QG Theory and Applications: Q-Vectors

Atmos 5110 Synoptic–Dynamic Meteorology I

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Suggested reading: Lackmann (2011), Section 2.3 (p. 48-50)

Motivation

Desire a form of the omega equation that avoids ambiguities arising from the two forcing terms being of opposite sign and the eliminates the need to examine multiple levels to evaluate the differential vorticity advection.

Advantages of the Q-vector form of the omega equation

1. Single forcing term
2. Can be evaluated on a single level

Disadvantages

1. Additional simplifying assumption (f plane in most uses, although this is not necessary)
2. Without explicitly plotting the Q-vectors, it's extremely difficult (perhaps impossible) to evaluate from traditional synoptic maps
3. Not as physically intuitive for beginning students

Q-vector omega equation

See me if you are interested in the full derivation (a good exercise for graduate students)

$$\left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega = -2\nabla\cdot\vec{Q}$$

where

$$\vec{Q} = -\frac{R}{\sigma p}\begin{pmatrix} \frac{\partial \vec{V}_g}{\partial x} \cdot \nabla\theta \\ \frac{\partial \vec{V}_g}{\partial y} \cdot \nabla\theta \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = -\frac{R}{\sigma p}\begin{pmatrix} \frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial x}\frac{\partial \theta}{\partial y} \\ \frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y} \end{pmatrix}$$

Interpretation

The left hand side is essentially the 3-D Laplacian acting on ω, as in the traditional omega equation. For sinusoidal (wave-like) patterns, the Laplacian can be approximated by a minus sign

$$\left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega \sim -\omega \propto w$$

Therefore,

$$w \propto -\omega \propto -2\nabla \cdot \vec{Q} \propto -\nabla \cdot \vec{Q}$$

So that,

- Q-vector convergence ($\nabla \cdot \vec{Q} < 0$) is associated with rising motion (w > 0)
- Q-vector divergence ($\nabla \cdot \vec{Q} > 0$) is associated with sinking motion (w < 0)

It is sometimes said that the Q-vector "points toward rising motion and away from sinking motion."

Important Caveat

The Q-vector is not to be confused with the velocity vector and Q-vector convergence and divergence are not directly related to the convergence and divergence of the wind.

Synoptic Application

In practice, we typically plot the Q-vectors and Q-vector divergence along with isotherms or isentropes at the level on which we are interested in evaluating vertical motion (e.g., 700-mb).

WTH is the Q-vector physically?

The Q-vector is the rate of change of the horizontal temperature gradient, $\nabla\theta$, following the geostrophic flow. This includes both changes in the magnitude of $\nabla\theta$ (i.e., the strength of a front) and the orientation of $\nabla\theta$ (i.e., the orientation of a front).

Class Activities:

1. Using the IDV Diagnostics -> QG-Omega-Qvector bundle, evaluate the large-scale vertical motion over North America over the past two days.
2. Perform a similar analysis using the IDV Diagnostics -> QG-Omega bundle and the traditional omega equation and gain an appreciation for the strengths and weaknesses of the two approaches.

Estimating the Q-vector on a weather map

The Q-vector is best estimated on a weather map utilizing a version of the Q-vector in which the x-axis is assumed to be parallel to the isotherms/isentropes with cold air on the left. In this case:

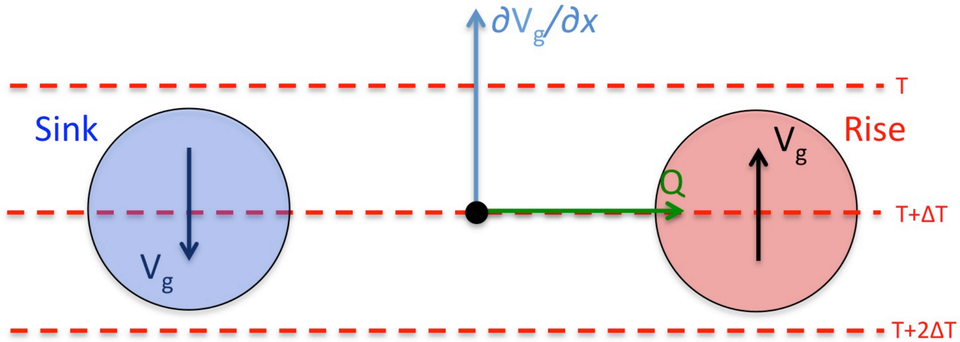
$$\vec{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial y}\right|(\hat{k} \times \frac{\partial \vec{V}_g}{\partial x})$$

and the direction of Q is perpendicular and to the right of the vector change of the geostrophic wind along an isotherm.

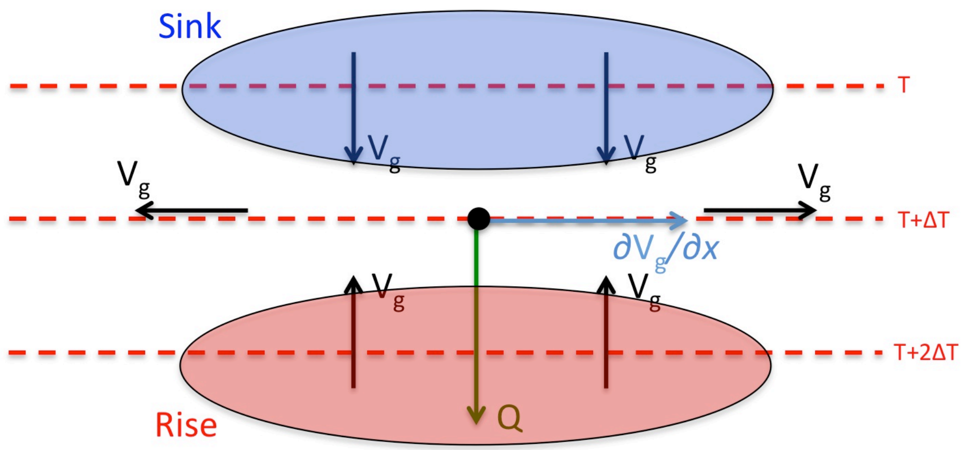
Thus, to determine the orientation of the Q-vector:

1. Find the vector change of the geostrophic wind along the isotherm
2. Rotate this vector 90° clockwise

Examples



$\partial V_g/\partial x$
T
Sink
V_g
Rise
V_g
Q
T+ΔT
T+2ΔT



Sink
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V_g
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T+ΔT
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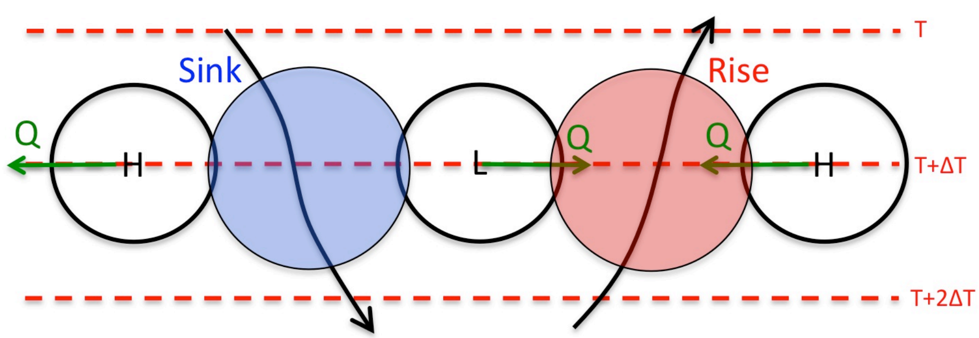

T
Sink
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Q
H
T+ΔT
T+2ΔT

Evaluating and Interpreting Vertical Motion: A Summary

1. Kinematic method
- Infer or calculate vertical velocity from horizontal wind fields and continuity equation
- Used to calculate vertical motion in hydrostatic models
- Sometimes applied subjectively with surface data, especially in data-rich regions, but difficult to apply to observational data where sparse
- Need to be cautious about divergence vs. diffluence/convergence vs. confluence

2. Omega equation
- Based on QG
- Can help infer large-scale vertical motion from traditional synoptic analyses
- Challenges arise from simplifying assumptions (e.g., LaPlacian), term cancellation, and the need to evaluate multiple levels for differential vorticity advection
- Doesn't account for circulations generated by many mesoscale systems (e.g., fronts, gravity waves, etc.)
- Smoothing needed with higher resolution model data

3. Q-vectors
- Based on QG
- Eliminates term cancellation and multi-level problems posed by traditional omega equation
- Extremely difficult to apply visually
- Doesn't account for circulations generated by many mesoscale systems (e.g., fronts, gravity waves, etc.)
- Smoothing needed with higher resolution model data

4. Raw model vertical velocity

- Based on kinematic method (hydrostatic model) or vertical momentum equation (nonhydrostatic model)
- "Full physics"
- Still not perfect – only as good as model physics, resolution (of both atmospheric and topographic features), and accuracy
- Model vertical velocity structure, especially at small scales, strongly dependent on resolution