Flow Kinematics

Kinematics - A branch of dynamics that describes the properties of pure fluid motion.

Why study it - Useful for understanding frontogenesis & other processes influenced by deformation.

- Consider the horizontal motion of a fluid element centered at any point in the fluid & moving with velocity U & V in the x & y direction

- Also, recall that the taylor expansion of a variable with two dimensions is given by

$$f(x,y) = f(0,0) + \left(\frac{\partial f}{\partial x}\right)_0 x + \left(\frac{\partial f}{\partial y}\right)_0 y + \cdots \text{ higher order terms}$$

- Then, the velocity can be approximated as

$$U = U_0 + \left(\frac{\partial U}{\partial x}\right)_0 x + \left(\frac{\partial U}{\partial y}\right)_0 y + \text{ higher order terms}$$

$$V = V_0 + \left(\frac{\partial V}{\partial x}\right)_0 x + \left(\frac{\partial V}{\partial y}\right)_0 y + \text{ higher order terms}$$

- Consider only the first order terms & expand by splitting up terms → i.e.,

$$U = U_0 + \frac{1}{2}\left(\frac{\partial U}{\partial x}\right)_0 x + \frac{1}{2}\left(\frac{\partial U}{\partial x}\right)_0 x + \frac{1}{2}\left(\frac{\partial U}{\partial y}\right)_0 y + \frac{1}{2}\left(\frac{\partial U}{\partial y}\right)_0 y$$

- Drop the 0 subscript & ADD $\frac{1}{2}\frac{\partial V}{\partial y}x - \frac{1}{2}\frac{\partial V}{\partial y}x$ and $\frac{1}{2}\frac{\partial V}{\partial x}y - \frac{1}{2}\frac{\partial V}{\partial x}y$ [like adding zero!]

$$U = U_0 + \frac{1}{2}\frac{\partial U}{\partial x}x + \frac{1}{2}\frac{\partial U}{\partial y}y + \frac{1}{2}\frac{\partial U}{\partial x}x + \frac{1}{2}\frac{\partial U}{\partial y}y + \frac{1}{2}\frac{\partial V}{\partial y}x - \frac{1}{2}\frac{\partial V}{\partial y}x + \frac{1}{2}\frac{\partial V}{\partial x}y - \frac{1}{2}\frac{\partial V}{\partial x}y$$

- Collecting terms

$$U = U_0 + \frac{1}{2}\underbrace{\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)}_{\delta}x + \frac{1}{2}\underbrace{\left(\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y}\right)}_{D_1}x + \frac{1}{2}\underbrace{\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)}_{D_2}y + \frac{1}{2}\underbrace{\left(\frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}\right)}_{-\zeta}y$$

- Similarly

$$V = V_0 + \frac{1}{2}\underbrace{\left(\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}\right)}_{D_2}x + \frac{1}{2}\underbrace{\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right)}_{\zeta}x + \frac{1}{2}\underbrace{\left(\frac{\partial V}{\partial y} + \frac{\partial U}{\partial x}\right)}_{\delta}y + \frac{1}{2}\underbrace{\left(\frac{\partial V}{\partial y} - \frac{\partial U}{\partial x}\right)}_{-D_1}y$$

⇒ The major kinematic properties of the horizontal wind field are given by:

$\delta = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}$ → Divergence

$D_1 = \frac{\partial U}{\partial x} - \frac{\partial V}{\partial y}$ → Stretching Deformation

$\zeta = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$ → Vorticity

$D_2 = \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}$ → Shearing Deformation

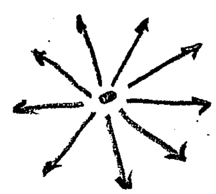
- The wind field at any time is comprised of these 4 components:

$$U = U_0 + \frac{1}{2}\left(\delta x + D_1 x + D_2 y - \zeta y\right)$$

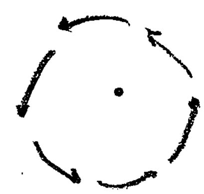
$$V = V_0 + \frac{1}{2}\left(D_2 x + \zeta x + \delta y - D_1 y\right)$$

- Examples of the four fields

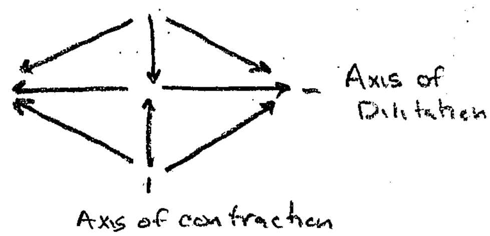
<u>Pure Divergence</u>



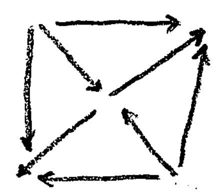
<u>Pure Vorticity</u>



<u>Stretching Deformation</u>



<u>Shearing Deformation</u>



- <u>Note</u>: Stretching & shearing deformation are the same – the only difference is the angle of the axis of dilitation relative to the x-axis. By rotating the coordinate system, one can always reduce either D_1 or D_2 to zero.

- <u>Example</u>: If D_2 is set to zero, the angle between the original x axis & the new one is given by:

$$\tan 2\psi = \frac{D_2}{D_1} = \frac{\partial v/\partial x + \partial u/\partial y}{\partial u/\partial x - \partial v/\partial y}$$

- The new set of axes (x', y') is called the <u>principle axes</u>

- The magnitude of the deformation in the rotated coordinate system is given by:

$$D_1' = \left(D_1^2 + D_2^2\right)^{1/2}$$

- The total deformation is called the <u>resultant deformation</u>

- Examples of Deformation from today's maps