QG Theory and Applications: Height Tendency Equation

Atmos 5110 Synoptic–Dynamic Meteorology I

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Suggested reading: Lackmann (2011), Section 2.4

Usage

The QG height tendency equation is used to understand and diagnose the development and decay of large-scale weather systems.

Derivation

Derived from the QG thermodynamic and vorticity equations. See Lackmann (2011) for details.

The Height tendency Equation

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi = -f_0\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) - \frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\vec{V}_g \cdot \nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right]$$

Yeah, it's daunting, but easier if you think of it as A=B+C where:

$$\chi = \frac{\partial\Phi}{\partial t} \propto \frac{\partial Z}{\partial t} \text{ (Geopotential Tendency)}$$

$$A = \left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi$$

$$B = -f_0\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) \text{ = Vorticity advection term}$$

$$C = -\frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\vec{V}_g \cdot \nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right] \text{ = Differential temperature advection term}$$

****Note that these terms are not synonymous with the differential vorticity advection and temperature advection terms of the omega equation*

Term A

Essentially the 3-D Laplacian acting on χ. For sinusoidal (wave-like) patterns, the Laplacian can be approximated by a minus sign.

$$A = \left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi \sim -\chi \propto -\frac{\partial Z}{\partial t}$$

Therefore,

$$-\frac{\partial Z}{\partial t} \propto -f_0 \vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) - \frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\vec{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

Term B

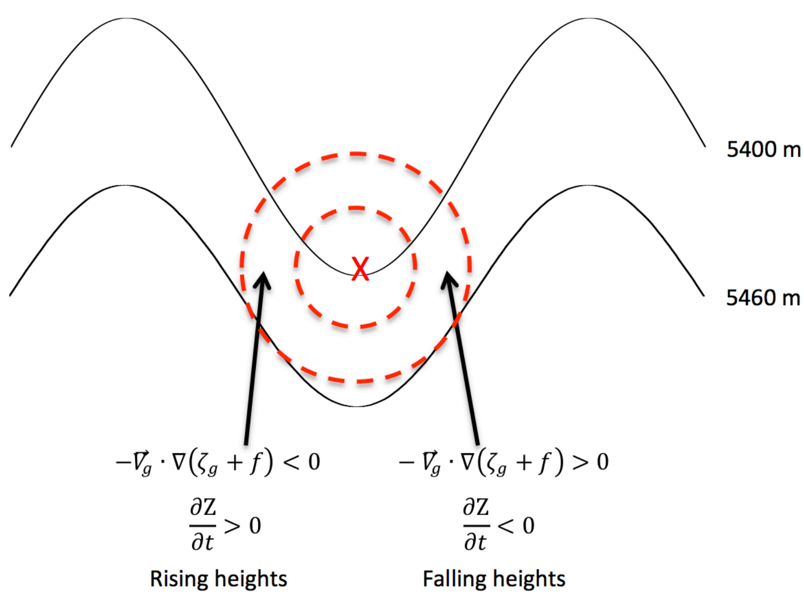
Vorticity advection term

$$-\frac{\partial Z}{\partial t} \propto -f_0 \vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)$$

$$\frac{\partial Z}{\partial t} \propto -\left[-f_0 \vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right]$$

Therefore

- Cyclonic vorticity advection (positive in NH) results in falling heights
- Anticyclonic vorticity advection (negative in NH) results in rising heights
- Typically evaluate @ 500 mb



Term C

Temperature advection term

$$-\frac{\partial Z}{\partial t} \propto -\frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\vec{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

$$\frac{\partial Z}{\partial t} \propto \frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\vec{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] \propto -\frac{\partial}{\partial z}\left[-\vec{V}_g \cdot \nabla \mathrm{T}\right]$$

Therefore

- Positive differential temperature advection results in height falls
- Negative differential temperature advection results in height rises

Examples

Lower tropospheric cold advection, no upper tropospheric temperature advection

300 mb $-\vec{V}_g \cdot \nabla \mathrm{T} = 0$

500 mb $\frac{\partial}{\partial z}\left(-\vec{V}_g \cdot \nabla \mathrm{T}\right) > 0 \rightarrow$ Height falls @ 500 mb

700 mb $-\vec{V}_g \cdot \nabla \mathrm{T} < 0$

Lower tropospheric warm advection, no upper tropospheric temperature advection

300 mb $-\vec{V}_g \cdot \nabla \mathrm{T} = 0$

500 mb $\frac{\partial}{\partial z}\left(-\vec{V}_g \cdot \nabla \mathrm{T}\right) < 0 \rightarrow$ Height rises @ 500 mb

700 mb $-\vec{V}_g \cdot \nabla \mathrm{T} > 0$

No lower tropospheric temperature advection, upper tropospheric cold advection

300 mb $-\vec{V}_g \cdot \nabla \mathrm{T} < 0$

500 mb $\frac{\partial}{\partial z}\left(-\vec{V}_g \cdot \nabla \mathrm{T}\right) < 0 \rightarrow$ Height rises @ 500 mb

700 mb $-\vec{V}_g \cdot \nabla \mathrm{T} = 0$

No lower tropospheric temperature advection, upper tropospheric warm advection

300 mb $-\vec{V}_g \cdot \nabla \mathrm{T} > 0$

500 mb $\frac{\partial}{\partial z}\left(-\vec{V}_g \cdot \nabla \mathrm{T}\right) > 0 \rightarrow$ Height falls @ 500 mb

700 mb $-\vec{V}_g \cdot \nabla \mathrm{T} = 0$

Term C in practice

- Warm advection above an upper-level trough tends to deepen the trough
- Cold advection beneath an upper-level trough tends to deepen the trough
- Best case for trough amplification is upper-level warm advection and lower-level cold advection
- Cold advection above an upper-level ridge tends to build the ridge
- Warm advection beneath an upper-level ridge tends to build the ridge
- Best case for ridge amplification is upper-level cold advection and lower level warm advection
- Rules of thumb are shortcuts – ultimately how the temperature advection changes with height is what matters

Class Activity

Using the IDV Diagnostics -> QG-HeightTend bundle, evaluate the right-hand terms and 500-mb height tendency over North America over the past two days. Be sure to examine the differential temperature advection by examining multiple levels and using the vertical probe.

Additional applications

The height tendency equation is sometimes used to diagnose low-level cyclone and anticyclone development:

- Warm advection above a surface cyclone (e.g., @ 850 mb) will tend to deepen the cyclone
- Cold advection above a surface anticyclone will tend to deepen the anticyclone

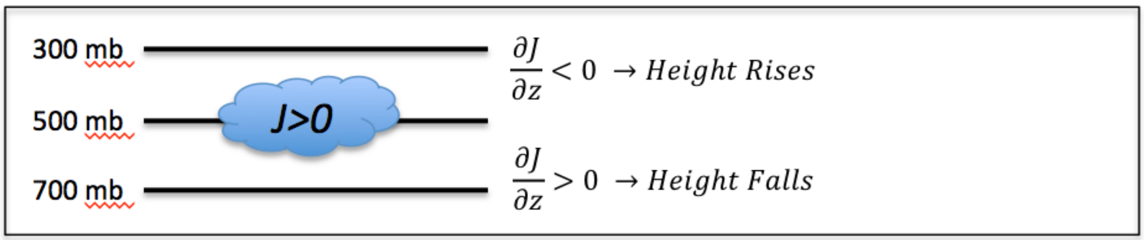
Diabatic effects

If diabatic effects are considered, then the height tendency equation contains an additional term that implies:

$$\frac{\partial Z}{\partial t} \propto -\frac{\partial J}{\partial z}, where\ J = diabatic\ heating\ rate$$

Therefore

- Positive differential diabatic heating yields height falls
- Negative differential diabatic heating yields height rises



<u>Synoptic Application</u>

A maximum in condensational warming in the lower-to-middle tropospheric height falls and upper-level height rises — important in cyclogenesis!