Potential Vorticity

Atmos 5110 Synoptic–Dynamic Meteorology I
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Suggested reading: Holton and Hakim (2013), section 4.5

Another perspective on the development and decay of large-scale weather systems is obtained from the conservation of potential vorticity (PV), which is conserved for adiabatic and frictionless flow:

\[ P = -g(\zeta + f) \frac{\partial \theta}{\partial p}; \quad \frac{DP}{Dt} = 0 \text{ for adiabatic, frictionless flow} \]

Components of Potential Vorticity

1. \((\zeta + f)\) = Absolute vorticity in isentropic coordinates
   - Isentropic coordinates use potential temperature \((\theta)\) for horizontal surfaces
   - Unless superadiabatic, \(\theta\) increases with height
   - \((\zeta + f)\) is the component of the absolute vorticity normal to the potential temperature surfaces

   ![Diagram of Potential Vorticity](image)

   - Usually (but not always), \(|\zeta + f| \approx |\zeta_e + f|\)
2. $-\frac{\partial \theta}{\partial p} = \text{Static Stability}$

- I find it easier to think about static stability in height coordinates, i.e.,

$$-\frac{\partial \theta}{\partial p} \propto \frac{\partial \theta}{\partial z}$$

- Given that $\theta$ increases with height:
  - Large $\frac{\partial \theta}{\partial z} \rightarrow$ Strong static stability
  - Small $\frac{\partial \theta}{\partial z} \rightarrow$ Weak static stability

**Class Activity:** Using the 5110->PV-StaticStability bundle, explore the profiles and cross sections of temperature and potential temperature to better understand static stability and the relationship to potential vorticity.
Physical Interpretation

- If PV is conserved, than an increase in \((\zeta_\theta + f)\) must be accompanied by a decrease in \(-\frac{\partial \theta}{\partial p}\) (or vice versa)

- For instance, if a fluid column is stretched, then the \(\zeta_\theta + f\) must increase

- If a fluid column is compressed, then \(\zeta_\theta + f\) must decrease
Example: Zonal flow with now initial relative vorticity impinging on a mountain barrier

<table>
<thead>
<tr>
<th>Position</th>
<th>$-\frac{\partial \theta}{\partial p}$</th>
<th>$f$</th>
<th>$\zeta$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-(\frac{\partial \theta}{\partial p})_0$</td>
<td>$f_0$</td>
<td>$0$</td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>Decreasing</td>
<td>Small Change</td>
<td>Increasing</td>
<td>Turns north</td>
</tr>
<tr>
<td>2</td>
<td>$-\frac{\partial \theta}{\partial p} &lt; 0$</td>
<td>$f_0$</td>
<td>$&gt;0$</td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td>Increasing</td>
<td>Inc than dec</td>
<td>Decreasing</td>
<td>Turns south</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\partial \theta}{\partial p} &gt; 0$</td>
<td>$f_0$</td>
<td>$&lt;0$</td>
<td></td>
</tr>
<tr>
<td>3-4</td>
<td>Decreasing</td>
<td>Decreasing</td>
<td>Increasing</td>
<td>Turns cyclonically</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{\partial \theta}{\partial p} &lt; f_0$</td>
<td>$&gt;0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-5</td>
<td>Increasing</td>
<td>Increasing</td>
<td>Decreasing</td>
<td>Overshoots $f_0$</td>
</tr>
<tr>
<td>5</td>
<td>$-\frac{\partial \theta}{\partial p}$</td>
<td>Oscillates</td>
<td>Oscillates</td>
<td>Wave train</td>
</tr>
</tbody>
</table>

Result: Windward ridge and lee trough
Example: Suppose we have equatorward moving air with no initial relative vorticity, but we allow the relative vorticity to increase or decrease.

Since potential vorticity is conserved

\[ P_i = P_f \]

\[ - \left[ g(\zeta + f) \frac{\partial \theta}{\partial p} \right]_i = - \left[ g(\zeta + f) \frac{\partial \theta}{\partial p} \right]_f \]

\( g = \text{constant and } \partial \theta \text{ doesn’t change} \Rightarrow \)

\[ \frac{\zeta_i + f_i}{\Delta p_i} = \frac{\zeta_f + f_f}{\Delta p_f} \]

Setting \( \zeta_i = 0 \) (no initial relative vorticity) and rearranging yields

\[ \frac{\Delta p_f}{\Delta p_i} = \frac{\zeta_f + f_f}{f_i} \]

Since the parcel is moving equatorward, \( f_f < f_i \), so we have several possibilities:

1. \( \zeta_f = 0 \Rightarrow \Delta p_f < \Delta p_i \Rightarrow \text{Column compresses} \)
2. \( \zeta_f < 0 \Rightarrow \Delta p_f << \Delta p_i \Rightarrow \text{Column compresses a lot} \)
3. \( \zeta_f > 0 \Rightarrow \Delta p_f \sim \Delta p_i \Rightarrow \text{No major change} \)
4. \( \zeta_f >> 0 \Rightarrow \Delta p_f > \Delta p_i \Rightarrow \text{Column stretches} \)

Vertical motion assumes bottom of column remains at the ground.
**Synoptic Application**

In the absence of other forcings:

- Northerly flow tends to be associated with subsidence
- Northerly flow with anticyclonic curvature tends to be associated with strong subsidence
- Northerly flow with cyclonic curvature tends to be associated with little vertical motion or weak ascent

**Class Question Review**

See classquestion.com