Boundary Layer Meteorology
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Neutral conditions

$$(\overline{u'w'})_s = u_*^2$$

$$u_*^2 = (\overline{u'w'})_s = K_m \frac{\partial \overline{U}}{\partial z}$$

u_*-Friction velocity
k-von Karman const.
z-height
z_0-roughness length

$$K_m = l^2 \left| \frac{\partial \overline{U}}{\partial z} \right| \qquad l = k \cdot z$$

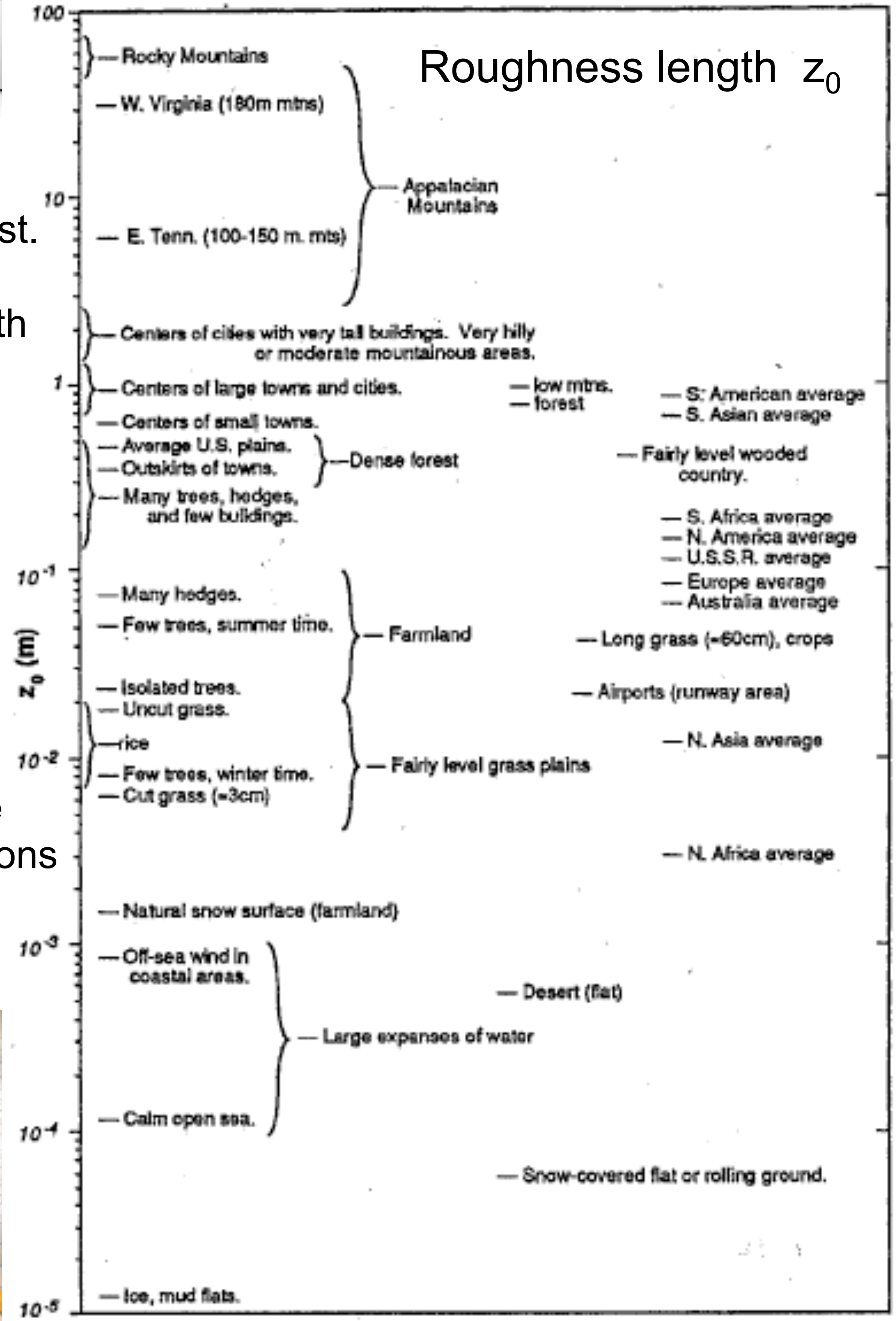
$$u_*^2 = k^2 z^2 \left| \frac{\partial \overline{U}}{\partial z} \right|^2 \rightarrow \left| \frac{\partial \overline{U}}{\partial z} \right| = \frac{u_*}{k \cdot z}$$

$$U(z) = \frac{u_*}{k} \log\left(\frac{z}{z_0}\right)$$

Logarithmic wind profile valid for **neutral** conditions

$$\rho \cdot (\overline{u'w'})_s = \tau$$

surface stress



Aerodynamic bulk formula ($\tau = \rho C_D \cdot U^2$)

$$\rho \cdot \overline{(u'w')}_s = \tau$$ surface stress

$$\overline{(u'w')}_s = u_*^2$$

$$U(z) = \frac{u_*}{k} \log\left(\frac{z}{z_0}\right)$$

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$$u_* = \frac{k \cdot U(z)}{\log\left(\frac{z}{z_0}\right)}$$

$$\overline{(u'w')}_s = \frac{k^2 \cdot [U(z)]^2}{\left[\log\left(\frac{z}{z_0}\right)\right]^2}$$

$$\tau = \rho \cdot \overline{(u'w')}_s = \rho \frac{k^2}{\left[\log\left(\frac{z}{z_0}\right)\right]^2} \cdot [U(z)]^2$$

drag coefficient C_D

$$\tau = \rho \cdot C_D \cdot U^2$$

Drag Coefficient (C_D)

$$\tau = \rho \cdot C_D \cdot U^2$$ surface stress

$$(\overline{u'w'})_s = \frac{\tau}{\rho}$$

In practice the drag coefficient is given usually with respect to the wind speed at z=10m and for neutral conditions (C_{DN10})

Typical values of the drag coefficient over the land are significantly larger than over the water

$C_{D\ land} \approx 7\times10^{-3}$

$C_{D\ water} \approx 1\times10^{-3}$

Transfer Coefficients

$$\tau = \rho \cdot (\overline{u'w'})_s = \rho \cdot C_D \cdot U^2$$ bulk formula for momentum

$$\rho \cdot (\overline{w'a'})_s = \rho \cdot C_a \cdot U(z_r) \cdot \left[a_0 - a(z_r)\right]$$ bulk formula for scalar 'a'

transfer coefficient for moisture

$$\rho \cdot (\overline{w'q'})_s = \rho \cdot C_E \cdot U(z_r) \cdot \left[q_0 - q(z_r)\right]$$ bulk formula for moisture

transfer coefficient for heat

$$\rho \cdot (\overline{w'\theta'})_s = \rho \cdot C_H \cdot U(z_r) \cdot \left[\theta_0 - \theta(z_r)\right]$$ bulk formula for heat

Velocity scales:

- Friction velocity:

$$u_* = \left[\overline{u'w'}^2 + \overline{v'w'}^2\right]^{\frac{1}{4}} \qquad u_*^2 = \left(\overline{u'w'}\right)$$

For one-dimensional case

- Convective velocity scale (Deardorff velocity):

$$w_* = \left[\frac{g \cdot z_i}{T_v}\overline{w'\theta_S'}\right]^{\frac{1}{3}}$$

z_i – height of capping inversion (PBL height)
T_v – virtual temperature
ϑ – potential temperture

Length scales:

- Monin-Obukhov length — L
- Stability parameter: — $\zeta = z/L$
- Height of capping inversion (PBL height): — z_i
- Aerodynamic roughness length — z_0
- Height above the surface — z

Monin-Obukhov Length: Height proportional to the height above the surface at which buoyant production of turbulence first equals mechanical (shear) production of turbulence.

$$L = \frac{-u_*^3}{k \cdot B_0} = \frac{-u_*^3}{w_*^3}\frac{z_i}{k}$$

$$L = \frac{-u_*^3}{k \frac{g}{T_v}\left(\overline{w'\theta'}\right)_s}$$

$$B_0 = \overline{w'b'}_0 = \frac{-u_*^3}{k \cdot L}$$

z_i – height of capping inversion (PBL height)
T_v – virtual temperature
ϑ – potential temperature
k – von Karman constant (0.41)
B_0- surface buoyancy flux

For unstable atmosphere $L < 0$, so $\zeta < 0$
For neutral atmosphere $L \to \infty$, so $\zeta = 0$
For stable atmosphere $L > 0$, so $\zeta > 0$

Universal similarity functions and eddy viscosities

Universal similarity functions relate the fluxes of momentum and sensible heat to their mean gradients

universal similarity function for momentum
$$\phi_m(\varsigma) = \frac{k \cdot z}{u_*}\left(\frac{\partial \overline{u}}{\partial z}\right)$$

eddy viscosity for momentum
$$K_m = \frac{-\overline{u'w'}}{\frac{\partial u}{\partial z}} = \frac{u_*^2}{\frac{u_*\phi_m(\varsigma)}{k \cdot z}} = \frac{u_* \cdot k \cdot z}{\phi_m(\varsigma)}$$

universal similarity function for heat
$$\phi_h(\varsigma) = \frac{k \cdot z}{\theta_*}\left(\frac{\partial \overline{\theta}}{\partial z}\right)$$

eddy viscosity for heat
$$K_h = \frac{-\overline{u'\theta'}}{\frac{\partial \theta}{\partial z}} = \frac{u_*\theta_*}{\frac{\theta_*\phi_h(\varsigma)}{k \cdot z}} = \frac{u_* \cdot k \cdot z}{\phi_h(\varsigma)}$$

$\phi_m(\varsigma) < 1$ for unstable conditions

$\phi_m(\varsigma) = 1$ for neutral conditions

$\phi_m(\varsigma) > 1$ for stable conditions

In neutral or stable stratification $\phi_m=\phi_h$ $(1/K_m=1/K_h)$

Pressure perturbations do not affect the eddy transport of momentum relative to heat and other scalars $Pr_t=1$.

$$\phi_h = \begin{cases} 1 & \text{for} \quad \varsigma = 0 \quad (\text{neutral}) \\ 1+5\varsigma & \text{for} \quad 0 \le \varsigma < 1 \quad (\text{stable}) \end{cases}$$

$$\phi_m = \begin{cases} 1 & \text{for} \quad \varsigma = 0 \quad (\text{neutral}) \\ 1+5\varsigma & \text{for} \quad 0 \le \varsigma < 1 \quad (\text{stable}) \end{cases}$$

In unstable stratification $\phi_h < \phi_m$ ($K_h > K_m$)

Eddy diffusivity for scalars is more than for momentum (universal similarity function for momentum (Φ_m) is greater than for scalars (Φ_h)).

$$\phi_m = \left\{ [1 - 16\varsigma]^{-\frac{1}{4}} \qquad \mathit{for} \quad 0 \le \varsigma < 1 \quad (\mathit{stable}) \right\}$$

$$\phi_h = \left\{ [1 - 16\varsigma]^{-\frac{1}{2}} \qquad \mathit{for} \quad 0 \le \varsigma < 1 \quad (\mathit{stable}) \right\}$$

Wind and thermodynamic profiles

- For all cases we can use one formula with stability correction function defined below:

$$\overline{U}(z) = \left(\frac{u_*}{k}\right)\cdot\left[\log\left(\frac{z}{z_0}\right) + \psi_M\left(\frac{z}{L}\right)\right]$$

Ψ_M – stability correction function

L – Monin-Obukhov length

$$\psi_M = \int_0^{\varsigma}\left[1-\phi_m(\varsigma')\right]d\varsigma'/\varsigma'$$

$$\psi_M = \begin{cases} \left(\dfrac{4.7\cdot z}{L}\right) & for \quad \dfrac{z}{L} > 0 \quad (stable) \\ 0 & for \quad \dfrac{z}{L} = 0 \quad (neutral) \\ -2\ln\left[\dfrac{1+x}{2}\right] - \ln\left[\dfrac{1+x}{2}\right] - \ln\left[\dfrac{1+x^2}{2}\right] + 2\tan^{-1}(x) - \dfrac{\pi}{2} & for \quad \dfrac{z}{L} < 0 \quad (unstable), \end{cases}$$

$$where: \; x = \left[1 - 15\frac{z}{L}\right]^{\frac{1}{4}}$$