Much like aerosol size distributions it turns out that rain and snow distributions can be closely parameterized by a simple exponential fit. The fit for rain drops is called the Marshall-Palmer distribution after the two scientists who first recognized this distribution based on a summer’s observations in Ottawa Canada published in 1948. This distribution of droplets is characterized by the following equation

$$\frac{dN(D)}{dD} = N_0 e^{-\Lambda D}$$

where $D$ is the droplet diameter and $N$ the concentration and $\Lambda$ is the slope of the rain distribution of a semilogarithmic plot. Marshall and Palmer showed that the slope can be related to the rainfall rate by

$$\Lambda (R) = 41R^{-0.21}$$

where $R$ is the rainfall rate and has units of $cm^{-1}$. Strangely $N_0$ appears to be nearly independent of $R$ and has an approximate value of $N_0 = 0.08 \, cm^{-4}$ (see Fig. 10.2 in Rogers and Yau)

The equivalent equations for snow turn out to be

$$\Lambda (R) = 25.5R^{-0.48}$$

$$N_0 = 3.8 \times 10^{-2} R^{-0.87}$$

(See Fig. 10.9 in Rogers and Yau)

**Relationship to Radar Return**

In radar imagery, the reflectivity factor scale is usually in units of dBz, which is equivalent to the quantity $10 \log Z$, where $Z$ is measured in units of $mm^6/m^3$ and it can be shown (not in this course) that

$$Z = \int_0^\infty \frac{dN(D)}{dD} D^6 dD$$

since $Z$ is proportional to the sixth moment of the size distribution it is primarily sensitive to precipitation sized particles. Substituting the Marshall-Palmer distribution for rain drops we find that

$$Z = N_0 \frac{6!}{\Lambda^7} = N_0 \frac{6!}{(41)^7} R^{1.47}$$

which agrees approximately with empirical relationships that show

$$Z = 200R^{1.6}$$

For snow the relationship is approximately

$$Z = 2000R^2$$