Journal of the Meteorological Society of Japan, Vol. 82, No. 5, pp. 1389–1397, 2004

Mesoscale Assimilation of TMI Rainfall Data with 4DVAR: Sensitivity Studies

Zhaoxia PU

Goddard Earth Sciences and Technology Center, University of Maryland, Baltimore County, NASA Goddard Space Flight Center, Greenbelt, Maryland, USA

and

Wei-Kuo TAO

Laboratory for Atmosphere, NASA Goddard Space Flight Center, Greenbelt, Maryland, USA

(Manuscript received 20 October 2003, in final form 5 July 2004)

Abstract

Sensitivity studies are performed on the assimilation of TRMM (Tropical Rainfall Measuring Mission) Microwave Imager (TMI) derived rainfall data into a mesoscale model, using a four-dimensional variational data assimilation (4DVAR) technique. A series of numerical experiments is conducted to evaluate the impact of TMI rainfall data on the numerical simulation of Hurricane Bonnie (1998). The results indicate that rainfall data assimilation is sensitive to the error characteristics of the data, and the inclusion of physics in the adjoint model. In addition, assimilating the rainfall data alone is helpful for producing a more realistic eye and rain bands in the hurricane, but does not ensure improvements in hurricane intensity forecasts. Further study indicated that it is necessary to incorporate TMI rainfall data together with other types of data, such as wind data into the model, in which case the inclusion of the rainfall data further improves the intensity forecast of the hurricane.

1. Introduction

The Tropical Rainfall Measuring Mission (TRMM) is a joint Japan-U.S. project to measure rainfall over the global tropics. With the world's first precipitation radar aboard a satellite, the TRMM satellite has provided the first detailed and comprehensive dataset on the four-dimensional distribution of rainfall and latent heating over the tropics (between 35°N and 35°S). TRMM offers a unique opportunity to improve the understanding of tropical meteorology, and to evaluate the impact of rainfall data on tropical weather forecasts. Early studies have demonstrated that assimilation of TRMM microwave imager (TMI) derived rainfall data into large-scale global models is beneficial for the analysis of the atmospheric general circulation (Hou et al. 2000), and also consequently can have significant impact on mesoscale forecasts (e.g., Supertyphoon Paka in 1997 (Pu et al. 2002)), as the global analysis provide initial guess filed and boundary conditions for the mesoscale model. Instead of direct assimilation of rainfall data into the global model, as in Hou et al. (2000) and Pu et al. (2002), this paper evaluates the impact of TMI rainfall on mesoscale forecasts via the *direct assimilation* of TMI-derived rainfall rates *into the mesoscale regional model* itself, using a four-dimensional variational data assimilation (4DVAR) technique.

Corresponding author and current affiliation: Zhaoxia Pu, Department of Meteorology, University of Utah, 135 S 1460 E, Rm. 819, Salt Lake City, UT 84112, USA.
E-mail: pu@met.utah.edu

The description of the model and the 4DVAR system, and that of the hurricane case and the TMI data are addressed in section 2 and 3, respectively. Detailed numerical results, including sensitivity studies and forecast impacts, are given in section 4. A summary and discussion are given in section 5.

2. Description of the mesoscale model and 4DVAR system

The Penn State University/National Center for Atmospheric Research (PSU/NCAR) mesoscale forecast model (MM5) and its adjoint system are used in this study. The MM5 is a limited-area, non-hydrostatic primitive equation model with multiple options for various physical parameterization schemes (Dudhia 1993; Grell et al. 1995). The model employs a terrain-following σ vertical coordinate, where σ is defined as $\sigma = (p - p_{\mathrm{top}})/(p_{\mathrm{sfc}} - p_{\mathrm{top}})$, where p is pressure, and p_{sfc} and p_{top} are the pressures at the surface and model top, respectively. Physics options used for the forecast model in this study include the Grell cumulus parameterization, a simple ice microphysics scheme (Dudhia 1993), the Blackadar high-resolution planetary boundary layer parameterization scheme (Blackadar 1976, 1979; Zhang and Anthes 1982), and a cloud atmospheric radiation scheme (Dudhia 1993). The land surface temperature is predicted using surface energy budget equations as described in Grell et al. (1995). For a more detailed description of MM5, see Dudhia (1993) and Grell et al. (1995).

The MM5 adjoint modeling system (Zou et al. 1998) is employed in the data assimilation experiments. For the variational data assimilation system, the physics options in the adjoint model are the Grell cumulus parameterization, a simple ice microphysics scheme (Dudhia 1989), and the Blackadar high-resolution planetary boundary layer parameterization scheme. Application of the MM5 adjoint model to a variety of mesoscale weather systems has been demonstrated in papers by Kuo et al. (1996) and Zou and Kuo (1996).

In general, a 4DVAR system tries to minimize the following cost function:

$$J(x_0) = \sum_{k=1,m} J_k + J_b, \tag{1}$$

where x is the analysis variable and the subscript "0" denotes the initial state. J_b is the background term, and J_k is the contribution to the cost function from an individual type of data. The subscript k denotes the type of data and m is the total number of available data types. For example, the contributions from one arbitrary type of observation can be described as follows:

$$J_k(x_0) = \sum_{i=0,\Delta} (H_i(M_i(x_0)) - O_i)^T \times W(H_i(M_i(x_0)) - O_i), \tag{2}$$

where O is the observation data, "i" denotes the "ith" time step for the non-linear forecast model M, at which the observations are available, and $i \in (0, \Delta)$, while Δ is the number of time steps in the assimilation window. W is a weighting factor that depends on the statistical error characteristics of the observational data. H is a so-called observation operator (possibly no-linear), which transfers the grid-space model variable x to the observational type. In order to minimize the cost function, the adjoint of the tangent linear model of the nonlinear forecast model is required (Talagrand 1987).

In this study, J_b is a simple background term measuring the distance between the model state and the MM5 analysis based on the large-scale ECMWF analysis (first guess field). Only approximated variances are included in the background weighting matrix.

The effectiveness of the 4DVAR technique for assimilation of precipitation observations has been addressed by Zou and Kuo (1996) and Zupanski and Messinger (1995). However, this paper will further investigate strategies on the assimilation of TMI data particularly.

3. Hurricane case and TMI rainfall data

The TRMM Microwave Imager (TMI) is one of several TRMM satellite sensors. The TMI measures the horizontal distribution of rainfall by receiving microwaves emitted or scattered by raindrops and ice particles in nine microwave channels. At NASA Goddard Space Flight Center, the TMI microwave radiances are used to retrieve surface rainfall rate information, via the Goddard Profiling (GPROF) algorithm. The basis for the rainfall retrieval algorithm is the Bayesian technique, described in Kummerow et al. (1996) and Olson et al. (1996, 1999). In order to evaluate the impact of TMI rainfall data on mesoscale forecasts, these retrieved surface rainfall data are assimilated into the MM5 model.

The TMI footprints usually cover the global tropics (35°N–35°S) in a 24-hour period. However, during a typical mesoscale analysis period (usually a 3 h or 6 h period) and for a specific regional domain, there are only limited TMI observations available. In most cases, the TMI samples only about twice a day for a certain region, and the time interval between the swathes may exceed 6 h. Considering the data availability during a 6 h analysis cycle, Hurricane Bonnie (1998) was selected from several storm cases to perform the sensitivity studies in this paper.

There were two TRMM swathes that passed over Bonnie in the Atlantic Ocean, with a time interval of about 6 h. The two overpasses were around 1139 UTC 22 August 1998, and 1807 UTC 22 August 1998 (Fig. 1), respectively. At the time, Bonnie was a category 1 hurricane based on the Saffir-Simpson intensity scale, having recently developed from a tropical storm. 1200 UTC 22 August 1998 was selected as the initial time for the experiments. A 6 h data assimilation window was set for the period 1200 UTC–1800 UTC 22 August 1998.

Based on TMI-derived surface rainfall data, being typically defined as an hourly "rain rate" and the actual data availability, the TMI-derived rainfall was treated as "hourly rainfall". Therefore, Eq. (2) can be written as follows:

$$J_1(x_0) = \sum_{i=0,\Delta} \left(\sum_{j=0}^{Tj} CR_{ij} - RR_i \right)^T \times W_{RR} \left(\sum_{j=0}^{Tj} CR_{ij} - RR_i \right), \tag{3}$$

where T_j is the number of time steps in a one hour time period, RR the retrieved TMI rain rate, and CR the model-generated rainfall in one time step, W_{RR} is a weighting factor that depends on the statistical error characteristics of the rain rate data.

4. Numerical experiments and results

The data assimilation experiments were conducted at 36 km horizontal grid resolution. The model domain is shown in Fig. 1. The model vertical structure comprises 27 σ levels with the top of the model set at a pressure of 50 hPa. The σ levels are placed at values of 1.0, 0.99, 0.98, 0.96, 0.93, and 0.89, and then decrease to 0.01 at an interval of 0.04. For the experiments, the first guess of initial field and boundary conditions are derived from ECMWF global analyses.

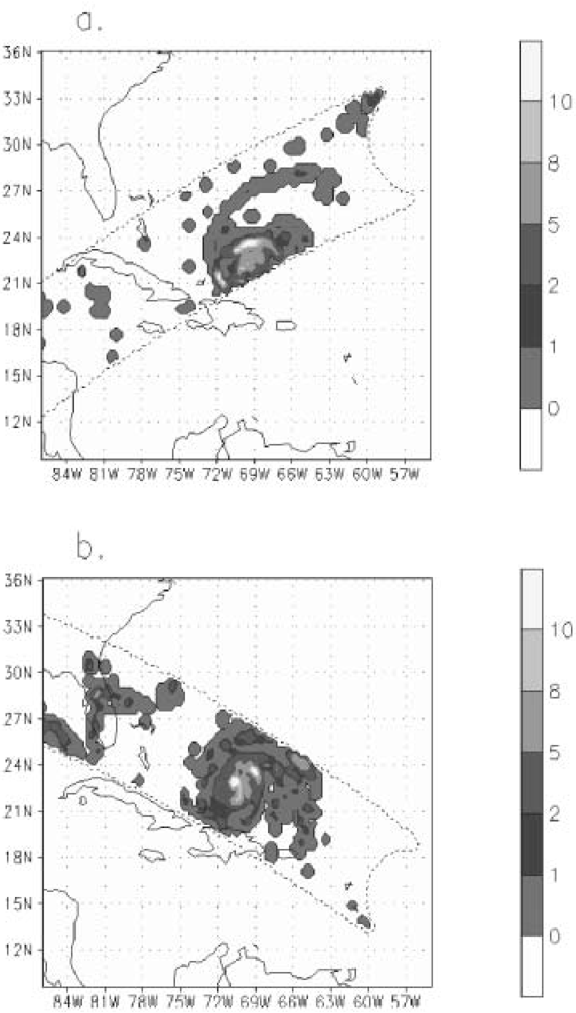

Fig. 1. Rain rates (mm/hr) for two TMI swathes that passed over the Hurricane Bonnie (1998) around a) 1139 UTC 22 August 1998 and b) 1807 UTC 22 August 1998.

4.1 Sensitivity studies

Numerical experiments were conducted to test strategies for assimilating TMI-retrieved rainfall rates. Two groups of sensitivity studies

Table 1. Experimental Design

Experiment number	Definition of W term for rainfall	Physics included in adjoint	Bogus vortex	Rainfall assimilation
CTRL	No	Grell cumulus scheme Dudhia microphysics	No	No
1	Unified	Grell cumulus scheme Dudhia microphysics	No	Yes
2	20% error	Grell cumulus scheme Dudhia microphysics	No	Yes
3	Bauer	Grell cumulus scheme Dudhia microphysics	No	Yes
4	Bauer	Grell cumulus scheme	No	Yes
5		(Same as Experiment 3)		
6	Bauer	None	No	Yes
7	No	Grell cumulus scheme Dudhia microphysics	Yes	No
8	Bauer	Grell cumulus scheme Dudhia microphysics	Yes	Yes

were performed to test the sensitivity of specification of the error characteristics of the data and the inclusion of physics in the adjoint model to the TMI rainfall data assimilation. Table 1 lists the experimental configuration for all numerical experiments in this paper.

a. Sensitivity of the specified rainfall data error characteristics

The W factor in Eq. 2 (e.g., W_{RR} in Eq. 3) is a weighting factor that depends on the statistical error characteristics of the observations. To some extent, this factor represents how much the 4DVAR system would "trust" the observations. Because the correlations between the observations are usually unknown or difficult to define, the W matrix is often defined as a diagonal matrix. In the previous studies, W term usually set as either a constant number (e.g., Zou and Kuo 1996) based on the variances of the available observations, or the inversion of the error variances based on an assumed percentage of data error (e.g., Hou et al. 2000). Due to the large data sample, the error characteristics of TMI retrieved rain rate were specified by Bauer et al. (2002). In order to examine the impact of W on rainfall assimilation, three experiments were conducted with the specification of the error characteristics as follows: in Experiment 1, W was set up as a unified number and defined as the inversion of variances based on all available data (i.e., unified W); in Experiment 2, a 20% error was assumed for the retrieved rainfall rate, and W was defined as the inversion of the error variances (i.e., 20% error); and in Experiment 3, the error characteristics were specified following Bauer et al. (2002) and Olson (personal communication) as:

$$\text{over ocean} \quad \sigma_s = 1.357RR^{0.7},$$

$$\text{over land} \quad \sigma_s = 2.516RR^{0.558},$$

where RR is the retrieved rain rate and σ_s the standard deviation of the rain rate. The σ_s was obtained from a large sample of retrieved rainfall rate datasets.

Figure 2 shows the variation of the cost-function with the number of iterations. The definition of W mainly impacts the convergence of the minimization in terms of both cost-function reduction and speed of convergence. As a consequence, W effects how much information can be gained from the observations. The results show that it is obviously advantageous to use the error specification suggested by Bauer et al. (2002) and Olson (personal comm.) as it lead to a better convergence performance.

b. Inclusion of physics in the adjoint

For a common forecast model (forward model), the cumulus parameterization and microphysical processes usually help the model to

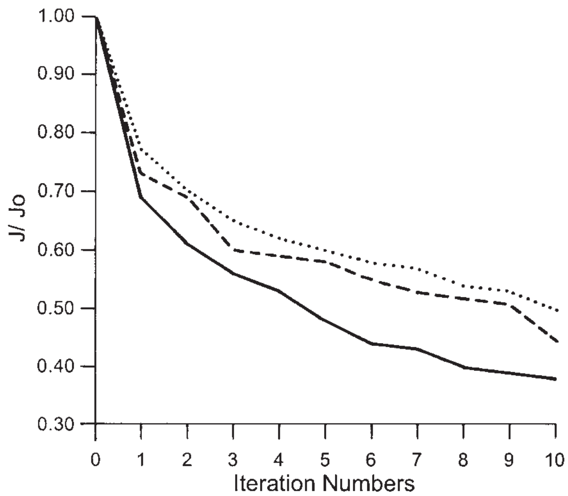

Fig. 2. Variation of normalized cost-function with iteration number for Experiment 1 (dotted line), Experiment 2 (dashed line) and Experiment 3 (solid line).

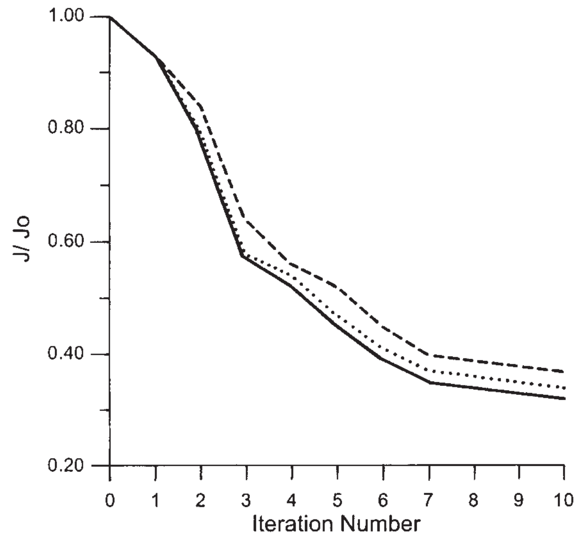

Fig. 3. Same as Fig. 2, but for Experiment 4 (dotted line), Experiment 5 (solid line) and Experiment 6 (dashed line).

produce a better rainfall forecast. However, due to the difficulties in deriving an adjoint model for the physics package, in some previous studies (e.g., Zou and Kuo 1996), not all of the physics processes were included in the adjoint model. In order to test the impact of including physics in adjoint models on data assimilation results, the following three experiments were conducted: in Experiment 4, the adjoint model include cumulus parameterization but not microphysics; in Experiment 5 (same as Experiment 3), both cumulus parameterization and microphysical processes are included in the adjoint model; and in Experiment 6, neither cumulus parameterization nor microphysical process are included in the adjoint model. For all Experiment 4∼6, the specified error characteristics followed those in Experiment 3.

Figure 3 shows the variation of the cost-function with the number of iterations. As expected, including all of the physics packages has the largest benefit in terms of both convergence and assimilation results.

4.2 Impact on forecasts

Figure 4 shows rainfall rates at the end of the data assimilation (6-h forecast) from Experiment 5, compared with the control experiment (CTRL) where rainfall data were not assimilated. Obviously, assimilating the rainfall data helps the model to produce a more realistic eye and rain bands in the hurricane. The results are quite encouraging. Twenty-four hour forecasts were conducted to test the impact of the rainfall data on the hurricane intensity forecast. Unfortunately, there was no improvement in the consequent forecasts. The forecasted track and intensity (in terms of maximum surface wind and minimum sea level pressure) were almost the same in cases both with and without the TMI rainfall data assimilation (figure not shown), indicating the impact of rainfall data on consequent forecasts is almost negligible. This is not consistent with previous results (Pu et al. 2002), in which the rainfall assimilation improved hurricane forecasts.

In order to explore additional strategies for rainfall data assimilation, an additional experiment was performed to assimilate rainfall data along with other data sets. As other conventional data is unavailable for this case, bogus vortex wind information is introduced into the assimilation process. Following Pu and Braun (2001), the bogus wind data was derived from the gradient balance equations as follows:

$$V_g^{\text{bogus}}(r) = [AB(p_n - p_c)\exp(-A/r^B)/\rho r^B]^{1/2}, \tag{4}$$

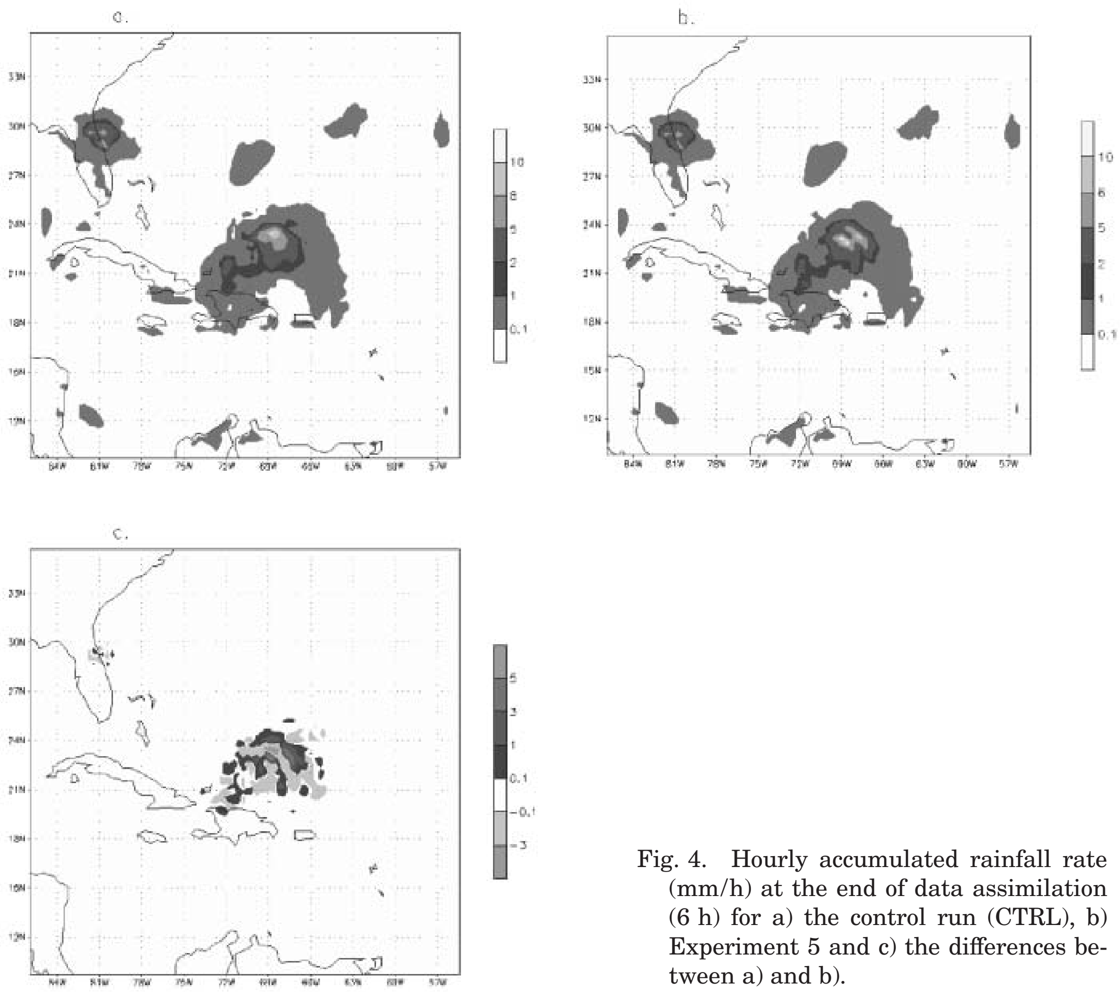

Fig. 4. Hourly accumulated rainfall rate (mm/h) at the end of data assimilation (6 h) for a) the control run (CTRL), b) Experiment 5 and c) the differences between a) and b).

where V_g^{bogus} is the gradient surface wind at radius r, ρ the air density (assumed constant at 1.15 g m^{-3}), p_c the central pressure and p_n the ambient pressure (theoretically at infinite radius, however, here taken from representative values in the hurricane environment). The scaling parameters A and B are defined by maximum wind information. By setting $dV_g^{\mathrm{bogus}}/dr = 0$, the radius of maximum surface wind (RMW) is $R_m = A^{1/B}$, and substitution back into (4) gives the maximum wind speed, $V_m = C(p_n - p_c)^{1/2}$, where $C = (B/\rho e)^{1/2}$ and e is the base of the natural logarithm.

Based upon the best available estimates (according to the report from the Hurricane Research Division/AOML/NOAA), the parameters defining the bogus vortex are given by $p_c =$ 980 hPa centered at (22.3°N, 69.8°W), $p_n =$ 1012 hPa, $V_m = 38.6$ m s^{-1}, and $R_m = 120$ km. The bogus wind information is defined on the grid points and extends out to a radius of 350 km. The surface wind is then extended into the vertical with a vertical profile following Pu and Braun (2001). For the experiments, the specified wind information is assimilated every 10 minutes within the first 30 minutes. During the data assimilation, constant weightings are giving to bogus data based on the inversion of the variances of the data.

Two sets of numerical experiments were performed in a 6-h assimilation window. In the first experiment (Experiment 7), only bogus wind information was assimilated into the model. In the second experiment (Experiment 8), the rainfall data are incorporated along with the bogus wind information.

Figures 5 and 6 show the rainfall rates at the end of the data assimilation (e.g., 6-h fore-

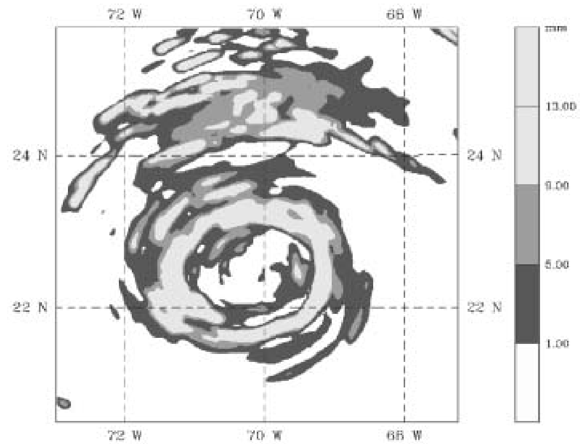

Fig. 5. Hourly accumulated rainfall rate (mm/h) at the end of data assimilation for assimilation of bogus wind only (Experiment 7).

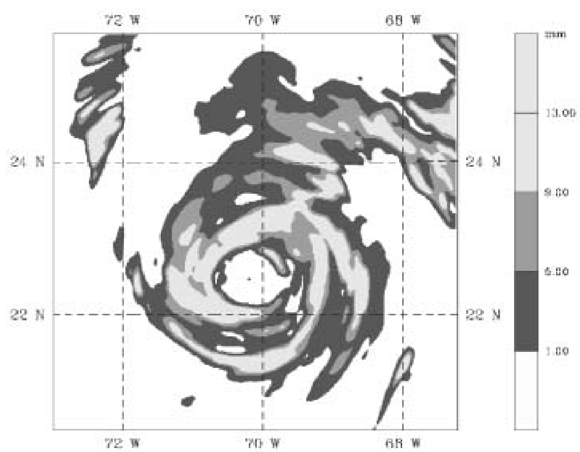

Fig. 6. Same as Fig. 5, but both bogus wind and rainfall data assimilated (Experiment 8).

casts) for Experiment 7 and 8, respectively. Compared with the TMI observations (Fig. 1b), the rainfall patterns in Fig. 8 are closer to the observed rainfall structure, indicating that assimilation of rainfall data further improves the asymmetric hurricane rainfall structure.

Further comparison is illustrated by histograms of the probability density function (PDF) of 1-h rainfall amounts at the end of the data assimilation (Fig. 7). The figure shows that rainfall is generally overestimated in the lower-

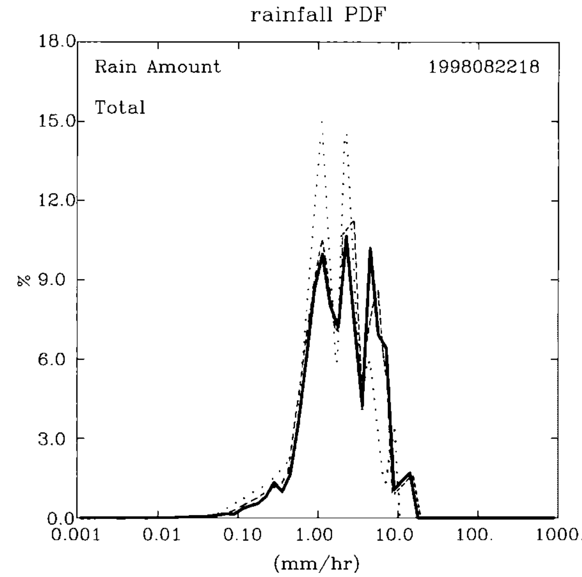

Fig. 7. Histograms of probability density functions of 1-h rainfall amounts at the end of data assimilation for Experiment 7 (dot line), Experiment 8 (dash line) and for TMI observations (solid line).

band of rainfall rate and underestimated in the higher-band of rainfall rate in the case without rainfall data assimilation. However, when the rainfall data are assimilated into the model, the spectrum of rainfall rates becomes more close to this feature from the observations. Both light and heavy rainfall rates (~10 mm/hr) are better represented compared to the case without rainfall assimilation.

Figure 8 shows the time variation of the forecast hurricane intensity in terms of the maximum surface wind and minimum sea-level pressure for the subsequent 24 h forecasts. It indicates that rainfall data assimilation is not only helpful for producing better vortex rainbands, but also improves subsequent forecasts. The differences in the forecasted track for two cases are negligible, but slight improvement has been seen in the case with rainfall assimilation (figure not shown). The positive impact shown in this group of experiments indicates that it may be necessary to incorporate TRMM rainfall data together with other types of data, such as wind data, in order to further improve the intensity forecasts for hurricanes.

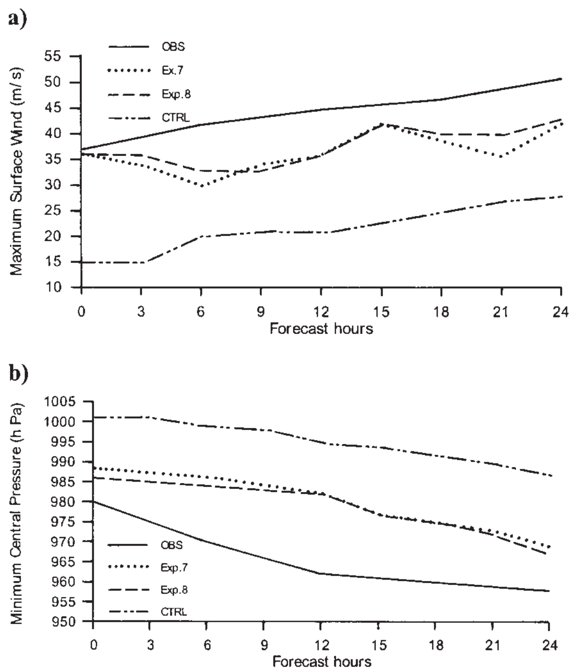

Fig. 8. Time series (3-h intervals) of a) maximum wind (m/s) at the lowest model level (about 50 m) and b) minimum sea-level pressure (hPa).

5. Summary and discussion

The main conclusions from the numerical experiments with TMI rainfall assimilation in this study are:

- Rainfall data assimilation is sensitive to the error characteristics of the data and the inclusion of physics in the forward and adjoint models, suggesting that it is necessary to use the full physics model in rainfall data assimilation and to take into account the error characteristic of the data;
- Assimilating the rainfall data alone produces a more realistic eye and rain bands in the hurricane, but does not ensure improvements in hurricane intensity forecasts. Numerical results indicate that it is necessary to incorporate TRMM rainfall data, together with other types of data such as wind data into the model, in which case the inclusion of the rainfall data will further improve the intensity forecast of the hurricane.

In addition to the TMI data, a similar experiment was performed assimilating surface rainfall data derived (from the same algorithm, i.e. GPROF) from TRMM precipitation radar (PR) for Hurricane Bonnie for the same assimilation window. Fortunately, the PR swathes overlap the TMI swathes in both time and space, except that the PR swathes are much narrower than the TMI swathes (i.e., they cover one third of the TMI swathes, figure not shown). A data assimilation experiment, similar to Experiment 3, was performed with the PR data. The results are very similar to those from the TMI data assimilation, suggesting that the PR can also provide useful data sets for rainfall assimilation.

Future studies will be conducted to further confirm the above conclusions, and to explore the possibility of incorporating TMI and PR rainfall with other conventional and satellite data to improve mesoscale precipitation and storm forecasting. On the other hand, the coverage of both TMI and PR data is very limited for regional applications. Therefore, the use of merged multi-satellite data (e.g., Huffman et al. 2001; Huffman and Bolvin 2003) will be another alternative option for future study. However, since the Global Precipitation Mission (GPM) is under preparation, a large benefit to NWP could be obtained from that global precipitation data.

Acknowledgements

This work is supported by the NASA Headquarters Atmospheric Dynamics and Thermodynamics Program, and by the NASA TRMM/GPM project. The authors are grateful to Dr. R. Kakar (NASA/HQ) for his support of this research. We thank Dr. William Olson for his help in getting TMI- and PR-retrieved rainfall data and also to Mr. Stephen Lang for proofreading the English in the paper. Acknowledgement is also made to NASA Goddard Space Flight Center for computer time used for this research. Review comments from two anonymous reviewers are also highly appreciated.

References

Bauer, P., J.-F. Mahfouf, W.S. Olson, F.S. Marzano, S.D. Michele, A. Tassa, and A. Mugnai, 2002: Error analysis of TMI rainfall estimates over ocean for variational data assimilation. *Quart. J. Roy. Meteor. Soc.*, **128**, 2129–2144.

Blackadar, A.K., 1976: Modelng the nocturnal boundary layer. *Preprints, Third Symp. on Atmospheric Turbulence, Diffusion, and Air Quality*, Raleigh, Amer. Meteor. Soc., 46–49.

———, 1979: High resolution models of the planetary boundary layer. *Advances in Environmental Science and Engineering*, Vol. 1, No. 1, J. Pfafflin and E. Ziegler, Eds., Gordon and Breach, 50–85.

Dudhia, J., 1993: A nonhydrostatic version of the Penn State-NCAR mesoscale model: Validation tests and simulation of an Atlantic cyclone and cold front. *Mon. Wea. Rev.*, **121**, 1493–1513.

Grell, G.A., J. Dudhia, and D.R. Stauffer, 1995: A description of the fifth-generation Penn State/NCAR mesoscale model (MM5). NCAR Technical Note, NCAR/TN-398 + STR, 138 pp. [Available from NCAR Publications Office, P. O. Box 3000, Boulder, CO 80307-3000].

Huffman, G.J., R.F. Adler, M.M. Morrissey, D.T. Bolvin, S. Curtis, R. Joyce, B. McGavock, J. Susskind, 2001: Global precipitation at one-degree daily resolution from multisatellite observations. *Journal of Hydrometeorology*, **2**, 36–50.

——— and D.T. Bolvin, 2003: TRMM real-time multi-satellite precipitation analysis data set documentation. (Available at Code 912, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA)

Hou, A.Y., S.Q. Zhang, A.M. Da Silva, and W.S. Olson, 2000: Improving assimilated global datasets using TMI rainfall and columnar moisture observations. *J. Climate*, **13**, 4180–4195.

Kuo, Y.-H., X. Zou, and Y.R. Guo, 1996: Variational assimilation of precipitable water using a nonhydrostatic mesoscale adjoint model. Part I: Moisture retrieval and sensitivity experiments. *Mon. Wea. Rev.*, **124**, 122–147.

Kummerow, C., W.S. Olson, and L. Giglio, 1996: A simplified scheme for obtaining precipitation and vertical hydrometeor profile from passive microwave sensor. *IEEE Trans. Geosci. Remote Sens.*, **34**, 1213–1232.

Olson, W.S., C.D. Kummerow, G.M. Heymsfield, and L. Giglio, 1996: A method for combined passive-active microwave retrievals of cloud and precipitation profiles. *J. Appl. Meteor.*, **35**, 1763–1789.

———, W.S., C.D. Kummerow, Y. Hong, and W.-K. Tao, 1999: Atmospheric latent heating distributions in the Tropics derived from passive microwave radiometer measurements. *J. Appl. Meteor.*, **38**, 633–664.

Pu, Z.-X. and S.A. Braun, 2001: Evaluation of bogus vortex techniques with four-dimensional variational data assimilation. *Mon. Wea. Rev.*, **129**, 2023–2039.

———, W.-K. Tao, S. Braun, J. Simpson, Y. Jia, J. Halverson, W. Olson, and A. Hou, 2002: The impact of TRMM data on mesoscale numerical simulation of Supertyphoon Paka. *Mon. Wea. Rev.*, **130**, 2248–2258.

Talagrand, O., 1997: Assimilation of pbservations, and introduction. *J. Meteor. Soc. Japan*, **75**, 191–209.

Zhang, D.-L. and R.A. Anthes, 1982: A high-resolution model of the planetary boundary layer—Sensitivity tests and comparisons with SESAME-79 data. *J. Appl. Meteor.*, **21**, 1594–1609.

Zou, X., W. Huang, and Q. Xiao, 1998: A user's guide to the MM5 adjoint modeling system. NCAR TN-437+IA. MMM division, NCAR. [Available from NCAR Publications Office, P. O. Box 3000, Boulder, CO 80307-3000].

——— and Y.-H. Kuo, 1996: Rainfall assimilation through an optimal control of initial and boundary conditions in a limited-area mesoscale model. *Mon Wea. Rev.*, **124**, 2859–2882.

Zupanski, D. and F. Mesinger, 1995: Four-dimensional variational assimilation of precipitation data. *Mon. Wea. Rev.*, **123**, 1112–1127.