Circulation and Vorticity

Atmos 5110 Synoptic–Dynamic Meteorology I

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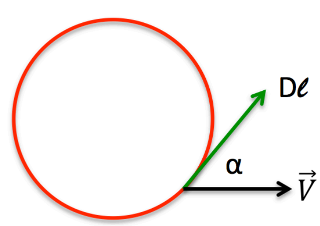
Suggested reading: Lackmann (2011), section 1.5

Holton and Hakim (2013), section 4.1–4.2

Characterizing and understanding the strength and tendency of rotation in atmospheric systems (e.g., cyclones, mesovorticies, tornadoes) is a critical component of meteorological analysis.

Circulation

Mathematical definition: The line integral about a contour of the component of the velocity vector that is locally tangent to the contour. Although this can be done in any plane, we concentrate on the horizontal.

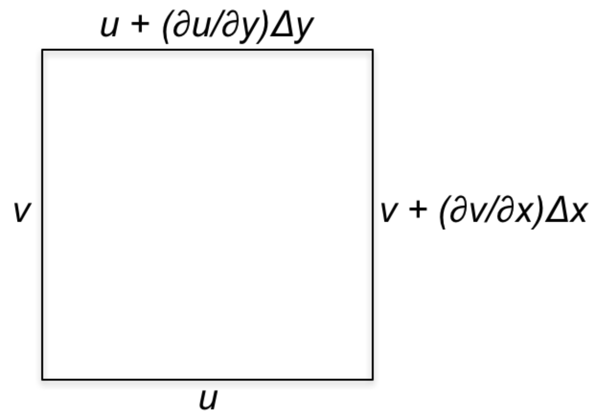


$$C \equiv \oint \vec{V} \cdot D\ell = \oint |\vec{V}| cos\alpha D\ell$$

Synoptic interpretation: Circulation is a measure of the extent to which a fluid exhibits rotary motion. By convention, it is defined to be positive for counterclockwise integration around the contour.

Component form example

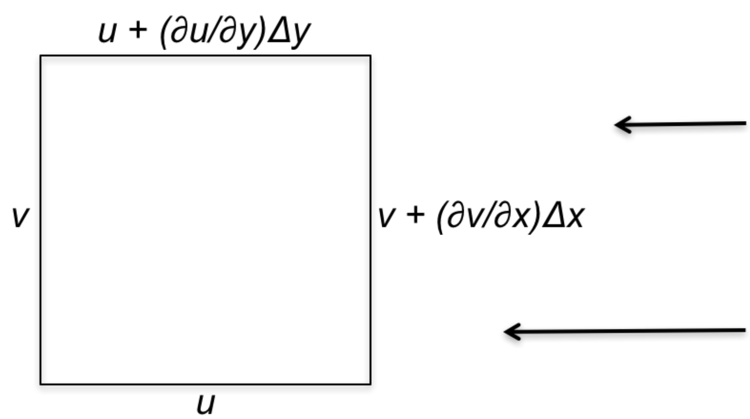
Based on integration around a square



$$C \equiv \oint \vec{V} \cdot D\ell = U\Delta x + \left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta y - \left(u + \frac{\partial u}{\partial y}\Delta y\right)\Delta x - v\Delta y$$

$$= (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\Delta x \Delta y$$

<u>Question:</u> What is the circulation about a 1000x1000 km square if there is an easterly flow that decreases in magnitude toward the north at 10 m s^{-1} $(1000\ km)^{-1}$.



$$C = (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\Delta x \Delta y$$

$$\frac{\partial v}{\partial x} = 0\ (no\ meridional\ wind);\ \frac{\partial u}{\partial y} = \frac{10\ ms^{-1}}{1000\ km}$$

$$C = \left(0 - \frac{10\ ms^{-1}}{1000\ km}\right)(1000\ km)^2 = -1x10^7 m^2 s^{-1}$$

Vorticity

Defined mathematically as the curl of the velocity vector

$$\vec{\omega} = \nabla \times \vec{U} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{\imath} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\hat{\jmath} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\right)\hat{k}$$

In synoptic meteorology, we typically consider only the vertical component of the vorticity

$$\zeta = \hat{k} \cdot \vec{\omega} = \hat{k} \cdot \nabla \times \vec{U} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

ζ is what meteorologists refer to as the relative vorticity. The absolute vorticity also includes the vertical component of the rotation of the Earth and is what is commonly plotted on synoptic charts.

$$\zeta_a = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f$$

What is vorticity?

Vorticity is a measure of the circulation per unit area (in other words, a fancy word for "spin"). Think of it as the circulation "density."

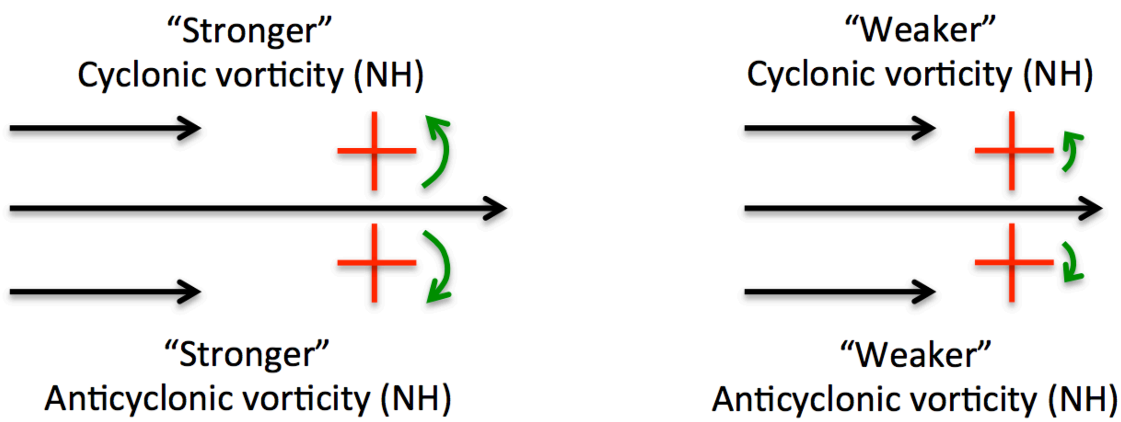
$$C \equiv \oint \vec{V} \cdot D\ell, \text{ and for a square } C = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\Delta x \Delta y$$

$$\Rightarrow C = \zeta \cdot A \qquad or \qquad \zeta = C/A$$

In other words, circulation is a *macroscopic* measure of rotation, whereas vorticity is a *microscopic* measure

Physical interpretation

One way to visualize vorticity is to place an imaginary pinwheel in the flow. The direction and speed of the spin correspond to the sign and magnitude of the vorticity, respectively.

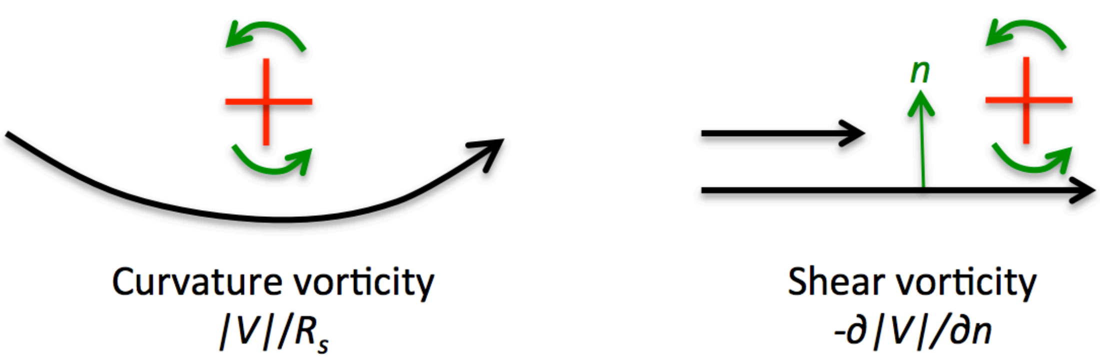


Relative vorticity components

Using natural coordinates, relative vorticity is sometimes broken down into to components:

1. Curvature vorticity: The turning of the flow along a streamline
2. Shear vorticity: The change of wind speed across a streamline

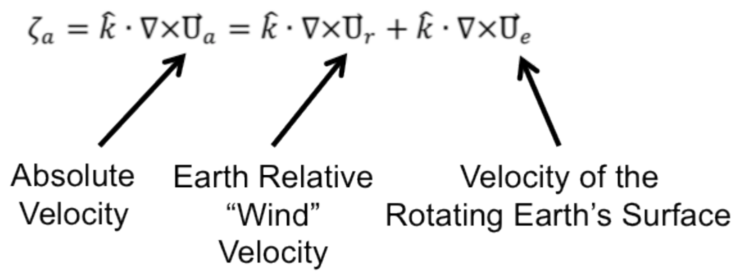
$$\zeta = \frac{|\vec{V}|}{R_s} - \frac{\partial|\vec{V}|}{\partial n} = Curvature\ vorticity + Shear\ vorticity$$



Absolute vorticity

Absolute vorticity also considers the effects of the Earth's rotation. The vorticity associated with the rotation of the Earth is sometimes called the planetary vorticity.

The absolute vorticity is thus based the curl of the absolute velocity, or the sum of the curl of the relative velocity and the curl of the velocity of the rotating Earth:

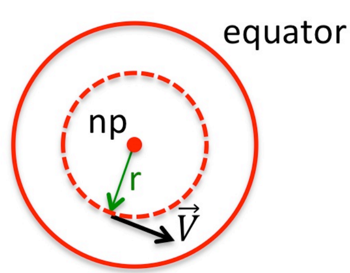


The relative vorticity is given by

$$\zeta = \hat{k}\cdot\nabla\times\vec{U}_r = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

But what are $\nabla\times\vec{U}_e$, the vorticity of the earth and $\hat{k}\cdot\nabla\times\vec{U}_e$, the vertical vorticity of the rotating Earth?

Start by determining the circulation around a latitude belt



$$C \equiv \oint \vec{V}\cdot D\ell$$

$\vec{V} = \Omega r$ based on solid body rotation

$\Rightarrow C = \oint \Omega r \cdot D\ell = \Omega r \oint D\ell$ since Ω and r are constant around a latitude circle

$$\oint D\ell = circumference = 2\pi r$$

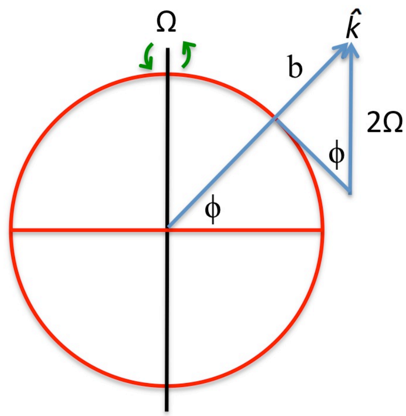
$$\Rightarrow C = \Omega r \cdot 2\pi r = 2\Omega\pi r^2$$

Recall that vorticity = C/A

$$\Rightarrow \nabla \times \vec{U}_e = \frac{C}{A} = \frac{2\Omega\pi r^2}{\pi r^2} = 2\Omega$$

Thus, the vorticity of the rotating earth (or any object experiencing solid body rotation) is 2Ω

However, we need to know the vertical component (i.e., $\hat{k} \cdot \nabla \times \vec{U}_e$)



$$b = \hat{k} \cdot \nabla \times \vec{U}_e = 2\Omega\sin\phi \text{ by geometry}$$
$$= f\text{, the coriolis parameter}$$

$$\Rightarrow \zeta_a = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f$$

Absolute vorticity on weather maps

- Commonly displayed rather than relative vorticity given the importance of planetary vorticity, *f*, in the dynamics of large-scale weather systems
- Typically expressed in units of 10^{-5} s^{-1}.
- Rare to see negative ζ_a on the synoptic scale
- Sometimes X and N are used to denote maxima and minima, respective

Class activity

Using the IDV Supernational bundle, identify vorticity maxima and minima and areas of positive and negative vorticity advection over the CONUS.

Class Question Review

Divergence, Deformation, Vertical Motion, Circulation & Vorticity

Questions 1-2: See classquestion

Question 3: On the image below at what location would you find a maximum in absolute vorticity?

Question 4: On the image below, at what locations would you find cyclonic vorticity advection?