Frontogenesis

What is Frontogenesis?

1. The formation of a front or frontal zone.

2. Mathematically, it is defined as the rate of change of the magnitude of the horizontal potential temperature gradient following parcel motion.

$$F \equiv \frac{D}{Dt}|\nabla_p \Theta|$$

↑ preferred over temperature since it is conserved

- To derive an expression for F, begin with

$$\frac{D}{Dt}\nabla_p\Theta = \frac{\partial}{\partial t}\nabla_p\Theta + u\frac{\partial}{\partial x}\nabla_p\Theta + v\frac{\partial}{\partial y}\nabla_p\Theta + w\frac{\partial}{\partial z}\nabla_p\Theta$$

$$= \nabla_p\frac{\partial\theta}{\partial t} + \nabla_p\left(u\frac{\partial\theta}{\partial x}\right) + \nabla_p\left(v\frac{\partial\theta}{\partial y}\right) + \nabla_p\left(w\frac{\partial\theta}{\partial z}\right) - \frac{\partial\theta}{\partial x}\nabla_p u - \frac{\partial\theta}{\partial y}\nabla_p v - \frac{\partial\theta}{\partial z}\nabla_p w$$

[NOTE: Based on $\nabla_p\left(v\frac{\partial\theta}{\partial y}\right) = \frac{\partial\theta}{\partial y}\nabla_p v + v\nabla_p\frac{\partial\theta}{\partial y}$ [Similarly for other variables]]

- Since $\nabla_p\frac{\partial\theta}{\partial t} + \nabla_p\left(u\frac{\partial\theta}{\partial x}\right) + \nabla_p\left(v\frac{\partial\theta}{\partial y}\right) + \nabla_p\left(w\frac{\partial\theta}{\partial z}\right) = \nabla_p\frac{D\theta}{Dt}$, then

$$\frac{D}{Dt}\nabla_p\Theta = \nabla_p\frac{D\Theta}{Dt} - \left(\frac{\partial\theta}{\partial x}\nabla_p u + \frac{\partial\theta}{\partial y}\nabla_p v + \frac{\partial\theta}{\partial z}\nabla_p w\right)$$

- Expanding yields

$$\frac{D}{Dt}\nabla_p\Theta = \frac{\partial}{\partial x}\left(\frac{D\theta}{Dt} - \frac{\partial\theta}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial x} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial x}\right)\hat{i}$$
$$+ \frac{\partial}{\partial y}\left(\frac{D\theta}{Dt} - \frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\right)\hat{j}$$

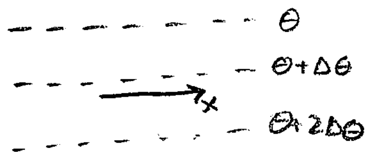
- To OBTAIN $\frac{D}{Dt}|\nabla_p\theta|$, take dot product of $\frac{D}{Dt}\nabla_p\theta$ with

$$\vec{n}_\theta = \frac{\nabla_p\theta}{|\nabla_p\theta|} = \frac{1}{|\nabla_p\theta|}\left(\frac{\partial\theta}{\partial x}\hat{i} + \frac{\partial\theta}{\partial y}\hat{j}\right),$$ where $\vec{n}_\theta$ is the unit vector in the direction of $\nabla_p\theta$

- This yields, with vector calc & Algebra:

$$\boxed{\begin{aligned} F = \frac{D}{Dt}|\nabla_p\theta| = \frac{1}{|\nabla_p\theta|}\Bigg[&\frac{\partial\theta}{\partial x}\left(\frac{\partial}{\partial x}\frac{D\theta}{Dt} - \frac{\partial\theta}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial x} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial x}\right) \\ &+ \frac{\partial\theta}{\partial y}\left(\frac{\partial}{\partial y}\frac{D\theta}{Dt} - \frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\right)\Bigg] \end{aligned}}$$

- To simplify, <u>ASSUME</u> X-AXIS is oriented along isotherms



- Then, $\partial\theta/\partial x = 0$, & we have

$$\frac{D}{Dt}|\nabla_p\theta| = \frac{\partial\theta/\partial y}{|\partial\theta/\partial y|}\left[\frac{\partial}{\partial y}\left(\frac{D\theta}{Dt}\right) - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\right]$$

Differential heating/cooling

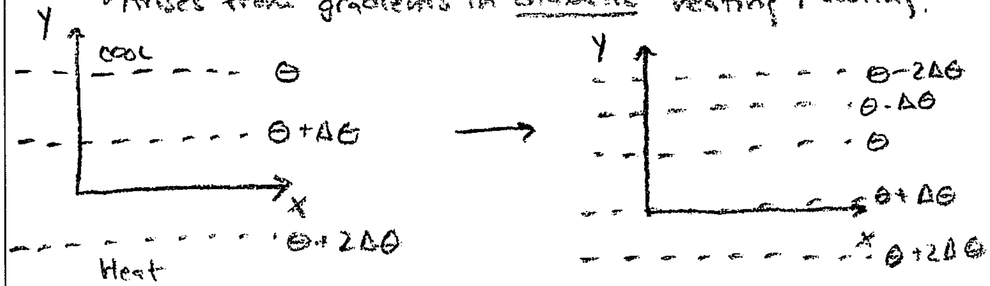
Confluence/Diffluence effects

Tilting effects

- These 3 processes are responsible for frontogenesis/frontolysis.

Differential Heating

• Arises from gradients in diabatic heating & cooling.

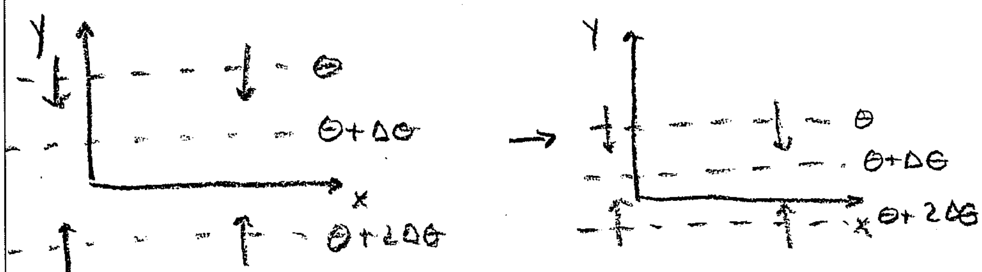


Frontogenesis

$$F = \frac{\partial\theta/\partial y}{|\partial\theta/\partial y|} \cdot \underbrace{\frac{\partial}{\partial y}\left(\frac{D\theta}{Dt}\right)}_{<0}$$

$\uparrow$ <0 $\quad \Rightarrow F>0 \rightarrow$ Frontogenesis.

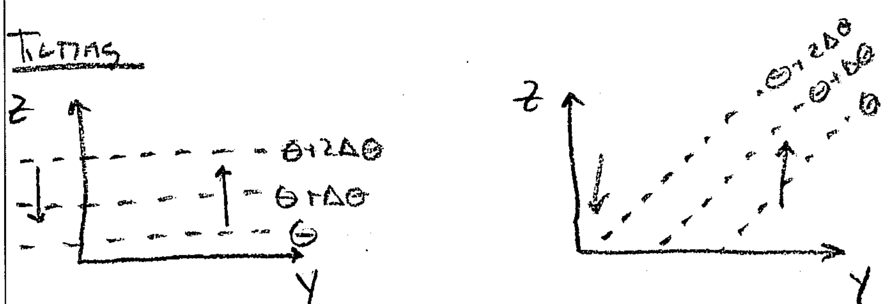
Confluence



$$F = \frac{\partial\theta/\partial y}{|\partial\theta/\partial y|} \cdot \underbrace{\left(-\frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y}\right)}_{<0}$$

$\uparrow$ <0 $\quad \Rightarrow F>0$ Frontogenesis!

Tilting



$$F = \frac{\partial\theta/\partial y}{|\partial\theta/\partial y|}\left(-\frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\right)$$

$\uparrow$ <0 $\quad$ $\uparrow$ <0 $\quad \Rightarrow F>0 \rightarrow$ Frontogenesis!

Frontogenesis in Terms of Basic Kinematic Properties

- Recall that

$$F = \frac{D}{Dt}|\nabla_p \theta| = \frac{1}{|\nabla_p\theta|}\left[\frac{\partial\theta}{\partial x}\left(\frac{\partial}{\partial x}\frac{D\theta}{Dt} - \frac{\partial\theta}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial x} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial x}\right) + \frac{\partial\theta}{\partial y}\left(\frac{\partial}{\partial y}\frac{D\theta}{Dt} - \frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y} - \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\right)\right]$$

- Assume $D\theta/Dt = 0$ & $w = 0$, then.

$$F = \frac{1}{|\nabla_p\theta|}\left[\frac{\partial\theta}{\partial x}\left(-\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial x}\right) + \frac{\partial\theta}{\partial y}\left(-\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y}\right)\right]$$

- Note that $\partial u/\partial x$, $\partial v/\partial x$, $\partial u/\partial y$, & $\partial v/\partial y$ can all be formulated in terms of basic kinematic properties. For example:

$$\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial u}{\partial x} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) = \frac{1}{2}(\delta + D_1)$$

- Similarly

$$\frac{\partial v}{\partial x} = \frac{1}{2}(\zeta + D_2) \qquad \frac{\partial u}{\partial y} = \frac{1}{2}(\zeta - D_2) \qquad \frac{\partial v}{\partial y} = \frac{1}{2}(\delta - D_1)$$

- Thus,

$$F = \frac{1}{2|\nabla_p\theta|}\left[\frac{\partial\theta}{\partial x}\left(-\frac{\partial\theta}{\partial x}(\delta + D_1) - \frac{\partial\theta}{\partial y}(\zeta + D_2)\right) + \frac{\partial\theta}{\partial y}\left(\frac{\partial\theta}{\partial x}(\zeta - D_2) - \frac{\partial\theta}{\partial y}(\delta - D_1)\right)\right]$$

$$= \frac{1}{2|\nabla_p\theta|}\left[-\left(\frac{\partial\theta}{\partial x}\right)^2(\delta + D_1) - \frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}(\zeta + D_2) + \frac{\partial\theta}{\partial y}\frac{\partial\theta}{\partial x}(\zeta - D_2) - \left(\frac{\partial\theta}{\partial y}\right)^2(\delta - D_1)\right]$$

Terms cancel

$\Rightarrow$ ζ does not directly affect frontogenesis!

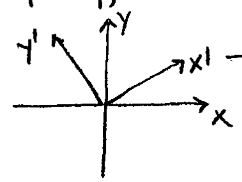
- Collecting terms

$$F = \frac{1}{2|\nabla_p\theta|}\left[-\delta\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right) - D_1\left(\left(\frac{\partial\theta}{\partial x}\right)^2 - \left(\frac{\partial\theta}{\partial y}\right)^2\right) - 2D_2\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}\right]$$

- Factoring out a $\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 = |\nabla_p\theta|^2$ yields

$$F = \frac{|\nabla_p\theta|}{2}\left[-\delta - D_1\frac{\left(\frac{\partial\theta}{\partial x}\right)^2 - \left(\frac{\partial\theta}{\partial y}\right)^2}{|\nabla_p\theta|^2} - 2D_2\frac{\partial\theta/\partial x\,\partial\theta/\partial y}{|\nabla_p\theta|^2}\right]$$

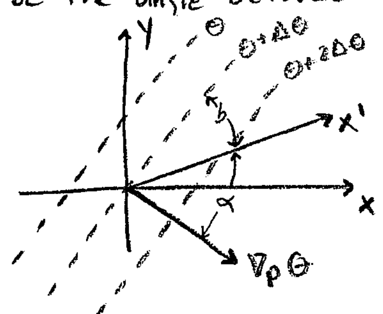
- To further simplify, rotate the coordinate system so THAT $D_2 \to 0$



$\to$ X' lies Along the Axis of dilitation

$$F = \frac{|\nabla_p \theta|}{2}\left[-\delta - D_1' \frac{\left(\frac{\partial\theta}{\partial x}\right)^2 - \left(\frac{\partial\theta}{\partial y}\right)^2}{|\nabla_p \theta|^2}\right]$$

- Now, let α be the angle between $\nabla_p \theta$ & the Axis of dilitation (x')



& b be the angle between x' & the isentropes so that $\alpha + b = -90°$ since the gradient $\nabla_p \theta$ is at right angle to the isentropes

- With alot of algebra & geometry, the expression Above simplifies to

$$\boxed{F = \frac{|\nabla_p \theta|}{2}\left[D\cos 2b - \delta\right]}$$

where

D = Resultant deformation and $= \left[D_1^2 + D_2^2\right]^{1/2}$

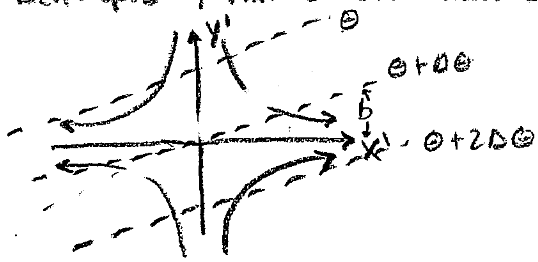
b = the Angle between the isentropes & the Axis of dilitation

δ = Divergence

Synoptic Application

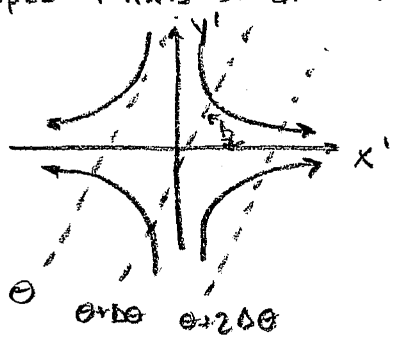
$$F = \frac{|\nabla_p \theta|}{2}\left[D\cos 2b - \delta\right]$$

1. Divergence ($\delta > 0$) creates frontolysis ($F < 0$) regardless of the orientation of the isentropes
2. Convergence ($\delta < 0$) produces Frontogenesis ($F > 0$) regardless of the orientation of the isentropes
3. Deformation produces frontogenesis ($F > 0$) if the angle between the isentropes & Axis of dilitation is $< 45°$ ($|b| < 45°$)



Frontogenesis

4. Deformation produces frontolysis ($F < 0$) if the angle between the isentropes & Axis of dilitation is $> 45°$ ($|b| > 45°$)



Frontolysis.

5. Vorticity does not directly contribute to frontogenesis / frontolysis. It can only indirectly contribute by rotating the isentropes & altering their Angle relative to the Axis of dilitation
6. For purely geostrophic motion (i.e., $\delta = 0$), only deformation produces frontogenesis & frontolysis.

Dynamical Consequences of Frontogenesis/Frontolysis

Fundamental properties of Quasigeostrophic (large-scale) flows

1. The atmosphere wants to be in thermal wind balance

$$\frac{\partial u_g}{\partial p} = -\frac{R}{f}\frac{\partial T}{\partial y}\ln\left(\frac{P_L}{P_U}\right) \qquad \frac{\partial v_g}{\partial p} = \frac{R}{f}\frac{\partial T}{\partial x}\ln\left(\frac{P_L}{P_U}\right)$$

i.e. the vertical wind shear is proportional to the horizontal temperature gradient

2. Geostrophic deformation acting to increase the temperature gradient forces the atmosphere away from thermal wind balance $\Rightarrow$ the temperature gradient becomes too large for the wind shear.

3. Ageostrophic motions (u_{ag}, v_{ag}, w) arise to help maintain thermal wind balance

Mathematical Proof

- Begin with the Q.G. Momentum eq.

$$\frac{D_g \vec{V}_g}{Dt} = -f_0(\hat{k}\times\vec{V}_a) - \beta y(\hat{k}\times\vec{V}_g)$$

- Assume no ageostrophic circulation to maintain thermal wind balance, that $\beta = 0$, & consider only zonal wind component. Then,

$$\frac{D_g u_g}{Dt} = 0$$

- Differentiate with respect to p

$$\frac{\partial}{\partial p}\left(\frac{D_g u_g}{Dt}\right) = \frac{\partial}{\partial p}\left(\frac{\partial u_g}{\partial t} + u_g\frac{\partial u_g}{\partial x} + v_g\frac{\partial u_g}{\partial y}\right)$$

Note: No vertical adv in Q.G. System, plus $w = 0$ by above assumptions

$$= \underset{①}{\frac{\partial}{\partial t}\left(\frac{\partial u_g}{\partial p}\right)} + \frac{\partial u_g}{\partial p}\frac{\partial u_g}{\partial x} + \underset{②}{u_g\frac{\partial}{\partial x}\left(\frac{\partial u_g}{\partial p}\right)} + \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y} + \underset{③}{v_g\frac{\partial}{\partial y}\left(\frac{\partial u_g}{\partial p}\right)}$$

$$1+2+3 = \frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial p}\right)$$

$\Rightarrow$

$$\frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial p}\right) = -\frac{\partial u_g}{\partial p}\frac{\partial u_g}{\partial x} - \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y}$$

By continuity, $\partial u_g/\partial x = -\partial v_g/\partial y$

$\Rightarrow$

$$\boxed{\frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial p}\right) = \frac{\partial u_g}{\partial p}\frac{\partial v_g}{\partial y} - \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y}}$$

- Now, take $\partial/\partial y$ of the Q.G. Thermodynamic energy eq. → remember that $\omega = 0$, $\vec{V}_{a_g} = 0$. Then,

$$\frac{D_g T}{Dt} = 0$$

- Take $\partial/\partial y$

$$\frac{\partial}{\partial y}\left[\frac{D_g T}{Dt}\right] = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial t} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y}\right)$$

$$= \underset{1}{\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial y}\right)} + \frac{\partial u_g}{\partial y}\frac{\partial T}{\partial x} + \underset{2}{u_g \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial y}\right)} + \frac{\partial v_g}{\partial y}\frac{\partial T}{\partial y} + \underset{3}{v_g \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)}$$

$$1 + 2 + 3 = \frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) \Rightarrow$$

$$\frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) = -\frac{\partial u_g}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial v_g}{\partial y}\frac{\partial T}{\partial y}$$

- Rearranging

$$\frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) = -\frac{\partial T}{\partial y}\frac{\partial v_g}{\partial y} - \frac{\partial T}{\partial x}\frac{\partial u_g}{\partial y}$$

- By thermal wind

$$\frac{\partial T}{\partial x} = -\frac{f_0 p}{R}\left(\frac{\partial v_g}{\partial p}\right) \qquad \frac{\partial T}{\partial y} = \frac{f_0 p}{R}\left(\frac{\partial u_g}{\partial p}\right)$$

- Substituting yields

$$\boxed{\frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) = \frac{-f_0 p}{R}\left[\frac{\partial u_g}{\partial p}\frac{\partial v_g}{\partial y} - \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y}\right]}$$

- This gives us two equations

$$\frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial p}\right) = \frac{\partial u_g}{\partial p}\frac{\partial v_g}{\partial y} - \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y}$$

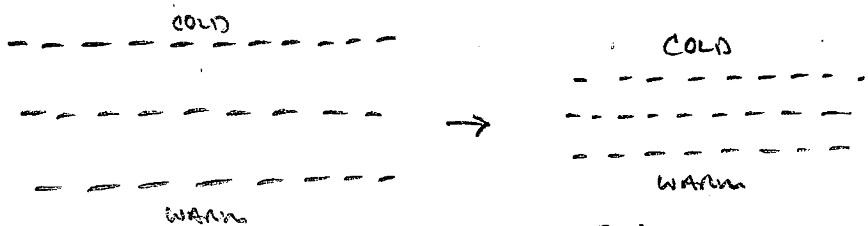
$$\frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) = -\frac{S_o p}{R}\left[\frac{\partial u_g}{\partial p}\frac{\partial v_g}{\partial y} - \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y}\right]$$

- Multiplying top eq by $S_o p/R$ & Adding gives

$$\frac{S_o p}{R}\frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial p}\right) + \frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) = 0$$

$$\Rightarrow \quad \frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) \propto -\frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial p}\right) \propto \frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial z}\right)$$

- What does this mean?

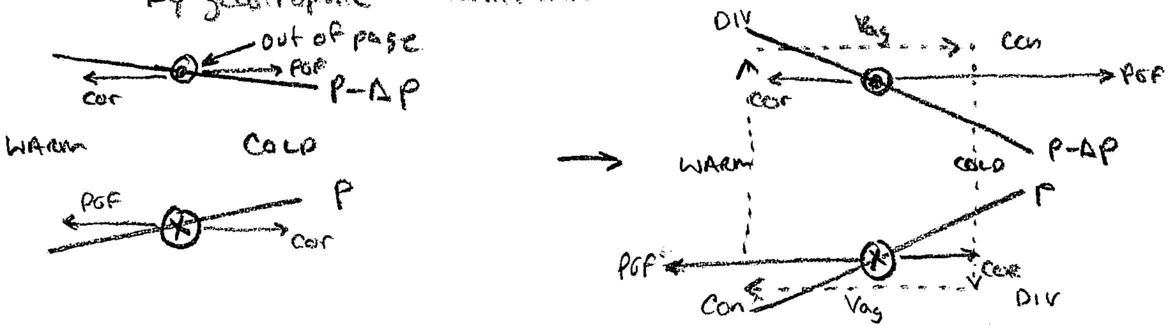


$\frac{\partial T}{\partial y} < 0$ $\qquad$ $\partial T/\partial y << 0 \Rightarrow \frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) < 0$

1. Based on the equation above, if we increase the temperature gradient, then $\frac{D_g}{Dt}\left(\frac{\partial T}{\partial y}\right) < 0$ [Since $\partial T/\partial y < 0$ in this example] & therefore the wind shear must decrease $\left[\frac{D_g}{Dt}\left(\frac{\partial u_g}{\partial z}\right) < 0 \text{ from eq.}\right]$ This is opposite what thermal wind balance wants!
2. Therefore, in the absence of ageostrophic motion, geostrophic motion acts to force the atmosphere away from thermal wind balance.
3. Ageostrophic motions arise to help maintain thermal wind balance.

Physical Proof

- Imagine a situation where the temperature gradient is changed impulsively by geostrophic deformation.



1. Change in temperature gradient changes thickness & pressure gradients → isobars become more steeply sloped

2. PGF overwhelms coriolis → oppositely directed ageostrophic winds develop at upper & lower levels

3. Convergence/Divergence & vertical motion develop → this is the so called <u>Ageostrophic secondary circulation</u>

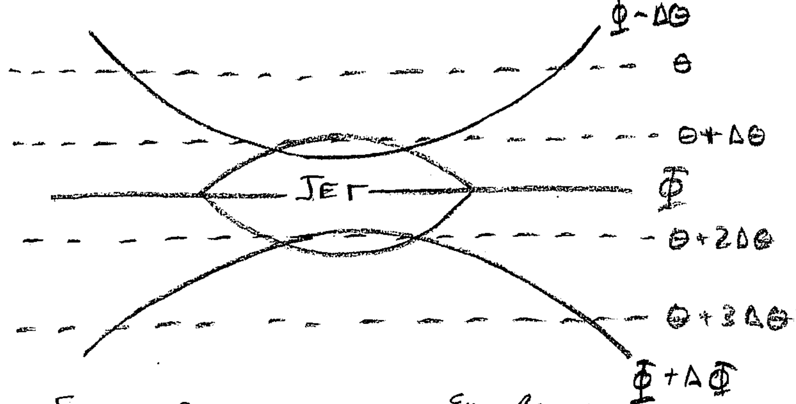
4. Secondary circulation acts to adjust the atmosphere back to thermal wind balance
   a. Vertical motion cools the warm air & warms the cold air, decreasing the temperature gradient
   b. Ageostrophic winds are affected by earths rotation & turn to right → winds aloft increase, winds at low levels decrese ⇒ shear increases,

- <u>Conclusion</u>: Ageostrophic secondary circulations arise to help maintain thermal wind balance since geostrophic deformation <u>Always</u> forces the atmosphere away from thermal wind balance.

The Jet Streak Model

- The classic example of ageostrophic secondary circulations is the 4-cell jet streak model.

- Assumes the jet streak is straight & that there is no along-jet temperature advection



Entrance Region
- geostrophic frontogenesis
- momentum advection destroys vertical shear

Exit Region
- geostrophic frontolysis
- momentum advection increases vertical shear.

∴ Something has to give!

- Ageostrophic secondary circulation can be inferred a number of ways → we will use Q.G.

A. Left entrance region → Cold advection & AVA ⇒ Subsidence
B. Right entrance region → Warm advection & CVA ⇒ Ascent
C. Left exit region → Warm advection & CVA ⇒ Ascent
D. Right exit region → Cold advection & AVA ⇒ Subsidence