Homework 9: Due April 8

1. 4.41

*From Problem 1.1 we have*

\[ F(\omega) = n(\omega) c\hbar \omega \]

where \( F(\omega) \) is the spectral irradiance of the source and \( n(\omega) \) the spectral number density of photons. Integrate over the visible:

\[ \int F(\omega) \, d\omega = \int n(\omega) c\hbar \omega \, d\omega = F_s \]

where \( F_s \) is the total visible irradiance of the source. From the mean value theorem

\[ F_s = h c \langle \omega \rangle N \]

where \( N \) is the total number density and \( \langle \omega \rangle \) is one value of \( \omega \) within the visible. We can choose this frequency to be in the middle of the spectrum given that the frequency varies by less than a factor of 2. The total number of photons in a volume \( V_c \) is \( NV_c \), which we set equal to 1. This gives

\[ F_s = \frac{h c \langle \omega \rangle}{V_c} \]

For \( V_c = 10^{-3} m^3 \), \( F_s \approx 10^{-7} \) W m\(^{-2} \), which is very small.

2. 4.10

The irradiance of sunlight illuminating Earth’s atmosphere is \( L_s \Omega_s \) where \( L_s \) is the solar radiance and \( \Omega_s \) is the solid angle subtended by the sun. Light scattered by the atmosphere (at any point) is proportional to this irradiance. At night the source of illumination is the full moon, which can be looked upon as simply a less luminous sun. The irradiance of sunlight on the moon is also \( L_s \Omega_s \). Assume that the moon is a diffuse reflector with reflectivity \( R \), and hence the radiance of moonlight is \( RL_s \Omega_s / \pi \), where \( \Omega_m \) is the solid angle subtended
by the moon at Earth. This solid angle is to good approximation \( \Omega_s \) (which is why we have eclipses). Therefore the ratio of the moonlit sky scattered radiance to the sunlit sky scattered radiance is

\[
\frac{R \Omega_s}{\pi} \simeq 2 \times 10^{-6}
\]

Thus, for the same film exposure the shutter has to be open \( 1/(2 \times 10^{-6}) = 5 \times 10^5 \) times longer, which is 1/2 hr. So we expect the exposure time required to photograph the moonlit blue sky to be at least half an hour, using the same film and aperture as for daylight photography.

3. 4.47

The radiance from the moon is

\[
L_{moon} = \frac{R_{moon}}{\pi} L_s \Omega_s
\]

The radiance of moonlight reflected on snowpack is

\[
L_{moon-snow} = L_{moon} \Omega_{moon} \frac{R_{snow}}{\pi}
\]

Since \( \Omega_{moon} \simeq \Omega_s \)

\[
\frac{L_{moon-snow}}{L_{moon}} = \frac{R_{snow} \Omega_s}{\pi} \simeq \frac{\Omega_s}{\pi} \simeq 10^{-5}
\]

So our eyes span at least 5 decades of sensitivity at night.

4. 5.15

More or less continuous cloud cover can only reduce radiation, at all solar wavelengths, at ground level. This is evident when you fly over clouds. But clouds can transform part of the direct solar radiation into indirect (diffuse) solar radiation. There may also be a psychological factor in that people take fewer precautions on cloudy days.

5. 5.32. As a corollary derive an equation for the sensitivity of the reflectivity of a cloud layer
as a function of droplet/drop size. Sketch this function.

Take the cloud to be of thickness $h$ and cross-sectional area $A$. Its total volume is therefore $Ah$. Assume that the cloud droplets (or rain drops) all have the same volume, denoted as $v$. The number of droplets (or drops) per unit volume is $N$, the volume of water per unit volume is $Nv$ and hence the total volume of water is $NvhA = V$. The optical thickness is $\tau = NC_{sca}h$. Combine these two equations to obtain

$$\tau = \frac{V C_{sca}}{A v}$$

We assume that the quantity $V/A$, the volume of water per unit cross-sectional area, is constant. Thus the optical thickness is proportional to the scattering cross-section per unit particle volume. From Fig. 3.11 it follows that for visible wavelengths the scattering cross-section per unit volume (of a sphere) is inversely proportional to its diameter (i.e. the scattering cross-section is proportional to diameter squared). Thus

$$\frac{\tau_{\text{rain}}}{\tau_{\text{cloud}}} = \frac{d_{\text{cloud}}}{d_{\text{rain}}}$$

If we take $d_{\text{cloud}} = 10 \mu m$ and $d_{\text{rain}} = 1000 \mu m$, the ratio of optical thicknesses is 1/100. No wonder we can see through rain shafts but not clouds!