Dynamics of Upper-Level (Rossby) Waves

Atmos 5110 Synoptic–Dynamic Meteorology I

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Suggested reading: Lackman (2011) section 1.5.3

Holton and Hakim (2013) section 5.7

The barotropic vorticity equation can be used to illustrate the important role that planetary rotation and the advection of planetary vorticity play in the behavior of upper-level waves.

Barotropic Vorticity Equation

Assuming the atmosphere is barotropic (i.e., density depends only on pressure) and the motion is purely horizontal (i.e., w=0, divergence=0), then the vorticity equation simplifies to what is known as the barotropic vorticity equation.

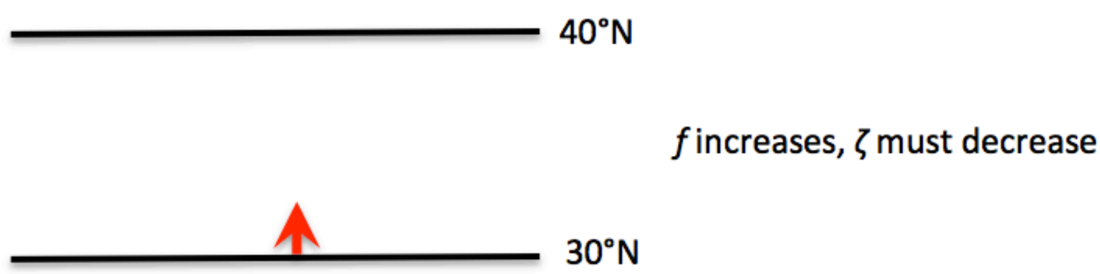
$$\frac{D(\zeta + f)}{Dt} = 0 \qquad or \qquad \zeta + f = constant$$

Physical interpretation: Absolute vorticity is conserved following parcel motion, which is purely horizontal..

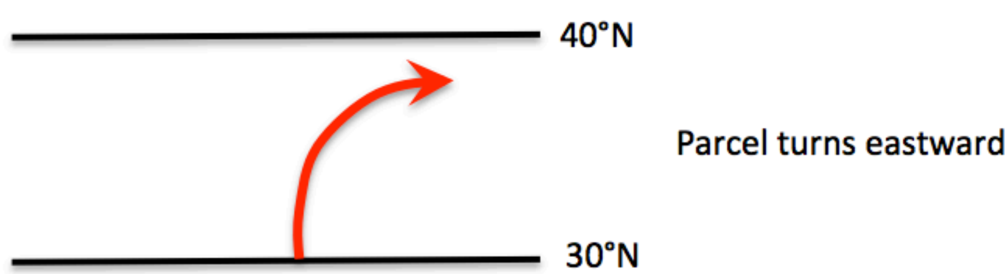
Advantages: This is the simplest model of large-scale fluid motion and it allows us to diagnose the role of planetary vorticity and planetary vorticity on large-scale weather systems.

Example

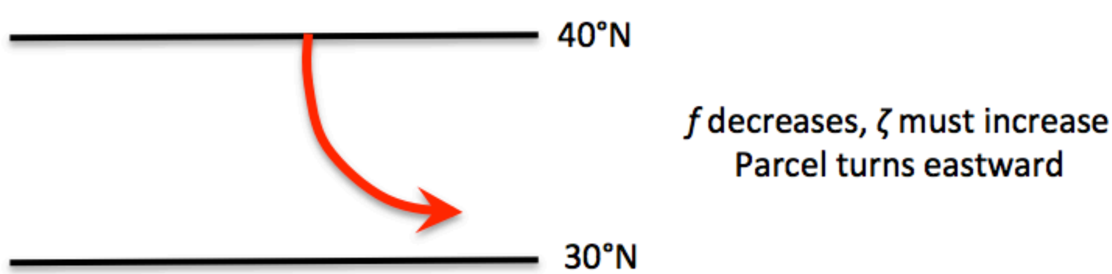
Imagine that we impulsively start southerly flow in the northern hemisphere with no initial relative vorticity



To conserve absolute vorticity, the parcel must acquire anticyclonic relative vorticity (i.e., $\zeta\downarrow$) since $f\uparrow$ as the parcel moves northward.



In contrast, northerly flow must acquire cyclonic relative vorticity (i.e., $\zeta\uparrow$) since $f\downarrow$ as the parcel moves southward.



Synoptic experience: Strong northerly outflow from the ITCZ or other area of tropical convection typically turns anticyclonically and can form a subtropical jet

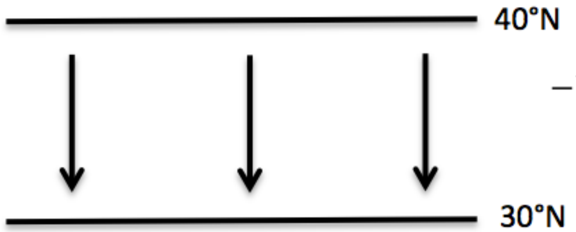
<u>Another Interpretation</u>

At a point, the figure vorticity field is determined by the advection of relative and planetary vorticity.

$$\frac{D(\zeta + f)}{Dt} = \frac{\partial(\zeta + f)}{\partial t} + \vec{V} \cdot \nabla(\zeta + f) = 0$$

$$\Rightarrow \quad \frac{\partial(\zeta + f)}{\partial t} = -\vec{V} \cdot \nabla\zeta - \vec{V} \cdot \nabla f$$

Example: For strong northerly flow with no initial relative vorticity, the relative vorticity at a point moist increase due to the advection of planetary vorticity.



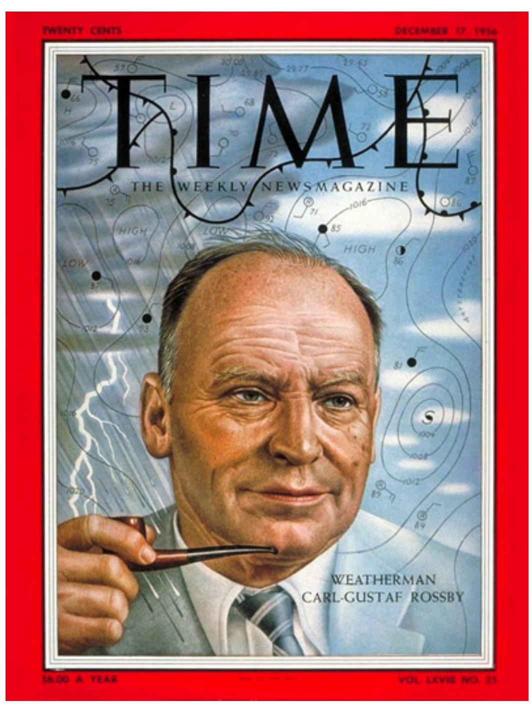
$$-\vec{V}\cdot\nabla f = -v\frac{\partial f}{\partial y} > 0 \Rightarrow \frac{\partial(\zeta + f)}{\partial t} > 0$$

$$since\ \ -\vec{V}\cdot\nabla\zeta = 0$$

Thus, the "carving out" of an upper-level trough by strong northerly flow can be viewed as the result of planetary vorticity advection

<u>Rossby Waves</u>

The barotropic vorticity equation can be used to improve our understanding of upper-level waves. The term Rossby Wave is used to describe a planetary-scale wave whose motion characteristics are influenced by the β effect (i.e., $\partial f/\partial y$). Named for Carl Gustav Rossby, the only meteorologist to appear on the cover of Time (Dec 17, 1956)



As shown in Lackmann (2001) section 1.5.3, the barotropic vorticity equation can be used to derive the *Rossby wave phase speed equation*:

$$c = U - \frac{\beta L^2}{4\pi^2}$$

where

U = mean zonal flow

$\beta = \partial f / \partial y$ (Beta effect)

and L = zonal wavelength (i.e., L_x)

Significance:

1. Rossby waves always propagate *upstream* (typically westward in the midlatitudes) relative to the mean flow

$$\frac{\beta L^2}{4\pi^2} > 0 \Rightarrow C < U$$

   This means that the wind moves through a Rossby wave. Relative to an air parcel, Rossby waves move upstream.

   Synoptic application: Upper-level waves moves slower than the flow.

2. The phase speed of a long wave is slower than a short wave since

$$as\ L\ increases, \frac{\beta L^2}{4\pi^2}\ increases, and\ c = U - \frac{\beta L^2}{4\pi^2}\ decreases$$

   Synoptic application: Short waves progress downstream more rapidly than long waves. Short waves appear to be steered by the long waves.

Class activity

Using the IDV Global-10day and Supernational bundles, examine the evolution of upper-level troughs and ridges and find evidence of upstream wave propagation and short-waves progressing downstream more rapidly than long waves.

Physical Interpretation of the Rossby Wave

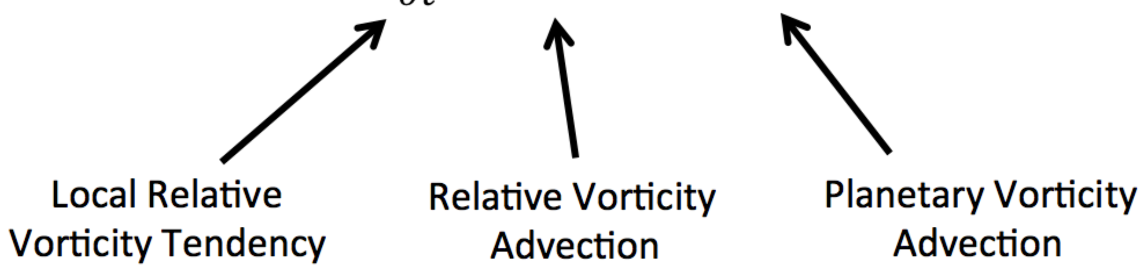
Rossby wave movement is the result of a battle between (1) relative vorticity advection and (2) planetary vorticity advection. Recall that

$$\frac{D(\zeta + f)}{Dt} = \frac{\partial(\zeta + f)}{\partial t} + \vec{V} \cdot \nabla(\zeta + f) = 0$$

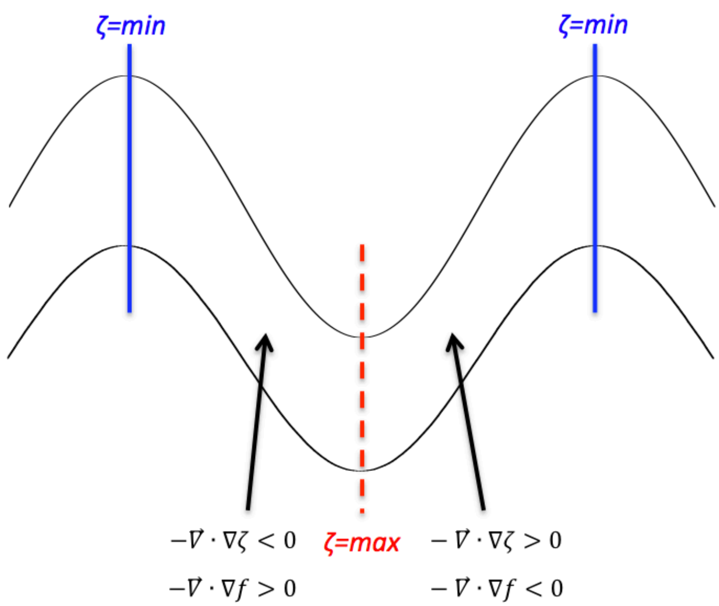
$$\Rightarrow \quad \frac{\partial(\zeta + f)}{\partial t} = -\vec{V} \cdot \nabla\zeta - \vec{V} \cdot \nabla f$$

However, $\frac{\partial f}{\partial t} = 0$, therefore

$$\frac{\partial \zeta}{\partial t} = -\vec{V} \cdot \nabla\zeta - \vec{V} \cdot \nabla f$$



If *f* was constant, the wave speed would simply be controlled by the advection of relative vorticity and the waves would move with the mean flow. However, in a Rossby wave, the planetary vorticity advection acts to cause the wave to move upstream, and the wave moves slower than the mean flow.



Thus, Rossby waves propagate upstream because of the influence of the planetary vorticity advection.

Long waves move more slowly than short waves because the planetary vorticity advection becomes more dominant as the wavelength increases

$$\frac{|-\vec{V}\cdot\nabla f|}{|-\vec{V}\cdot\nabla\zeta|} \sim \frac{U(f/L)}{U(U/L^2)} \sim \frac{fL}{U} \;\;\Rightarrow\;\; as\ L\ increases, \frac{|-\vec{V}\cdot\nabla f|}{|-\vec{V}\cdot\nabla\zeta|}\ increases$$

Stationary waves and pattern progression

What controls the wave number of a stationary wave and whether or not a pattern is slowly evolving or very progressive?

Begin with the Rossby wave phase speed equation:

$$c = U - \frac{\beta L^2}{4\pi^2}$$

Substituting

$$L = \frac{2\pi a cos\phi}{n}$$

Yields

$$c = U - \frac{\beta a^2 cos^2\phi}{n^2}$$

From this equation, the wave number of a stationary wave, n_s, can be found by setting c to 0 and solving for n_s

$$n_s = \left(\frac{\beta a^2 cos^2\phi}{U}\right)^{1/2}$$

Substituting

$$\beta = \frac{\partial f}{\partial y} = \frac{2\Omega cos\phi}{a}$$

Yields

$$n_s = \left(\frac{2\Omega a cos^3\phi}{U}\right)^{1/2}$$

Possibilities for a given wave

- $n=n_s \rightarrow$ wave is stationary ($c=0$)
- $n>n_s \rightarrow$ wave progresses ($c>0$)
- $n<n_s \rightarrow$ wave retrogrades ($c<0$)

Synoptic application

1. $n_s \propto (1/U)^{1/2} \Rightarrow$ as U increases, n_s gets smaller. Thus, as the mean zonal flow increases, there are fewer stationary or retrogressive waves, and the overall pattern becomes more progressive.

2. $n_s \propto \cos^{3/2}\phi \Rightarrow$ as latitude (ϕ) increases, $\cos^{3/2}\phi$ decreases and n_s gets smaller. Thus, a wave of a given wave number is more likely to be progressive at high latitudes than low latitudes (Note, however, that some of the most important blocks occur in the mid-to-high latitudes).

Class Question Review

See classquestion.com