar{P}_{[\omega_1, \omega_2]} = \int_{\omega_1}^{\omega_2} P_{\text{onesided}}(\omega) d\omega$$