QG Theory and Applications: Omega Equation

Atmos 5110 Synoptic–Dynamic Meteorology I

Instructor: Jim Steenburgh

jim.steenburgh@utah.edu

801-581-8727

Suite 480/Office 488 INSCC

Suggested reading: Lackmann (2011), Section 2.3

Usage

The omega equation is used to diagnose large-scale vertical motion.

Derivation

Derived from the QG thermodynamic and vorticity equations. See Lakcmann (2011) for details.

The Omega Equation

$$\left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + \mathrm{f}\right)\right] + \frac{1}{\sigma}\nabla^2[\vec{V}_g \cdot \nabla\left(-\frac{\partial\Phi}{\partial p}\right)]$$

Yeah, it's daunting, but easier if you think of it as A=B+C where:

$$A = \left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega$$

$$B = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + \mathrm{f}\right)\right]$$ = Differential vorticity advection term

$$C = \frac{1}{\sigma}\nabla^2[\vec{V}_g \cdot \nabla\left(-\frac{\partial\Phi}{\partial p}\right)]$$ = Temperature advection term

Term A

Essentially the 3-D Laplacian acting on ω. For sinusoidal (wave-like) patterns, the Laplacian can be approximated by a minus sign.

$$A = \left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega \sim -\,\omega \;\propto w$$

Therefore,

$$w \propto -\omega \propto \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + \mathrm{f}\right)\right] + \frac{1}{\sigma}\nabla^2[\vec{V}_g \cdot \nabla\left(-\frac{\partial\Phi}{\partial p}\right)]$$

<u>Term B</u>

Differential vorticity advection term

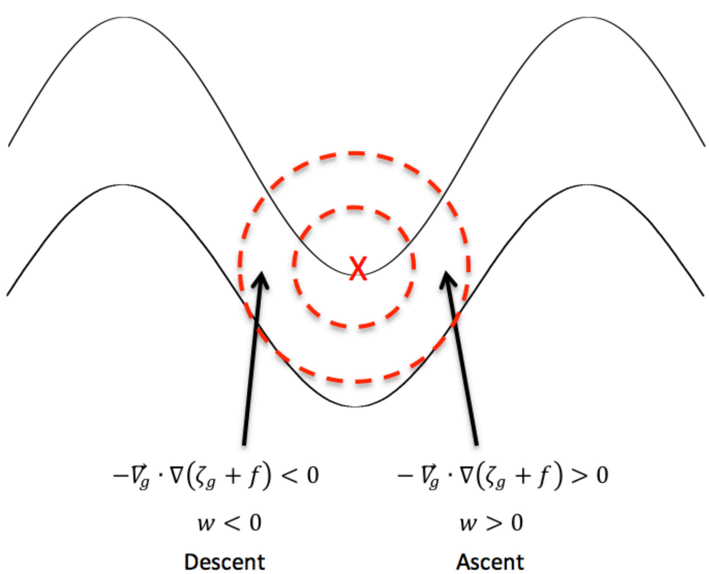
$$w \propto \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + \mathrm{f}\right)\right] \propto \frac{\partial}{\partial z}\left[-\vec{V}_g \cdot \nabla(\zeta_g + \mathrm{f})\right]$$

Therefore

- Cyclonic vorticity advection (positive in NH) increasing with height indicates ascent
- Anticyclonic vorticity advection (negative in NH) increasing with height indicates descent

Typically we assume that the vorticity advection at low levels is weak so that

- Cyclonic vorticity advection (CVA) @ 500 mb indicates ascent
- Anticyclonic vorticity advection (AVA) @ 500 mb indicates descent



In practice, we often use $-\vec{V}_g \cdot \nabla(\zeta + \mathrm{f})$ rather than $-\vec{V}_g \cdot \nabla\left(\zeta_g + \mathrm{f}\right)$ since most maps display the 500-mb height and absolute vorticity (rather than geostrophic absolute vorticity)

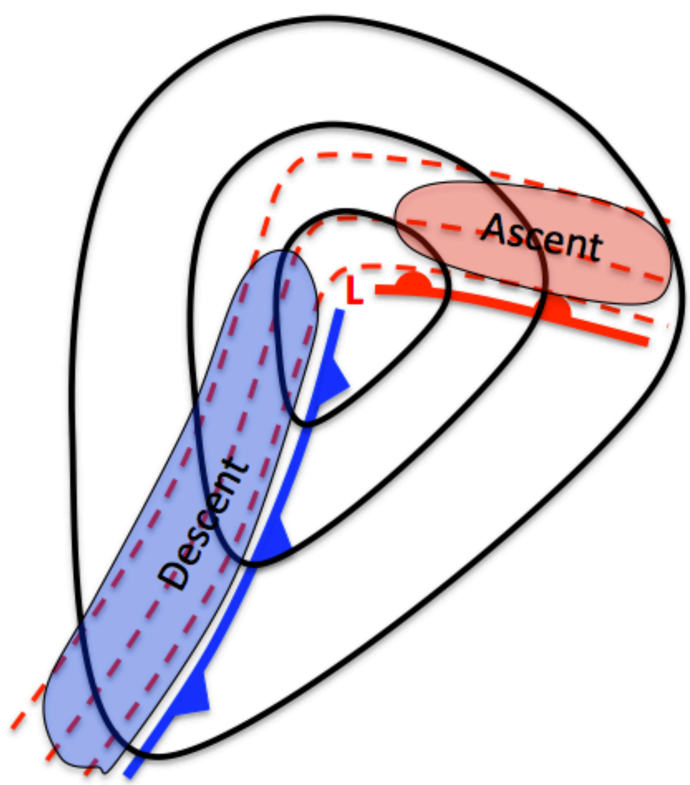
<u>Term C</u>

Temperature advection term

$$w \propto \frac{1}{\sigma}\nabla^2\left[\vec{V}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] \propto \nabla^2[\vec{V}_g \cdot \nabla T] \propto -\nabla^2[-\vec{V}_g \cdot \nabla T]$$

Therefore

- A maximum in warm advection indicates ascent
- A maximum in cold advection indicates descent
- Laplacian operator is such that any maximum in temperature advection indicates ascent and any minimum indicates descent



In practice, we often diagnose $-\vec{V} \cdot \nabla T$ rather than $-\vec{V}_g \cdot \nabla T$ since many maps display the total wind in either vector or barb form.

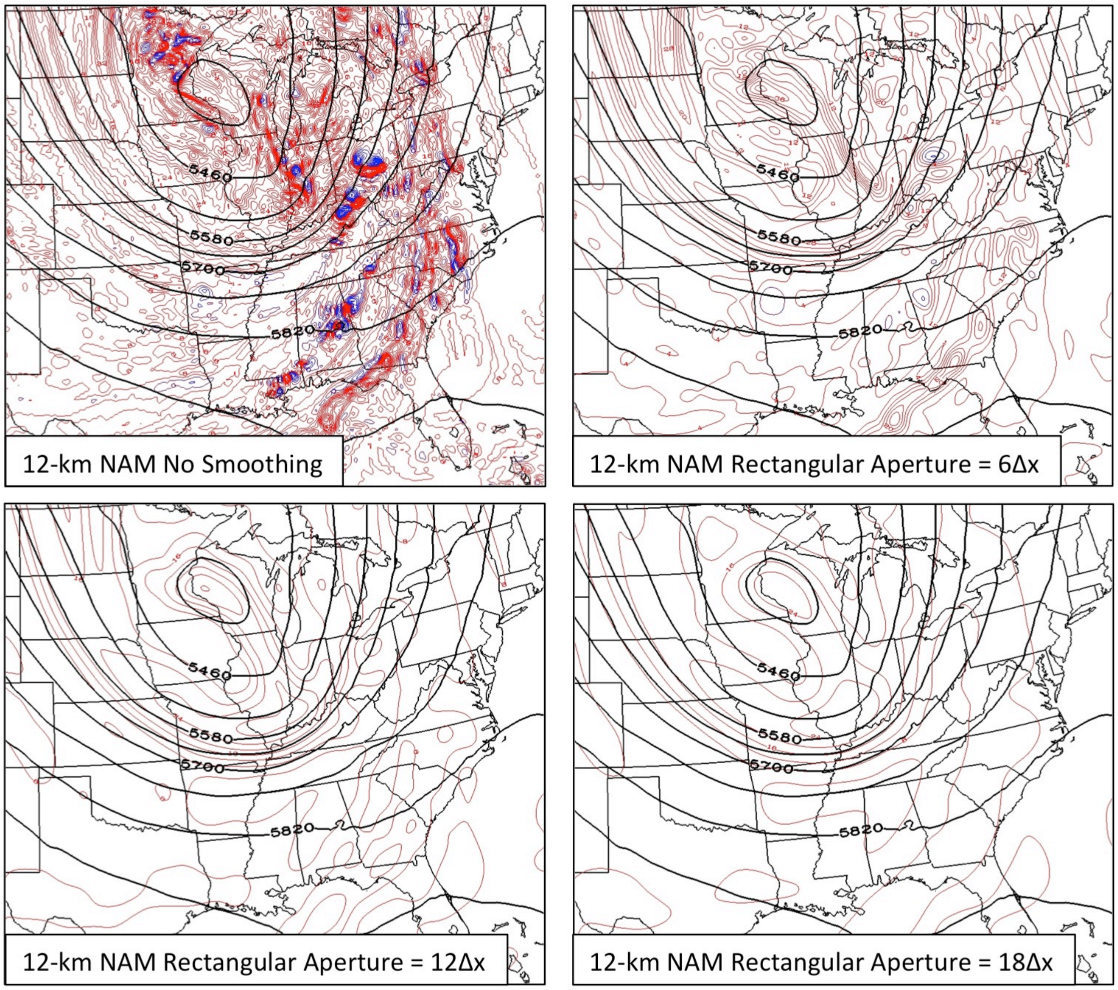
Use of the Omega Equation in practice

- Usually concerned with lower-to-middle tropospheric vertical motion since this is what affects the development of clouds, precipitation, and surface weather systems.
- Typically evaluate 500-mb vorticity advection (assuming 1000-mb advection is small or zero) and either 850-mb or 700-mb temperature advection.
- Key questions to consider (Lackmann 2011)
    - How intense is the forcing for ascent (or descent if interested in subsidence – e.g., formation of wintertime cold pools in west)?
    - Do the right-hand terms reinforce or cancel?
        - Ambiguities develop when they cancel
    - For cloud and precipitation forecasting, is there sufficient moisture available?
    - Are there non-QG mechanisms that might be important (e.g., mountain effects, sub-QG circulation systems such as fronts and gravity waves) and either reinforce or obscure the QG signal?

Application challenges with high-resolution model guidance

- QG information can be swamped by the detailed predictions produced by high-resolution model guidance
- The use of filters that remove small-scale features is often necessary to see the forest through the trees
- Alternatively, the omega equation could be solved using "successive overrelaxation" to obtain omega, which is a natural smoother

The image below compares NAM forecasts of 500-mb height and absolute vorticity, the latter with no smoothing or smoothing with a rectangular aperture filter with a 6Δx, 12Δx, and 18Δx cutoff. Note how filtering the small-scale information enables one to identify the larger-scale forcing.



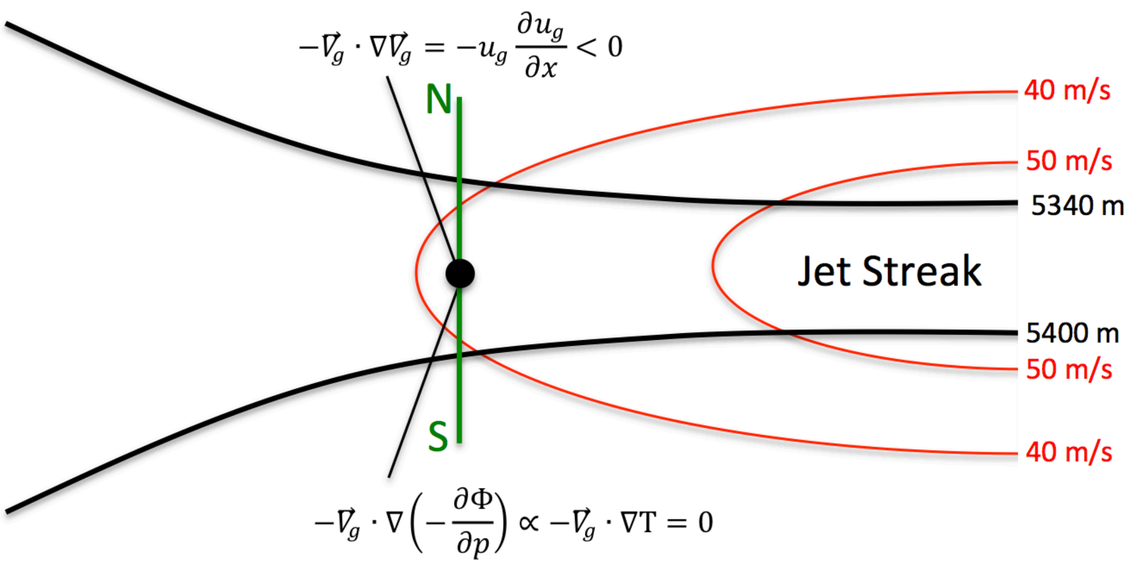
<u>Class Activities</u>:

1. Using the IDV Diagnostics -> QG-Omega bundle, evaluate the right-hand terms and the vertical motion over North America over the past two days. Be sure to examine the differential vorticity advection by examining multiple levels and using the vertical probe.
2. Using IDV, generate a plot of 500-mb height and absolute vorticity using the latest 12-km NAM forecast. Filter the absolute vorticity using the rectangular aperature filter, varying the cutoff from lower to higher values.

Understanding QG vertical motion

Discussion below based on Durran and Snellman (1987) and Lackman (2011).

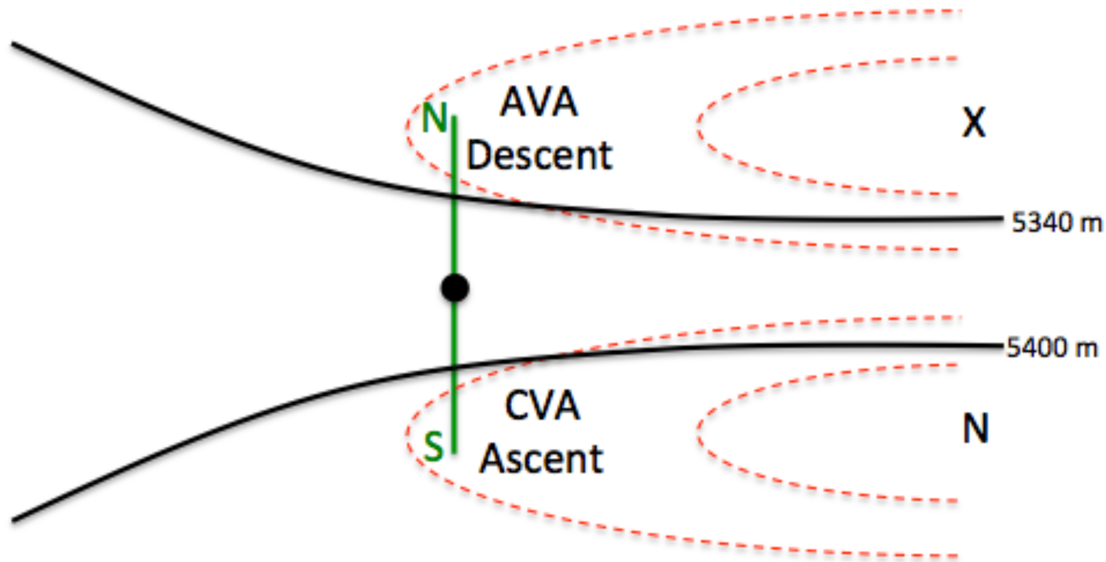
Consider a situation where the 1000-mb height is 0 m with a jet streak located at 500-mb. In this instance, the 500-mb height contours also represent 1000–500 mb thickness contours. What happens to the thermal wind balance in the entrance region of the jet?



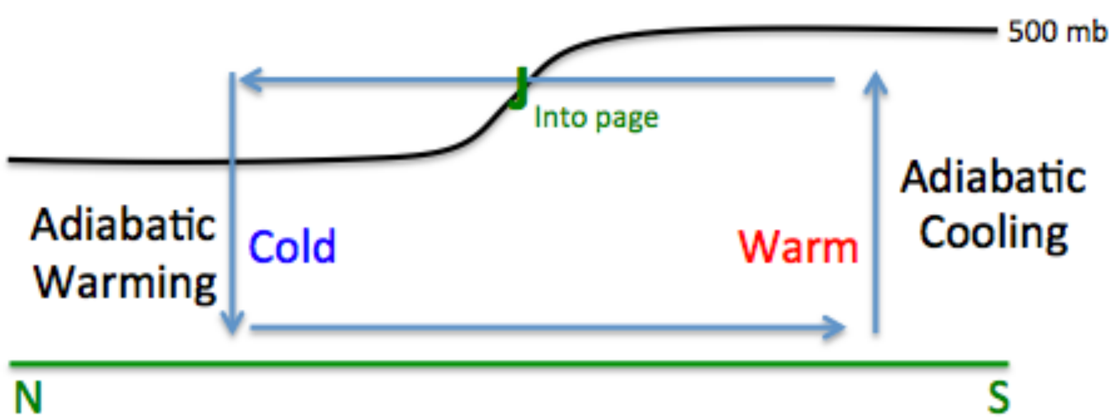
The geostrophic momentum advection in the entrance region weakens the wind shear, but there is no temperature advection to weaken the temperature gradient and maintain thermal wind balance.

Given that the atmosphere is usually very close to thermal wind balance, how is thermal wind balance maintained in such a situation?

Evaluate the vertical motion using the QG-omega equation.



Diagnose the resulting ageostrophic secondary circulation in the plane of cross section N-S.



This circulation helps to restore thermal wind balance by:

- Decreasing the horizontal temperature gradient by adiabatically cooling the warm air and adiabatically warming the cold air

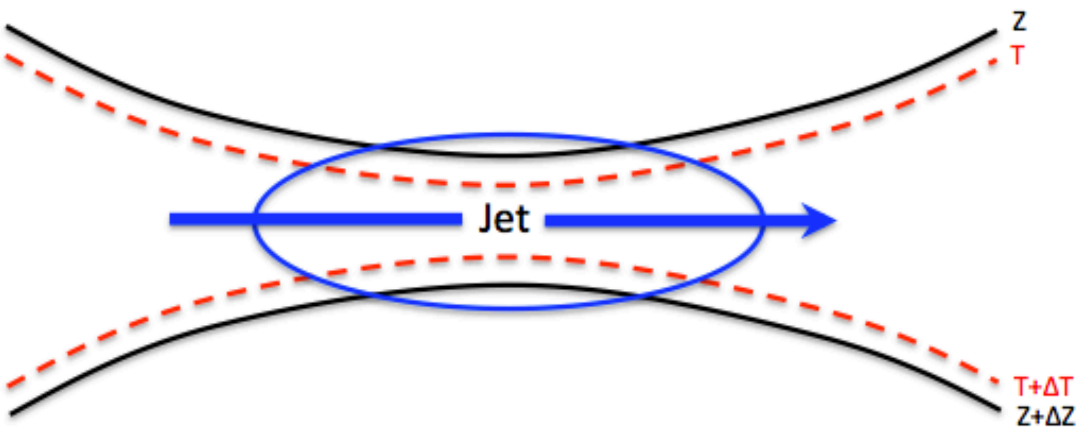
- The coriolis force acting on the horizontal ageostrophic flow also helps by increasing the shear that the geostrophic momentum advection is weakening:
    - The upper-level ageostrophic wind is turned into the page, increasing the eastward upper level flow
    - The lower-level ageostrophic wind is turned out of the page, resulting in westward lower level flow

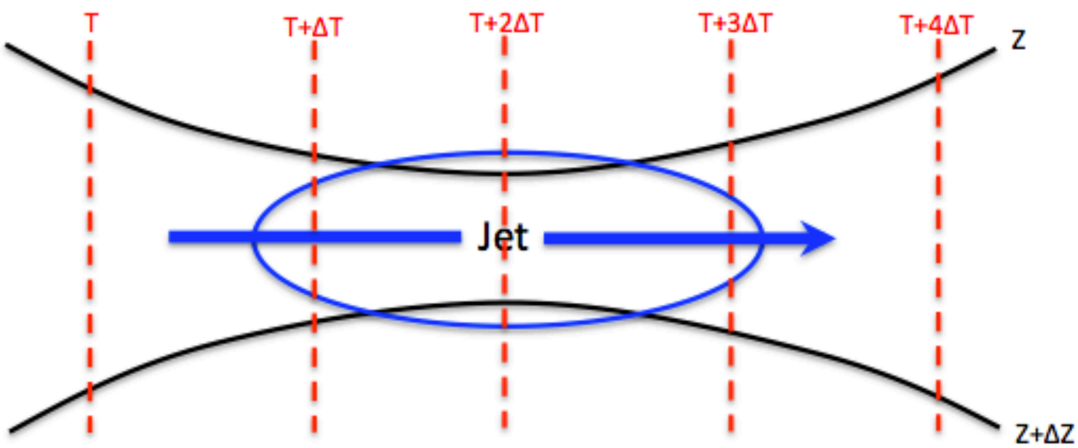
<u>Key Points</u>

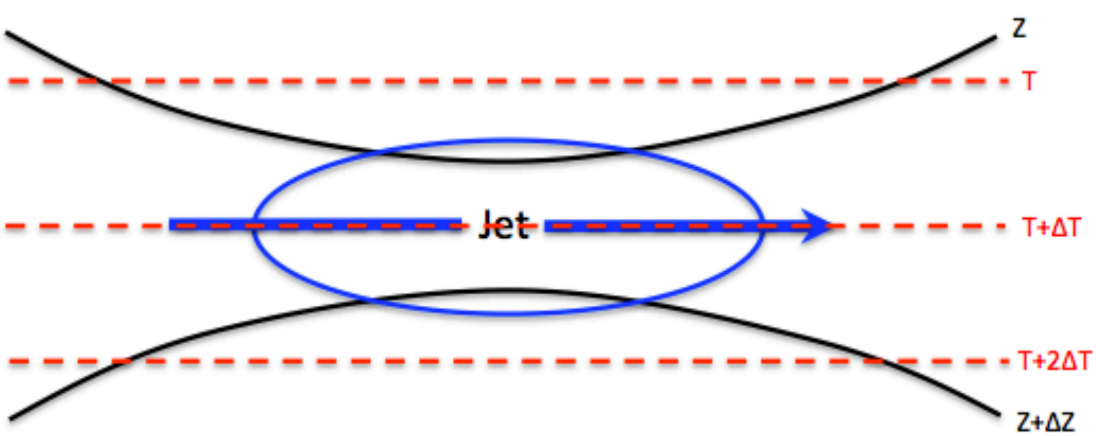
- The large scale vertical motion can be diagnosed from the differential vorticity advection and the horizontal temperature advection
- This is typically done using the 500-mb absolute vorticity advection and the 850 or 700-mb temperature advection
- The resulting vertical motion field represents a component of the ageostrophic secondary circulation that develops to maintain thermal wind balance

Class Activity

Diagnose the ageostrophic secondary circulation in the entrance and exit regions of the jet streaks below and compare with that of the "ideal" jet streak at the top.







Class Question Review

See classquestion.com and refer to the analyses below for 0000 UTC 17 October 2019.